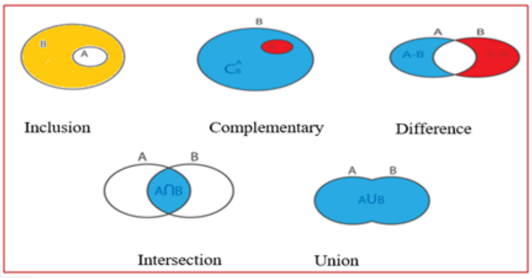
Chapter II : Sets, Relation, and Applications



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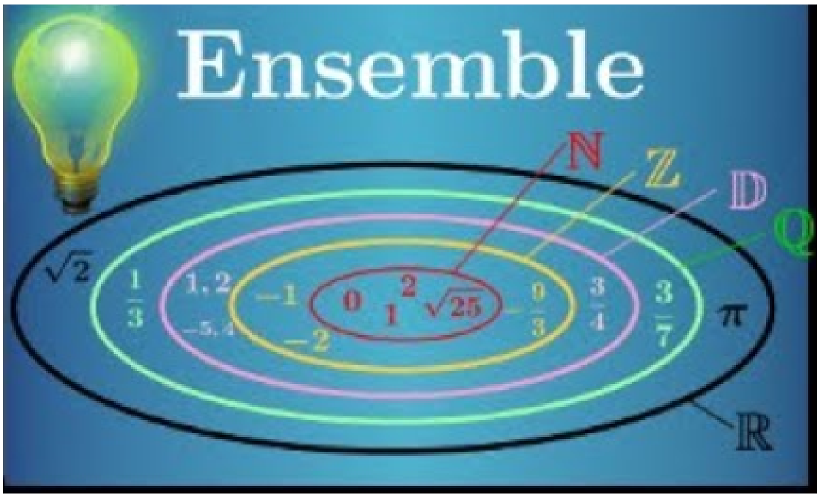
I Chapter II : Sets, Relation, and Applications

1. Set Theory



Definition: A set[*] is a collection of mathematical objects brought together according to one or more common properties .

Example

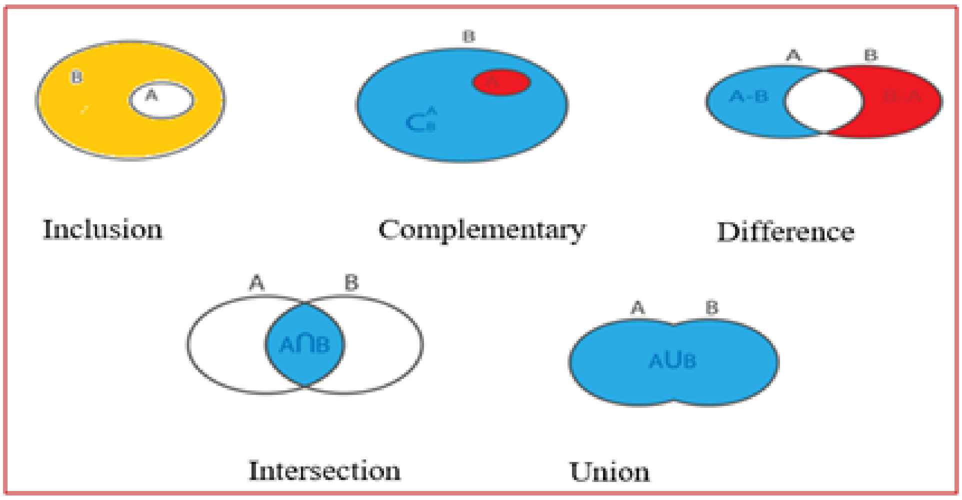
*We designate by IN the set of natural numbers $N = 0, 1, 2, 3, ...n$.

*The empty set is denoted: ∅ which does not contain any element.

1.1. Operations on sets

The following section presents the most important set operations, as well as their symbolic notation.

