THE SPHERICAL MIRROR AND MIRROR PLAN

❖ DESCRIPTION

A spherical mirror is a spherical cap with center C and vertex S made reflective. The axis of symmetry is the optical axis of the mirror. This axis is usually oriented from left to right because light arrives from the left (by convention). There are two types of spherical mirrors:

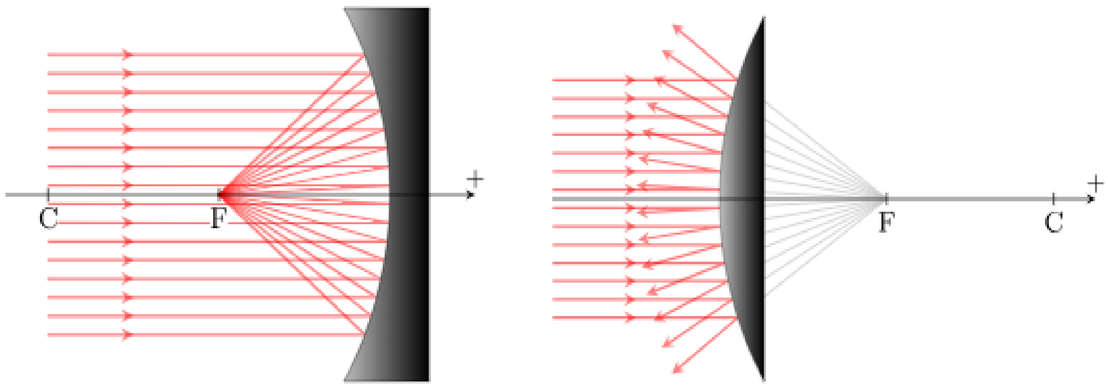


Figure 4.1: Concave and convex spherical mirrors.

- the concave mirror is a spherical mirror such that $\overline{SC} < 0$
- the convex mirror is a spherical mirror such that $\overline{SC} > 0$

In the case of spherical mirrors, the principle of inverse return of light implies: $f = f'$

- The conjugation relation for a spherical mirror:

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{1}{\overline{SF}} = \frac{1}{f'} \qquad \overline{SF} = \overline{SF'} = f'$$

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}}$$

- The magnification

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$$

- The focal lengths $\overline{SF}$ and $\overline{SF'}$ have the expressions:

$$\overline{SF} = \overline{SF'} = \frac{\overline{SC}}{2}$$

Case of the mirror plan:

For a plane mirror $\overline{SC} = \infty$

The conjugation relation is then written: $\overline{SA'} = -\overline{SA}$

The object and the image are equidistant from the mirror.

The magnification γ in this case is: $\gamma = 1$

The image $A'B'$is the same size as the object AB

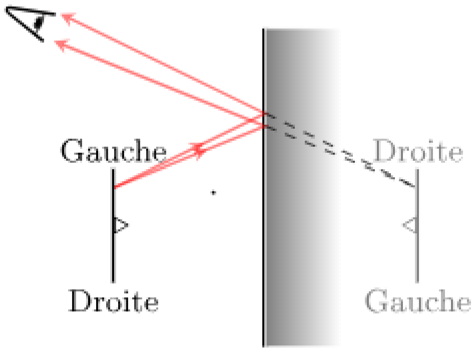


Figure 4.2: Formation of an image with a plane mirror. The image is reversed (left/right)

Plane mirrors

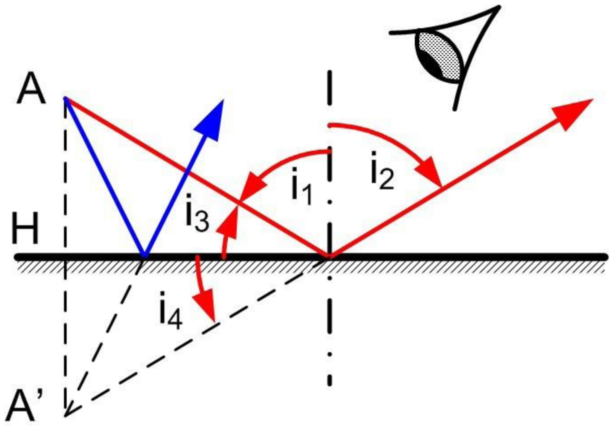


Figure 4.3 Formation of an image of point A with a plane mirror.

