III-1. Introduction

In the previous chapter, devoted to kinematics, we carried out a geometric description of the movement (position, speed, acceleration), in different coordinate systems. The movement has been studied without worrying about the agents which cause it.

Dynamics is the science that studies (or determines) the causes of the movements of these bodies.

III-2. Quantity of movement:

III-2.1. Definition:

When called the quantity of movement of a material point M of mass m and speed $\vec{v}$, the product:

$$\vec{p} = m.\vec{v} \qquad \text{(III.1)}$$

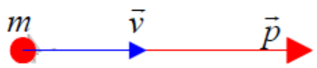


Figure III.1 *Quantity of movement*

[p]=MLT^{-1} ; if measured in kg.m.s^{-1} , for example. Like velocity, momentum depends on the reference frame considered.

Momentum is a vector quantity that has the same direction as speed.

III-2.2 Conservation of momentum

If there is a change in the speed or quantity of motion it implies that the particle is not free.

Suppose there are two free particles that are only subject to mutual influences, so they are isolated from the rest of the universe:

At time t : $\vec{p} = m_1.\vec{v_1} + m_2.\vec{v_2}$

At time t' : $\vec{p'} = m_1.\vec{v'_1} + m_2.\vec{v'_2}$

Experiments have proven that $\vec{p} = \vec{p'}$, i.e. the entire momentum of a system composed of two particles, subject only to their mutual influences, remains constant.

III-3. Center of inertia or barycenter

Also called moment of inertia or center of gravity. It was first defined by the physicist Archimedes.

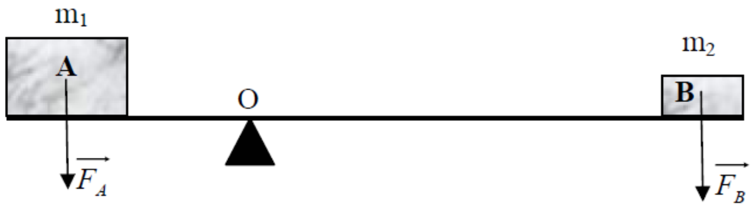


Figure III.2 *Center of inertia*

To obtain the relationship giving the center of inertia point of any system, let's study the equilibrium of the system shown in the following figure:

For the system to be in equilibrium, the sum of the moments of the forces relative to O must be zero:

$$\sum \overrightarrow{M^{0}_{\vec{F}_i}} = \vec{0} \quad \Rightarrow \quad \overrightarrow{M^{0}_{\vec{F}_A}} + \overrightarrow{M^{O}_{\vec{F}_B}} = \vec{0}$$

$$\Rightarrow \quad \overrightarrow{OA} \wedge \vec{F}_A + \overrightarrow{OB} \wedge \vec{F}_B = \vec{0}$$

$$\Rightarrow \quad \overrightarrow{OA} \wedge m_1 \vec{g} + \overrightarrow{OB} \wedge m_2 \vec{g} = \vec{0}$$

$$\Rightarrow \quad m_1 \overrightarrow{OA} \wedge \vec{g} + m_2 \overrightarrow{OB} \wedge \vec{g} = \vec{0}$$

$$\Rightarrow \quad \left(m_1 \overrightarrow{OA} + m_2 \overrightarrow{OB}\right) \wedge \vec{g} = \vec{0}$$

$$\Rightarrow \quad m_1 \overrightarrow{OA} + m_2 \overrightarrow{OB} = \vec{0}$$

(III.2)

Mathematicians have generalized this equality for any system represented by the figure below.

$$m_1 \overrightarrow{GM_1} + m_2 \overrightarrow{GM_2} + \ldots\ldots\ldots m_i \overrightarrow{GM_i} = \vec{0}$$

$$\Rightarrow \quad \sum_i m_i \overrightarrow{GM_i} = \vec{0}$$

(III.3)

On the other hand,

$$\overrightarrow{OG} + \overrightarrow{GM_i} = \overrightarrow{OM_i}$$

$$\Rightarrow \quad \overrightarrow{GM_i} = \overrightarrow{OM_i} - \overrightarrow{OG}$$

(III.4)

So

$$\sum_i m_i \left(\overrightarrow{OM_i} - \overrightarrow{OG}\right) = \vec{0}$$

(III.5)

