Beam Analysis
''Analysis of Statically Determinate Beams"

Table des matières

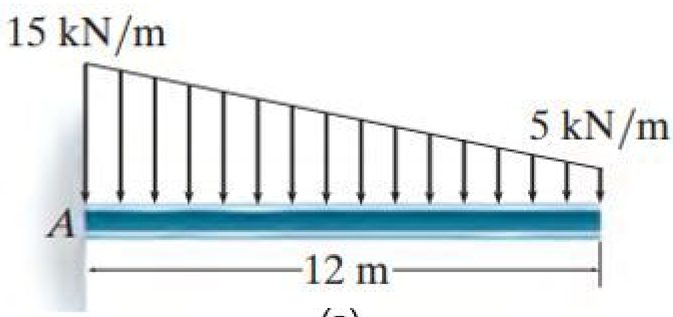
I

1. Internal loading developed in a beam (Section Method)

1. 1.1. Determine the member's support reactions

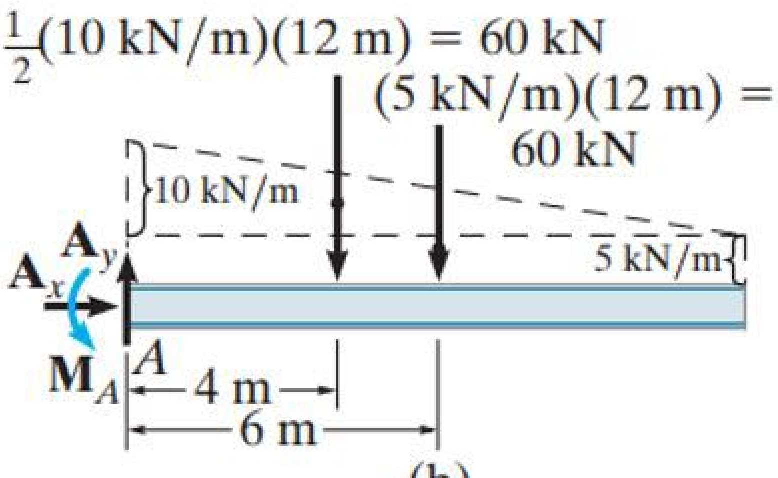
Exemple

Determine the reactions on the beam in the Figure (a)



(a)

Solution:



(b)

Equations of equilibrium:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 60 - 60 = 0 \qquad A_y = 120\ \text{kN}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad -60(4) - 60(6) + M_A = 0 \quad M_A = 600\ \text{kN} \cdot \text{m}$$

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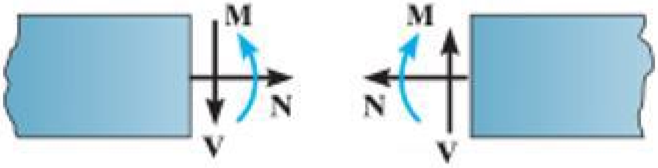
2. 1.2. Free-Body diagram

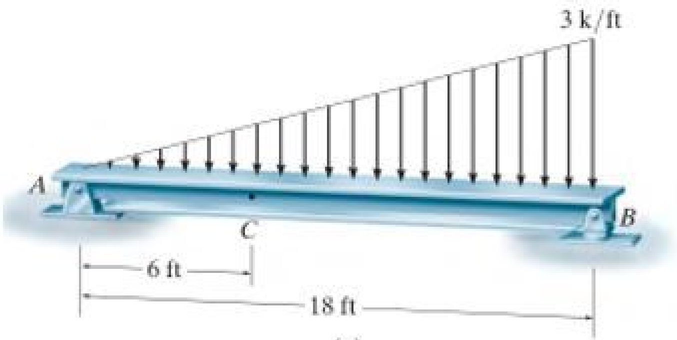
- Keep all distributed loadings, couple moments, and forces acting on the member in their exact location, then pass an imaginary section through the member, perpendicular to its axis at the point where the internal loading is to be determined.
- After the section is made, draw a free-body diagram of the segment that has the least number of loads on it. At the section indicate the unknown resultants N, V, and M acting in their positive directions.