Let an object point be A and its projection on the mirror H. Any ray out of A follows the law ($i2 = -i1$). The prolongation of the reflected ray cuts the line AH at a point AH'. On figure 4.3 we easily demonstrate that if the absolute values of i1 and i2 are equal, those of i3 and i4 also are and HA=HA'. Point A' is the symmetry of A in relation to the mirror. This is true for all luminous rays issued of A, the image is stigmatic, figure 4.3 show a real object and a virtual image. If we inverse the direction of the luminous rays, the object becomes A', virtual, and the image becomes A, real.

Spherical mirrors

A spherical mirror is a concave or convex reflecting surface defined by the center of curvature C and a vertex S located on the surface. The curvature ray is $R = \overline{SC}$.

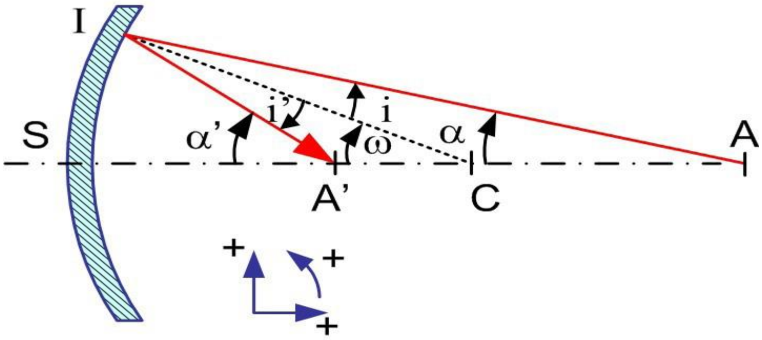


Figure1

Let us consider a point A of the line SC. A luminous ray arising from A reflects itself on a point I of the mirror et cuts its line SC in A'. If we carry out the same operation as for the diopters, we have:

$$i = i' \quad \textbf{refraction law}$$

$$\alpha + i = \omega \quad and \quad \alpha' = i' + \omega$$

Therefore: $\omega - \alpha = \alpha' - \omega$

$$\begin{cases} \tan\omega \approx \omega = \dfrac{\overline{HM}}{\overline{CH}} = \dfrac{\overline{HM}}{\overline{CS}} \\ \tan\alpha \approx \alpha = \dfrac{\overline{HM}}{\overline{AH}} = \dfrac{\overline{HM}}{\overline{AS}} \\ \tan\alpha' \approx \alpha' = \dfrac{\overline{HM}}{\overline{A'H}} = \dfrac{\overline{HM}}{\overline{A'S}} \end{cases}$$

And

$$\frac{\overline{HM}}{\overline{CS}} - \frac{\overline{HM}}{\overline{AS}} = \frac{\overline{HM}}{\overline{A'S}} - \frac{\overline{HM}}{\overline{CS}}$$

$$\frac{1}{\overline{CS}} - \frac{1}{\overline{AS}} = \frac{1}{\overline{A'S}} - \frac{1}{\overline{CS}}$$

$$\frac{2}{\overline{CS}} = \frac{1}{\overline{A'S}} + \frac{1}{\overline{AS}}$$

The expression below is analogous to the diopters by replacing n' by –n. We therefore have, as for the diopters, $\overline{SA} = P$, $\overline{SA'} = P'$, $\overline{SC} = R$

$$\frac{1}{P'} + \frac{1}{P} = \frac{2}{R}$$

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{P'}{P}$$

Image construction in a mirror

From point B of object AB, the ray going through C (Red) reflects to itself, the ray going through F (Green) reflects itself in parallel to the axis, the ray parallel to the axis (Blue) reflects itself by going through F and the ray going through S (Orange) reflects itself symetrically in relation to the axis.
B' is at the intersection of emerging rays, A' is on the **projection** of B' on the axis.