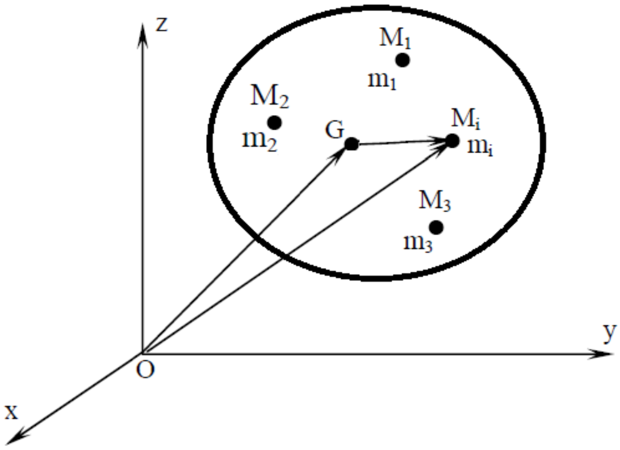


Figure III.3 *System in equilibrium, no motion.*

$$\Rightarrow \sum_i m_i \overrightarrow{OG} = \sum_i m_i \overrightarrow{OM_i}$$

$$\Rightarrow \overrightarrow{OG} = \frac{\sum_i m_i \overrightarrow{OM_i}}{\sum_i m_i} = \frac{\sum_i m_i \overrightarrow{OM_i}}{M}$$

(III.6)

M: represents the total mass of the system in equilibrium.

III-4. Newton's laws of mechanics

In his "Mathematical Principles of Natural Philosophy", published in 1687, Isaac Newton distinguished himself by unifying and transferring mechanics through the statement of three laws that explain quantitatively how any state of motion is spread out.

III-4.1. Principle of inertia

Also known as Newton's First Law.

In a Galilean reference frame (R), the center of inertia of any mechanically isolated (or pseudo-isolated) material system is either at rest or in uniform rectilinear motion.

Galilean reference frame (***R***)

Isolated (or pseudo-isolated)

$$\vec{v} = \vec{cste} \Rightarrow \vec{p} = m.\vec{v} = \vec{cste} \Rightarrow \frac{d\vec{p}}{dt} = \vec{0}$$

This principle leads to the law of conservation of the total momentum of an isolated or pseudo-isolated system.

Remarks :

- A frame of reference in uniform rectilinear motion relative to a galilean frame of reference is galilean.
- The choice of a Galilean frame of reference depends on the mechanical system to be studied.

Examples of Galilean reference points :

Copernicus landmark, terrestrial landmark, ...

III-4.2. Notion of Force

Also known as Newton's second law.

Any cause capable of modifying the momentum vector of a material point in a Galilean reference frame is called a ***FORCE***.

Different types of force exist:

- Interaction forces at a distance (gravitational forces);
- Electromagnetic forces;
- Nuclear forces;
- Contact forces (Friction forces);
- etc....

In a Galilean reference frame (R), the vector sum of the forces exerted on a material point M of mass m is equal to the derivative with respect to time of the momentum of this point.

$$\sum \vec{F} = \frac{d\vec{p}_{M/(R)}}{dt}$$

(III.7)

The momentum of M in (R) is $\vec{p}_{M/(R)} = m.\vec{v}_{M/(R)}$.

The system's momentum vector corresponds to the velocity vector of the system's center of inertia multiplied by the total mass (see 2.1a). As mass is an invariant, it is possible to give another form to this principle.

Galilean reference frame ***(R)***

Non-isolated system $\sum \vec{F} \neq \vec{0}$

$$\sum \vec{F} = \frac{d\,\vec{p}_{M/(R)}}{dt} = \frac{d\left(m.\vec{v}\right)}{dt} = m.\frac{d\,\vec{v}}{dt} = m.\vec{a}$$

This principle tells us only about the motion of the system's center of inertia and not about the motion of the other points in the system.

Remarks:

- 1N=1Kg.m.s^{-2}
- This 2ème law is independent of the 1ère law, which asserts the existence of a Galilean reference frame.

III-4.3. Principle of action and reaction

Also known as Newton's third law.

If we consider an isolated system consisting of two inertially interacting material points, we have:

$\vec{p}_1 = m_1.\vec{v}_1$ and $\vec{p}_2 = m_2.\vec{v}_2$

When two particles M_1 and M_2 are interacting, whatever the frame of reference and whatever their motion (or lack of motion), $\vec{F}_{1\to 2}$ is the total force exerted by particle M_1 on particle M_2 and $\vec{F}_{2\to 1}$ is the total force exerted by particle M_2 on particle M_1 . By virtue of the fundamental principle, we have :

The action of particle M_1 on particle M_2 is exactly opposite to the simultaneous action of particle M_2 on particle M_1 .