- ***Inclusion:*** $E \subset F$ if every element of E is also an element of F. In other words: $\forall x \in E(x \in F)$ We say then that E is a subset of F or a part of F.
- ***Union:*** for $A \cup B$=E{$x \in E$, $x \in A$ or $x \in B$}
- ***Intersection:*** $A \cap B$={x\in E , $x \in A$ and $x \in B$}
- ***Complementary:*** Complement of A or A^c, the set of all elements that are not in A.



✎ *Note*

- ***Equality of two sets:*** E = F if and only if $E \subset F$ and $F \subset E$
- ***Difference of two sets:*** The difference of two sets A, B is the set of elements of A which are not in B, denoted A – B
- ***Symmetrical difference:*** Let E be a non-empty set and A, $B \subset E$,the symmetric difference between two sets A, B is the set of elements which belong to A – B or B – A note $A \triangle B$.
  $A \triangle B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B$

1.2. Cardinality

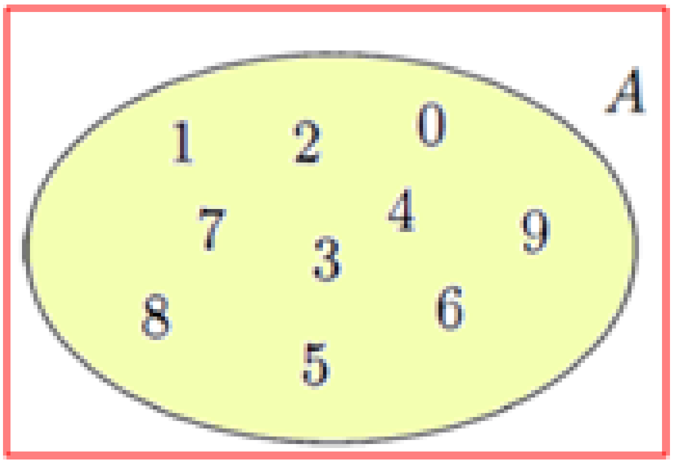
Definition: when the set E is finite, the number of elements it contains is called its cardinal. we note it card(E) .

⚠ *Warning*

when E is infinite, we set Card E = $+\infty$

1.3. Venn diagram

we can represent a set graphically using a Venn diagram.consider for example the set A defined in extension by $A = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,$ the Venn diagram of set A is presented by the following figure:



2. Equivalence Relation and Order Relation

2.1. Equivalence Relation

Definition: Let R be an equivalence relation* in the set E and $x, y, z \in E$ if only if it is reflexive, symmetric, and transitive (4)*.

- ***Reflexive:*** $(\forall x \in E), (xRx)$

- ***Symmetric:*** $(\forall x \in E), (\forall y \in E), (xRy \Rightarrow yRx)$

- ***Transitive:*** $(\forall x, y, z \in E), ((xRy) \wedge (yRz)) \Rightarrow (xRz))$

2.2. Order Relation

Definition: A relation is called an order relation if it is reflexive, anti symmetric, and transitive.

- ***Reflexive:*** $(\forall x \in E), (xRx)$

- ***Anti-symmetric :*** $(\forall x \in E), (\forall y \in E), (xRy) \wedge (yRz) \Rightarrow (x = y).$

- ***Transitive:*** $(\forall x, y, z \in E), ((xRy) \wedge (yRz)) \Rightarrow (xRz))$