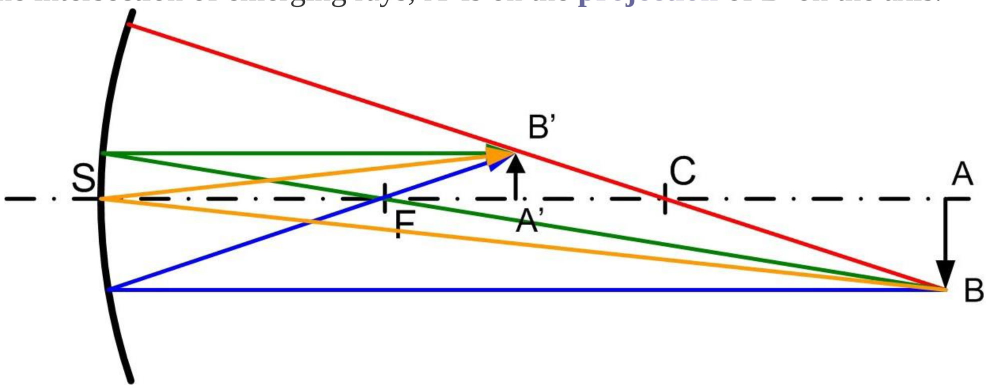


Figure 2 : *Concave Mirror*

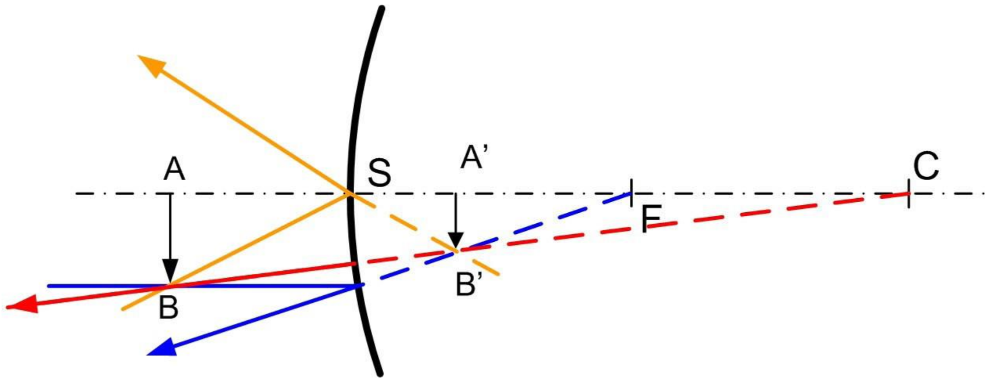
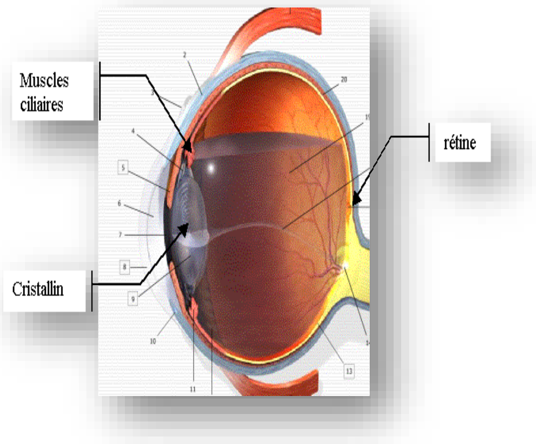


Figure 3 : *Convex Mirror*

THE EYE

➢ *REMINDERS ON THE STRUCTURE OF THE EYE*

The eye is an organ that allows humans to analyze light, which allows us to analyze the environment in which it is located.