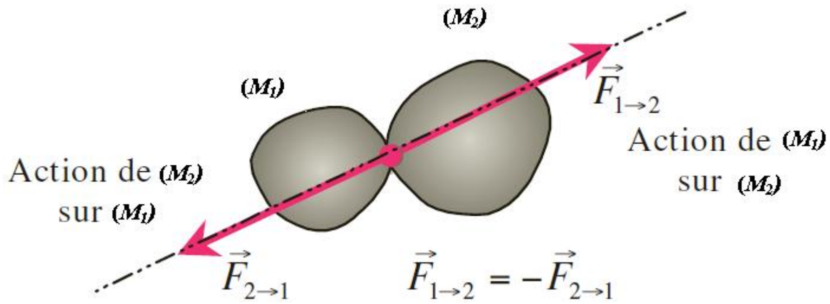


Figure III.4 *Illustration of the principle of reciprocal actions.*

III-5. Strengths

We have already seen that a force can be represented by a vector associated with a point corresponding to the point of application.

The application of the laws of mechanics requires a good balance of the forces exerted on a system, i.e. not to omit or add any force. To do this, the system must be clearly defined, so that we can then list the forces exerted on it by the outside world.

There are two types of force:

- *Distant interaction forces* (actor and receiver are not in contact): examples include gravitational forces, electromagnetic forces, nuclear cohesive forces.
- *Contact forces:* examples of friction and tension forces.

III-5.1. Interaction forces at a distance

a. Newtonian gravitational forces

The force exerted by one mass M on another mass m is called the gravitational force or gravitational interaction force. This force of interaction follows a law enunciated by Newton in 1650.

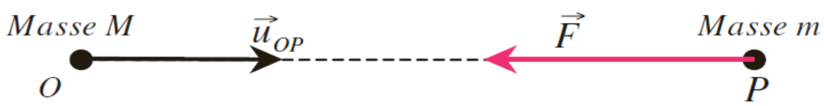


Figure III.5 *Gravitational force: action of mass M in O on mass m in P.*

The force $\vec{F}$ exerted by a mass M (point at O) on a mass m (point at P) such that OP = r, is written :

$$\vec{F} = \vec{F}_{M-m} = -G\frac{m.M}{r^2}\vec{u}_{OP}$$

(III.8)

with $\vec{u}_{OP}$ unit vector along OP and G = 6.67 . 10^{-11} m^3 .kg s^{-1-2} , the universal gravitational constant.

b. Coulombic interaction

The Coulombic interaction is the analogue of the gravitational interaction for electric charges.

$$\left|\overrightarrow{F_{qq'}}\right| = -\frac{1}{4\pi\varepsilon_0}\frac{q\,q'}{r^2} = K\frac{q\,q'}{r^2}$$

with

$$K = \frac{1}{4\pi\varepsilon_0} = 9\;10^9\;mc^2Kg\,s^{-2}$$

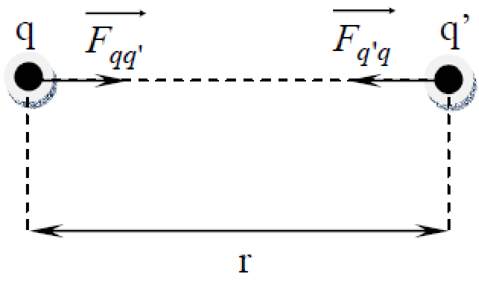


***Figure III.5** Coulombic interaction.*

c. Electromagnetic interaction

The force experienced by an electric charge placed in fields $\vec{E}$ (electric) and $\vec{B}$ (magnetic) is called the electromagnetic or Lorentz force:

$$\vec{F} = q\left(\vec{E} + \vec{v} \wedge \vec{B}\right)$$

(III.9)

III-5.2. Contact forces: Friction forces

a). Definitions

Experiment: Object in motion not subjected to a driving force always comes to rest, contrary to Newton's first law.

Certain interactions oppose movement: ***friction forces***

They are essential to everyday life: walking, wheeled vehicles, braking a car, gripping an object...

