Chapter II: stresses in materials

II-1 definition:

In materials science, **stress** is defined as the force applied to a material divided by the area over which the force is applied. It is a measure of the internal resistance of a material to deformation when subjected to an external force. The formula for stress (σ) is:

$$\sigma = \frac{F}{A}$$

Where:

- σ is the stress,
- F is the force applied,
- A is the cross-sectional area over which the force is distributed.

Stress can be classified into different types, including:

- **Tensile stress:** When the material is stretched (extended).
- **Compressive stress:** When the material is compressed.
- **Shear stress:** When the material experiences forces that cause layers to slide past each other.

II-2 Tensile:

When a force is applied perpendicular to the cross-sectional area of the bar (figure 8), the length increases by an amount under the action of external force F.

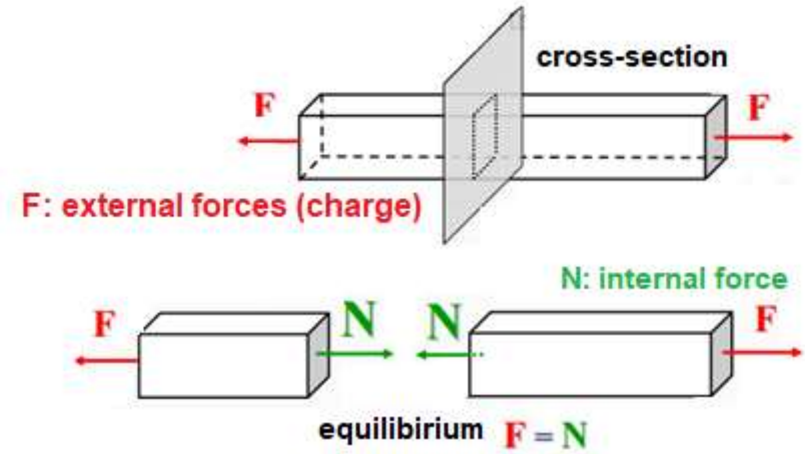


Fig. 8

The relationships of tensile is:

$$Normal\ stress\ (\sigma) = \frac{F}{A}$$

Units:

The basic units of tensile stress in S.I units i.e. (International System) are N / m2 (or Pa Pascal) MPa = 10^6 Pa, GPa = 10^9 Pa, KPa = 10^3 Pa

Sometimes (N/mm^2) units are also used, because this is an equivalent to MPa.

Strain in materials science refers to the deformation or displacement of material that occurs when it is subjected to stress. It is the measure of how much a material stretches, compresses, or deforms relative to its original size or shape. Strain is a **dimensionless** quantity because it is a ratio of change in lateral to the original length.

The formula for strain ($\mathcal{E}$) is:

$$tensile\ strain\ (\mathcal{E}) = \frac{\text{change in lateral (}\Delta\text{L)}}{\text{original length }\ L_0}$$

Where:

- $\mathcal{E}$ is the strain,
- ΔL is the change in length (final length minus original length),
- L_0 is the original length of the material.

❖ **Types of Strain:**

1. **Tensile strain**: Occurs when a material is stretched.
2. **Compressive strain**: Occurs when a material is compressed.
3. **Shear strain**: Occurs when a material is subjected to shear forces that cause it to deform in a sliding manner.

Strain is typically expressed in terms of **percentage** or as a decimal (e.g., 0.02 or 2% strain). It is important to note that strain is a **dimensionless** quantity, meaning it doesn't have units, unlike stress.

II-3 Stress-strain diagram

The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium carbon structural steel. Metallic engineering materials are classified as either ductile or brittle materials.