2.3. Equivalence class

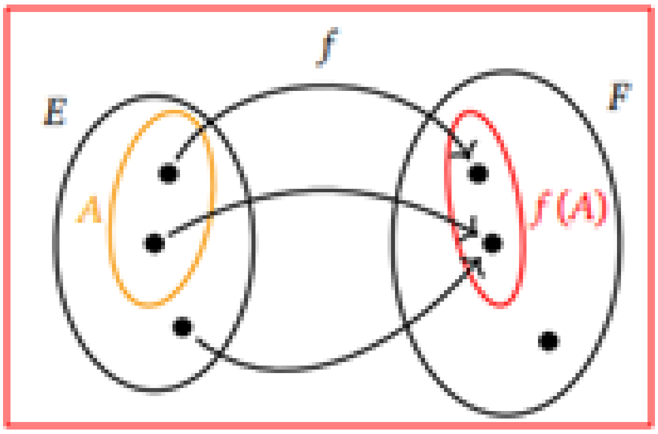
Let R be an equivalence relation; we call the equivalence class of an element $x \in E$ the set of elements $y \in E$ which are in relation R with x we denote C_x, where: $C_x = y \in E / xRy$

3. Application

3.1. Direct and Reciprocal image

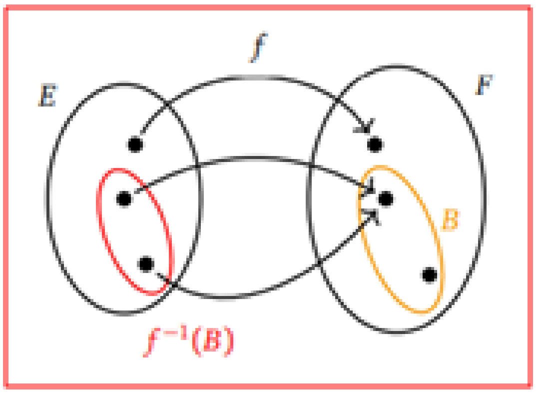
3.1.1. The direct image

Let f $E \longrightarrow F$ and $A \subset F$ we call image of A by f a subset of F, denoted $f(A)$ such that: $f(A) = f(x) \in F / x \in A$



3.1.2. The reciprocal image

Let f $E \longrightarrow F$ and $B \subset F$ we call the reciprocal image* of B by f, the part of E denoted f^{-1} (5)*.



4. Exercice : Test

[solution n°1 p.8]

Let A, B be two parts of a set E. We note C_A^E as the complement of A in E. What is the correct answer?

- $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (False)
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (True)
- $\overline{A \cup B} = A \cap B$ (False)

5. Exercice : Test

[solution n°2 p.8*]*

*Let E and F be two non-empty sets and f be a map from E to F. Let A, B be two subsets of E. What are the correct answers?

- ☐ $A \subset B \Rightarrow f^{-1}(A) \subset f^{-1}(B)$ (True)
- ☐ $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$ (True)
- ☐ $f^{-1}(A \cap B) \subset f^{-1}(A) \cap f^{-1}(B)$ (False)
- ☐ $f^{-1}(A \cup B) = f^{-1}(A) \cap f^{-1}(B)$ (False)

Exercises solution

> **Solution** n°1

Exercice p. 6

Let A, B be two parts of a set E. We note C_A^E as the complement of A in E. What is the correct answer?

- ○ $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (False)
- ◉ $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (True)
- ○ $\overline{A \cup B} = A \cap B$ (False)

> **Solution** n°2

Exercice p. 7

*Let E and F be two non-empty sets and f be a map from E to F. Let A, B be two subsets of E. What are the correct answers?

- [x] $A \subset B \Rightarrow f^{-1}(A) \subset f^{-1}(B)$ (True)
- [x] $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$ (True)
- [ ] $f^{-1}(A \cap B) \subset f^{-1}(A) \cap f^{-1}(B)$ (False)
- [ ] $f^{-1}(A \cup B) = f^{-1}(A) \cap f^{-1}(B)$ (False)

Glossary

Equivalence relation

We say that a relation is an equivalence relation if it isreflexive, symmetric, and transitive.)

Set

is well-determined collections that are completely characterized by their elements.

the reciprocal image

the reciprocal image is the map whose definition set is the set of parts of F and whose arrival set is the set of parts of E.

References

4 Alain Louveau, Christian Rosendal (2001)

5 J. Franchini et J. C. Jacquens (1996)