Two types of friction: ***Solid friction***

fluid friction

When friction: opposes the movement of an object: ***kinetic friction***

prevents a movement from starting: ***static friction***

b). Solid friction

In this figure, the solid block is in motion under the action of the driving force Fe .

$\vec{F}_e$: -Training strength ;

$\vec{R}_n$: -Reaction force ;

$\vec{F}$: -Friction force ;

φ : -angle of friction.

In module $F_e = \mu R_n$.

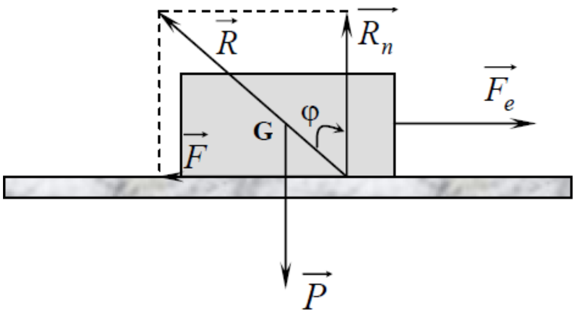


Figure III.6 *Solid friction.*

with: μ coefficient of friction: this is a constant that depends on the nature of the contact surface.

We have:

$$\frac{\vec{F}}{\vec{R}_n} = tg\phi = \mu_s = \mu_c \qquad \text{(III.10)}$$

where , μ_s μ_c are constants called static and kinetic friction coefficients.

c). Fluid friction

When a solid moves in a fluid (a gas such as air, or a liquid such as water), it is subjected to frictional forces from the fluid. The resultant of these actions is a force vector proportional to the object's velocity vector.

With k a positive constant, we have :

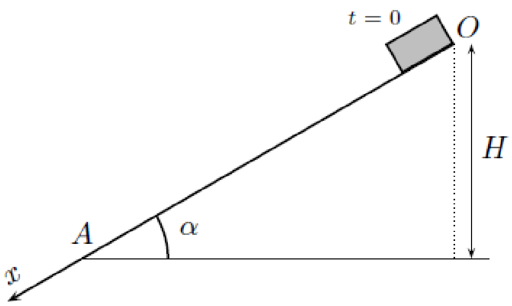
$$\vec{F} = -k\vec{\upsilon} \qquad \text{(III.11)}$$

This force only exists if there is movement.

III-6. Exercises

Exercise III.1

A solid M of mass m stands at the top of an inclined plane of angle α, with no initial velocity. Note H the distance from this initial point O to the horizontal plane, and g the gravity field (assumed constant).



1 . What is the sliding condition on the friction coefficient f *for* the solid to start moving at $t = 0$?

2 . Determine the acceleration of the solid at time t.

3 . Deduce the norm of the solid's velocity at point A.

Exercise III.2

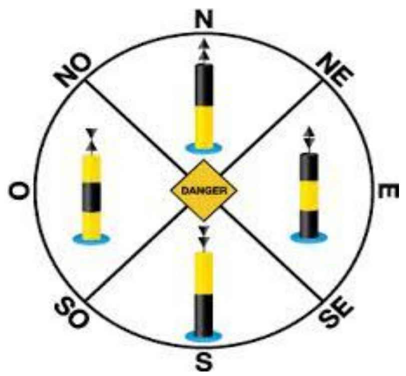
A body with mass $m_1 = 3.2$ kg moves westwards at a speed of 6.0 m/s. Another different body, with mass $m_2 = 1.6$ kg, is moving northwards at a speed of 5.0 m/s. The two bodies interact. After 2s, the first body moves in the N 30° E direction at a speed of 3.0 m/s. Show and calculate :

a)- The total momentum of the two particles before and after the 2s.

b)- The change in momentum of each particle, and the interacting forces.

c)- The change in speed of each particle.

d)- The magnitude and direction of the second body's velocity.



Exercise III.3

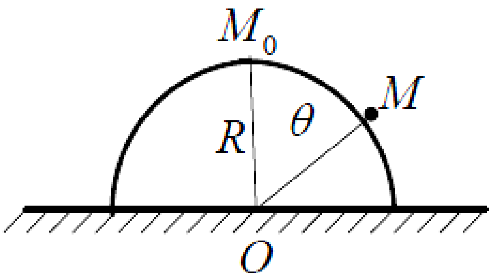
A half-sphere of radius $R = 2$ m and center O rests on a horizontal plane. A particle of mass m, starting from rest at point M_0 at the top of the half-sphere, slides under its own weight.