positive sign convention

Exemple

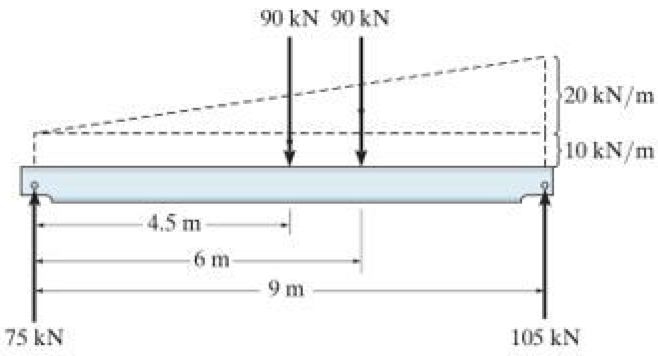
Determine the internal shear (V) and moment (M) acting at a section passing through point C in the beam shown in Figure (a)



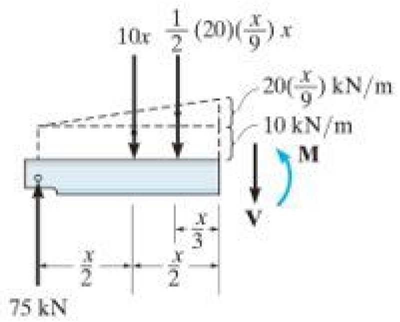
(a)

Solution:

1.Support Reactions:



2.Free-Body Diagram:



3. 1.3. Equations of Equilibrium

Exemple

Continue with the previous example

3.Equations Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad 75 - 10x - \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x\right] - V = 0$$

$$V = 75 - 10x - 1.11x^2$$

$$\circlearrowleft + \Sigma M_S = 0; \qquad -75x + (10x)\left(\frac{x}{2}\right) + \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x\right]\frac{x}{3} + M = 0$$

$$M = 75x - 5x^2 - 0.370x^3$$

II

2. Shear Force and Moment Diagrame (S.F. D & B.M.D)

1. 2.1. Shear Diagram

• Establish the V and x axes and plot the values of the shear at the two ends of the beam.

• Since the slope of the shear diagram at any point is equal to the intensity of the distributed loading w at the point.(Note that w is positive when it acts upward.)

• If a numerical value of the shear is to be determined at the point,

one can find this value either by using the method of sections as

discussed befor or by using this Eq:

$$\Delta V = \int w(x)\, dx$$

$$\left.\begin{array}{r}\text{Change in}\\ \text{Shear}\end{array}\right\} = \left\{\begin{array}{l}\text{Area under}\\ \text{Distributed Loading}\\ \text{Diagram}\end{array}\right.$$

, which states that the change in the shear force is equal to the area under the distributed loading diagram.

• Since w(x) is integrated to obtain V, if w(x) is a curve of degree n, then V(x) will be a curve of degree For example, if w(x) is uniform, V(x) will be linear.

2. 2.2. Moment Diagram

Establish the M and x axes and plot the values of the moment at the ends of the beam.

• Since the slope of the moment diagram at any point is equal to the intensity of the shear at the point.

• At the point where the shear is zero, and therefore this may be a point of maximum or minimum moment.

• If the numerical value of the moment is to be determined at a point, one can find this value either by using the method of

sections as discussed befor or by using Eq:

$$\Delta M = \int V(x)\,dx$$

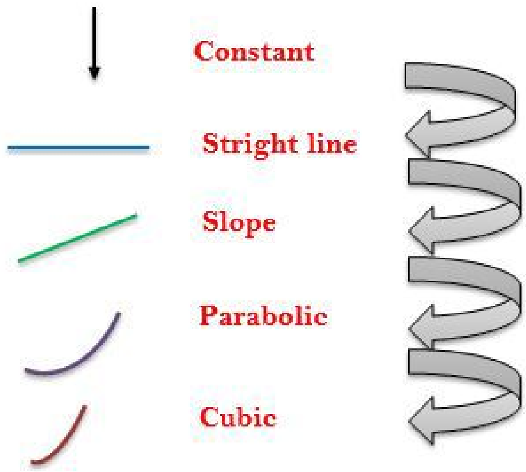
$$\left.\begin{matrix}\text{Change in}\\ \text{Moment}\end{matrix}\right\} = \left\{\begin{matrix}\text{Area under}\\ \text{Shear Diagram}\end{matrix}\right.$$

which states that the change in the moment is equal to the area under the shear diagram.

• Since V(x) is integrated to obtain M, if V(x) is a curve of degree n, then M(x) will be a curve of degree For example, if V(x) is linear, M(x) will be parabolic.