• **The retina** is comparable to a background screen on which the rays form images. When a group of rays coming from an object forms a beam converge on a point of the retina, a clear image is then interpreted by the brain.

• **The cornea** corresponds to a rigid surface allowing to converge the rays of light.

• **The crystalline lens** corresponds to a flexible lens which is deformable thanks to the ciliary muscles.

Figure 4.3: The eye.

1.EYE MODELING:

The eye can be modeled as a converging lens (cornea-crystalline assembly) with variable focal length OF' to accommodate the position of objects and form their images on the retina. The distance between this lens and the retina is called eye depth d. For a normal eye, the focal length is equal to the depth of the eye. The furthest point of distinct vision (the Punctum Remotum PR) is at infinity. The closest point of distinct vision (the Punctum Proximum PP) is at the minimum distance $d_m = 25cm$ from the lens.

We define the amplitude of accommodation by:

$$A = \mathrm{V_{max}} - \mathrm{V_{min}}$$

$$\mathrm{V_{max}} = \frac{n'}{d} - \frac{1}{\overline{PP}}$$

$$\mathrm{V_{min}} = \frac{n'}{d} - \frac{1}{\overline{PR}}$$

$$A = \frac{1}{\overline{PR}} - \frac{1}{\overline{PP}}$$

$n'$is the refractive index inside the eye after the lens. d is the distance between the cornea-lens assembly and the retina: $\frac{n'}{d} \approx 59\delta$. $\overline{PP}$ is the distance between the lens and the Punctum Proximum. $\overline{PR}$ is the distance between the lens and the Punctum Remotum PR.

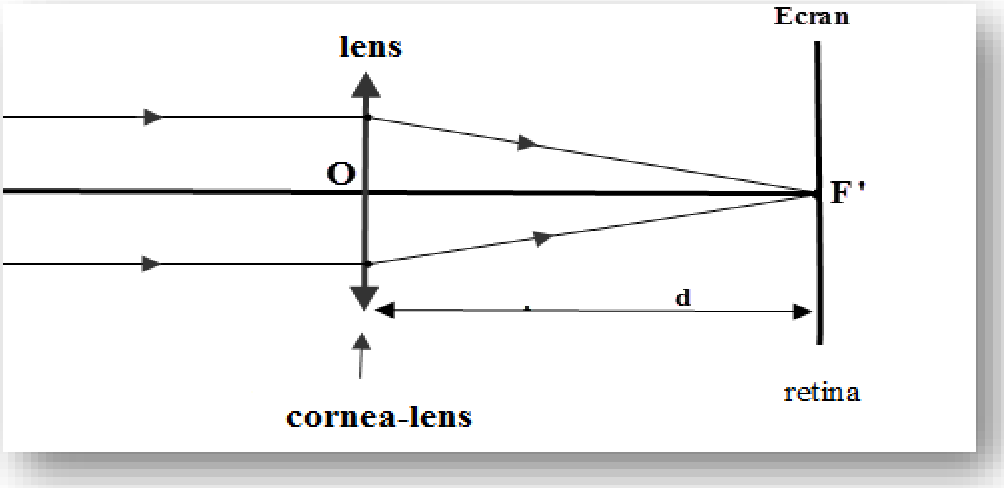


Figure 4.4: Representative diagram of a normal eye.