1/ Write the differential equation of motion of the particle as it slides, knowing that the coefficient of sliding on the surface of the sphere is μ.

2/ Neglecting friction:

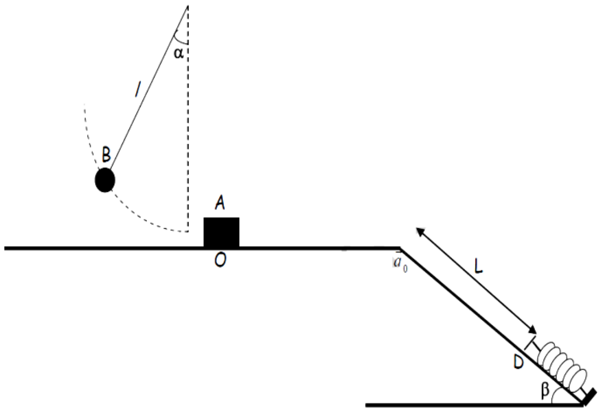
a) Where does the mass leave the dome?

b) What is its speed?



Exercise III.4

A ball *B* of mass m, attached to an inextensible wire of length *l*, is moved away from its equilibrium position by an angle α and left without initial velocity. As it passes through the vertical position, the ball strikes a body *A* of the same mass and stops. Body *A* slides along the *OCD* track of the figure. The *OC* = *d* section is a rough horizontal plane with a dynamic friction coefficient μ_d . The portion *CD* = *L*, perfectly smooth, is inclined at an angle β= 30° to the horizontal.



1- Draw the forces exerted on body A in a position between *O* and *C*.

2- Calculate the acceleration of body *A* between *O* and *C*. Deduce the nature of the motion.

3- Give the expression for the velocity of ball *B* just before it hits body A.

4- Using the conservation of momentum of the system, determine the velocity of body *A* after the interaction.

5- Express the velocity of body *A* at point *C as a* function of *g*, *l*, *d*, α and μ_d .

6- By what angle α_m must ball *B* be moved away so that body *A* arrives at *C* with zero velocity?

7- From point *C*, body *A* approaches part *CD* with zero velocity. It arrives on a perfect spring with no-load length l_0 and stiffness constant *k*.

- Show the forces exerted on A as the spring compresses.

- What is the value of the maximum spring compression?

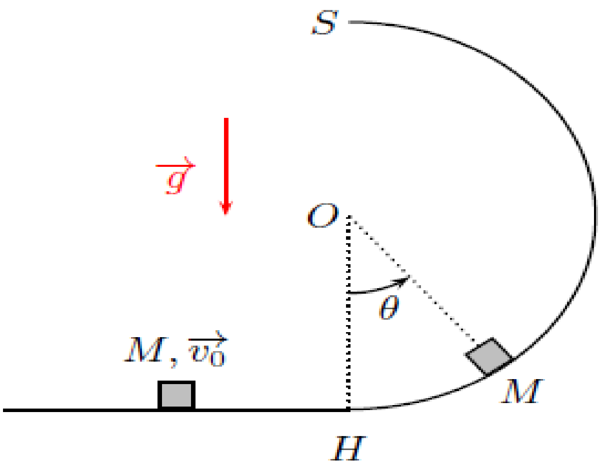
We give :

m = 200 g, d = 1 m, l = 10 cm, L = 1 m, μ_d = 0.1, g = 10 m/s2 and k = 140 N/m.

Exercise 4

A small car, comparable to a material point M of mass m, is launched at speed v_0 onto a flat horizontal track extended by a vertical semicircle of radius R.

The car slides without friction on the support, which it may leave (the link is not bilateral). Its position inside the semicircle is marked by the angle $\theta(t)$ formed by the radius OM with the descending vertical (OH).



1 . How does the car's speed vary until it reaches point H?

2 . Determine the expression for the angular velocity of the car when it is located in the semicircular track at the position marked by the angle θ, as a function of v_0 , g, R and θ.

3 . Determine the norm of the reaction of the semicircular track on the car (assuming contact maintained), as a function of m, v_0 , g, R and θ. How does it vary as a function of θ?

4 . At what condition on the launch speed v_0 will the car reach the top S of the runway without breaking contact with it?