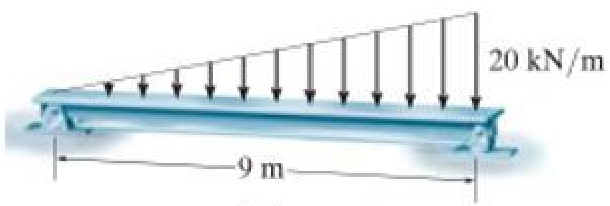
Important useful summary

Order of the Lines:



Exemple

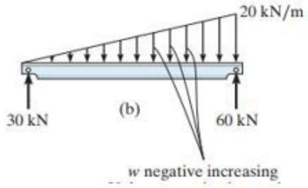
Draw the shear and moment diagrams for the beam in Figure (a) Draw the shear and moment diagrams for the beam in Figure (a)



(a)

Solution:

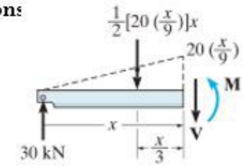
1.Support Reactions: the reactions have been calculated and are shown on the free-body diagram of the beam, Figure b.



2.Shear Diagram: at points x = 0, V = +30 KN and x = 9 m, V = -60 KN are first plotted. Note that the shear diagram (Figure c) starts with zero slope since w = 0 at x = 0 and ends with a slope of w = -20 KN.

The point of zero shear can be found by using the method of sections segment of length x, Figure e. We require V = 0, so that:

$$+\uparrow \Sigma F_y = 0; \qquad 30 - \frac{1}{2}\left[20\left(\frac{x}{9}\right)\right]x = 0 \qquad x = 5.20 \text{ m}$$



(e)

3.Moment Diagram: for the value of shear is positive but decreasing and so the slope of the moment diagram is also positive and decreasing. At x = 5.20 m, dM/dx = 0.

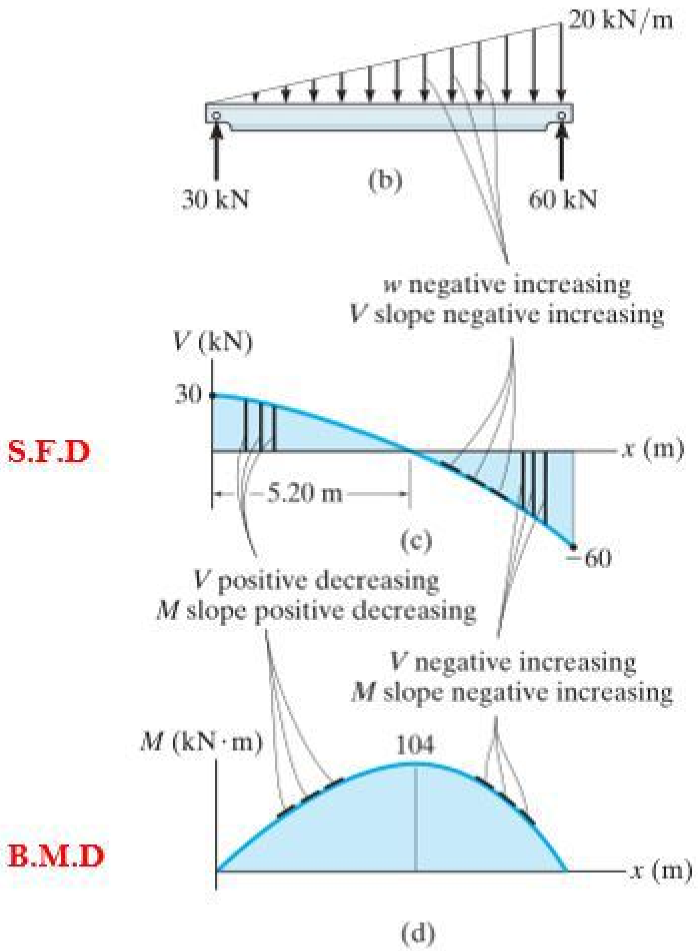
Likewise for 5.20 < x < 9 the shear and so the slope of the moment diagram are negative increasing as indicated.

The maximum value of moment is at x = 5.20 m since dM/dx = V = 0 at this point, Figure d.

From the free-body diagram in Figure e we have:

$$\circlearrowleft + \Sigma M_S = 0; \qquad -30(5.20) + \frac{1}{2}\left[20\left(\frac{5.20}{9}\right)\right](5.20)\left(\frac{5.20}{3}\right) + M = 0$$

$$M = 104 \text{ kN} \cdot \text{m}$$


20 kN/m
30 kN
(b)
60 kN
w negative increasing
V slope negative increasing
V (kN)
30
S.F.D
x (m)
5.20 m
(c)
−60
V positive decreasing
M slope positive decreasing
V negative increasing
M slope negative increasing
M (kN·m)
104
B.M.D
x (m)
(d)