2.THE DIFFRENT AREAS OF VISION

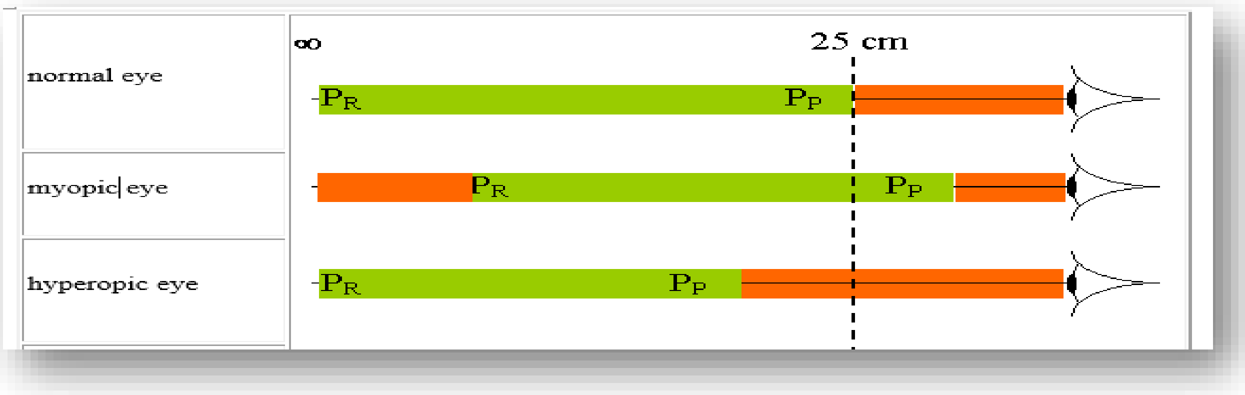


Figure 4.5: Representative the different areas of vision:

2.5 DEFECTS OF VISION AND THEIR CORRECTIONS:

4.5.1 Myopia (Short sightedness):

It is a kind of defect in human eye due to which a person can see near objects clearly but he cannot see the distant objects clearly. Myopia is due to :(i) Excessive curvature of cornea, (ii) Elongation of eye ball.

- In this case $OF' < d$ ($F'$is in front of the retina**).**

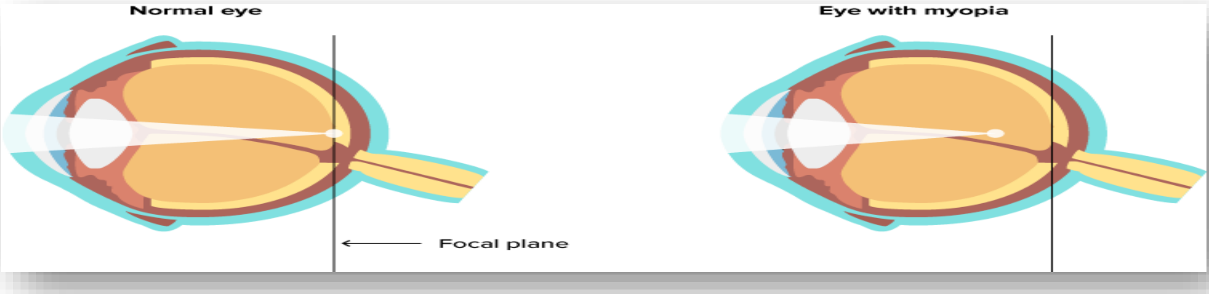


Figure 4.6: Eye normal and myopia.

Correction: Myopia or short-sightedness can be corrected by wearing spectacles containing concave lens.

4.5.2 Hypermetropia (Long sightedness):

I t is a kind of defect i n human eye due to which a person can see distant objects properly but cannot see the nearby objects clearly. I t happens due to: (i) Decrease in power of eye lens i .e., increase in focal length of eye l ens. (ii) Shortening of eye ball.

- In this case $OF' > d$ ($F'$is in behind of the retina**).**

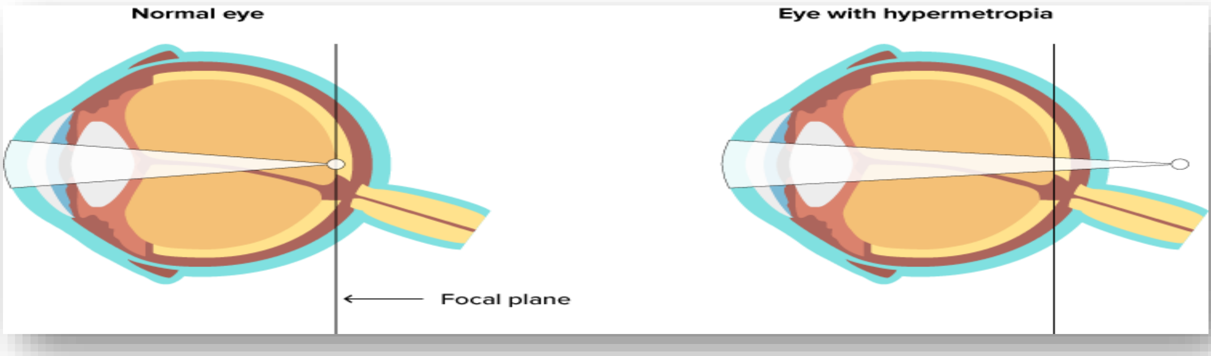


Figure 4.7: Eye normal and hypermetropia.

Correction: The near-point of an eye having hypermetropia is more than 25 cm. The condition of hypermetropia can be corrected by putting a convex lens i n front of the eye.

Presbyopia: is the loss of the power of accommodation with age.

- ✓ Astigmatism: the focal length is not the same in all viewing directions.

3.MAGNIFIYING GLASS

A magnifying glass is a convex lens that is used to produce a magnified image of an object. The lens is usually mounted in a frame with a handle. A magnifying glass can be used to focus light, such as to concentrate the sun's radiation to create a hot spot at the focus for fire starting.

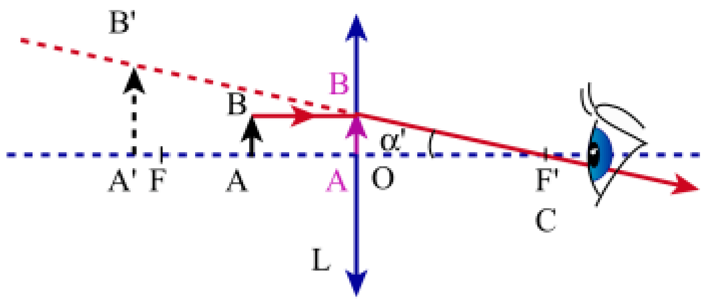


Figure 4.8: The magnifying glass.

- The algebraic distance $\overline{OA'}$of the image $A'B'$of an object AB is calculated from the conjugation relation:

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}}$$

- The magnification γ of the magnifying glass is calculated by the relation:

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$

- The power P of the magnifying glass is defined by the relationship:

$$P = \frac{\tan \alpha'}{AB}$$

α' is the angle with which the observer sees the image (Figure 2). Its unit is diopter (δ).

- The magnification of the magnifying glass is:$G = \frac{\tan\alpha'}{\tan\alpha}$

  $\alpha'$is the angle from which we observe the image $A'B'$

  α the angle at which we observe the object AB from a distance d_m where d_m represents the minimum distinct vision distance: $d_m = 25cm = 0.25m$.

- Relationship between G and P: $G = P.d_m$.

4.THE OPTICAL MICROSCOPE:

The optical microscope consists of two thin converging lenses (Figure 4.9):

- The lens with a focal length $\overline{OF'}$of a few millimeters gives a real enlarged and reversed imageA_1B_1 of the object AB. The magnification of the objective is defined by:

$$\gamma = \frac{\overline{A_1B_1}}{\overline{AB}} = \frac{\overline{O_1A_1}}{\overline{O_1A}}$$

- The eyepiece with a focal length O_2F_2' of a few centimeters works like a magnifying glass. It allows you to observe a virtual image $A'B'$enlarged and rejected to infinityA_1B_1. To do this, the image $A_1B_1$of the object AB must be in the focal plane of the eyepiece. The point A_1 must therefore merge with the object focal point F_2 of the eyepiece (Figure 4.9).

  The magnification G of the eyepiece is defined by the ratio:

$$G = \frac{\tan\alpha'}{\tan\alpha} = \frac{\frac{A_1B_1}{O_2F_2}}{\frac{A_1B_1}{d_m}} = \frac{d_m}{O_2F_2}$$

- The power of an optical microscope is defined by the relationship:

$$P = \frac{\tan\alpha'}{AB} = \frac{\frac{A_1B_1}{O_2F_2}}{AB} = \frac{A_1B_1}{AB}.\frac{1}{O_2F_2} = |\gamma|.\frac{1}{O_2F_2}$$

  We also demonstrate that: $P = \frac{\Delta}{O_1F_1'.O_2F_2}$

  Where .Δ= $F_1'F_2$ represents the optical interval of the microscope.

- The commercial magnification of the optical microscope is defined by the relationship:

$$G_C = \frac{\tan\alpha'}{\tan\alpha} = \frac{\frac{A_1B_1}{O_2F_2}}{\frac{AB}{d_m}} = \frac{d_m}{O_2F_2}.\frac{A_1B_1}{AB} = G.|\gamma|$$

  Noticed:

$$G_C = \frac{d_m}{O_2F_2}.\frac{A_1B_1}{AB} \ and \ P = \frac{A_1B_1}{AB}.\frac{1}{O_2F_2} \ \ from\ where\ G_C = P.d_m$$

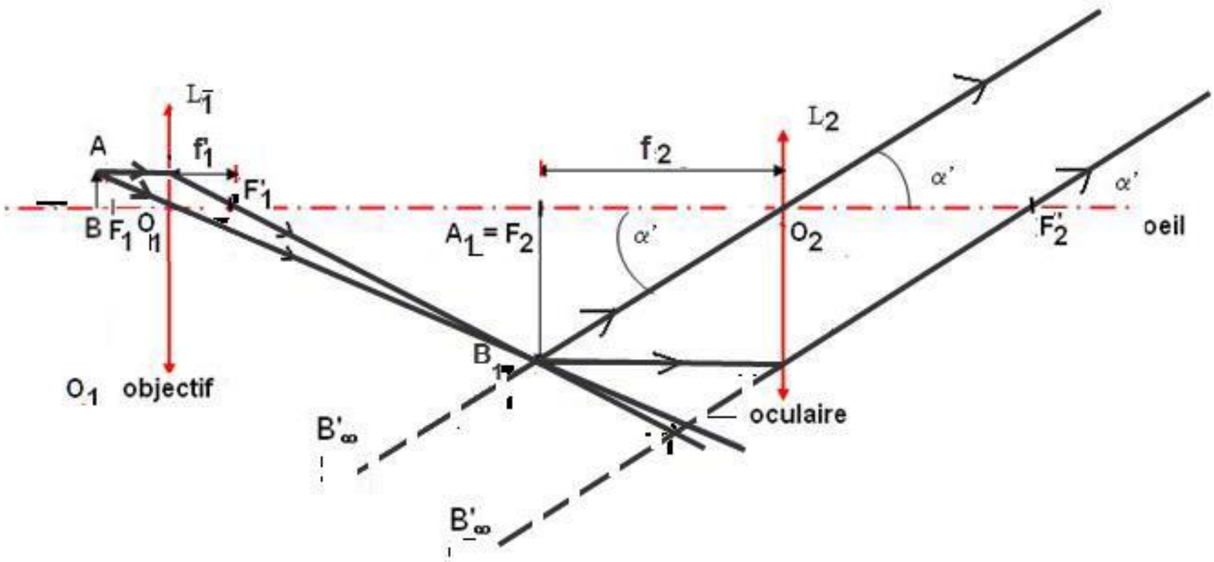


Figure 4.9: Building the image of an object through an optical microscope.

- The numerical aperture (maximum angle under which rays from the object can penetrate the optical system)
- Separating power is the ability to distinguish two adjacent points as distinct. The eye has the ability to distinguish particles with a diameter of up to $0.1\mu m$. However, they must be separated from each other by a distance of at least $5\mu m$. The resolving power of the eye is $5\mu m$.