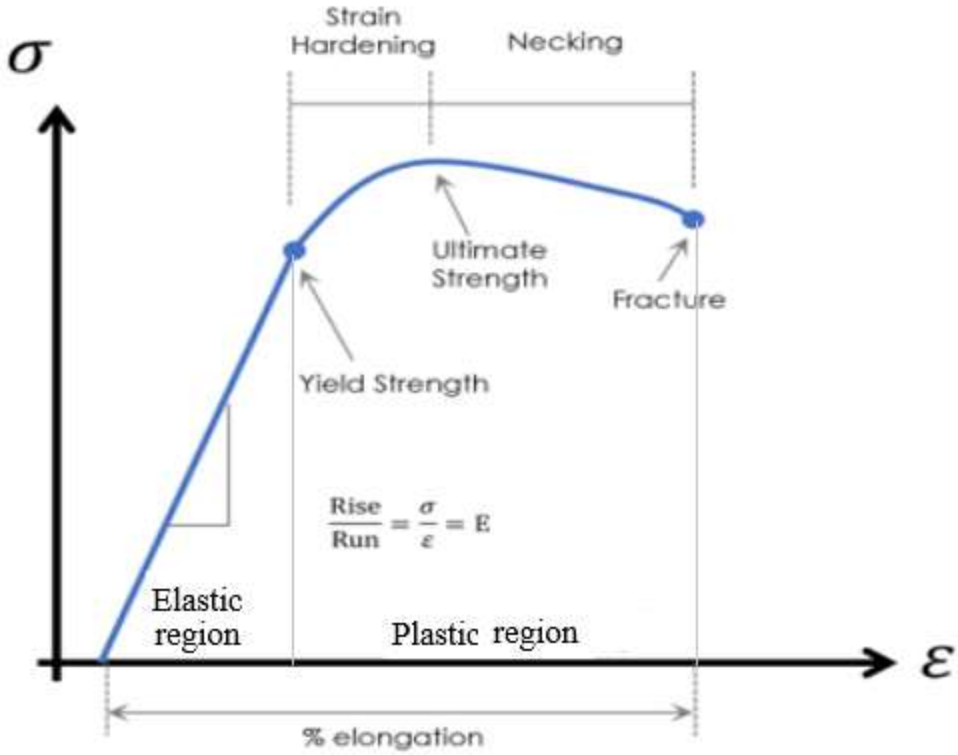
The stress-strain curve is a straight line. This linear relation between elongation and the axial force causing was first noticed by Sir Robert Hooke in 1678 and is called Hooke's

$$\sigma = E\,\boldsymbol{\varepsilon}$$

since $\sigma = F / A$ and $\varepsilon = \Delta L / L$, then $F / A = E\, \Delta L / L$. Solving for ΔL,

$$\Delta L = \frac{F\,L}{A\,E}$$

This can be graphically represented on a stress-strain diagram (note it only holds for the elastic region) as the rise-run ratio. Young's Modulus is material-dependent and can be found in tables



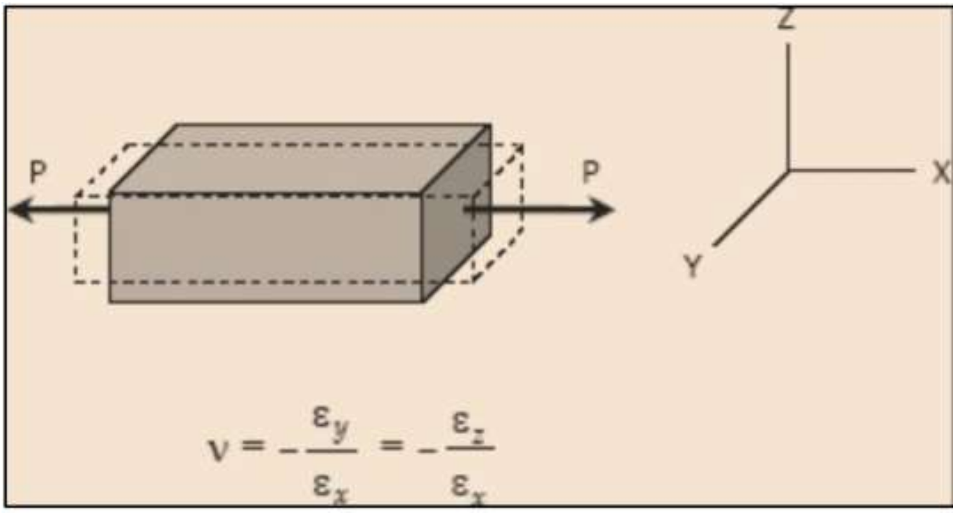
Elastic Region	Elastic region in which an applied stress will cause a strain that is non-permanent. When the stress is removed, the material will return to its original state (reducing any elongation to 0)
Plastic Region	Region in which applied stresses cause irreversible strains. When the stress is removed, the material will return to the elongation of the yield point but will not reenter the elastic region.
Yield Strength	Dividing point between elastic behavior and plastic behavior ("point of no return"). Any stress applied that is greater than the stress at the yield point results in bringing the material into the plastic region.

Brittle Material	A brittle material will experience miniscule elongation prior to failure.
Ductile Material	A ductile material will experience significant elongation ("necking") before failure. Particularly, it may have a long period in the plastic region before the fracture point.
Ultimate Strength	The greatest stress that is experienced in a material. Ductile materials will experience less stress as necking increases. Brittle materials will have a fracture point very near (if not identical to) the ultimate strength.
Fracture Point	The point at which the material separates, or breaks. This point will be close to the ultimate stress for low ductility materials and further from the ultimate stress for high ductility materials.

II-4 Poisson's Ratio :

If a bar is subjected to a tensile loading there will be an increase in length of the bar in the direction of the applied load, but there is also a decrease in a lateral dimension perpendicular to the load. It has been observed that for elastic materials, the lateral strain is proportional to the longitudinal strain. The ratio of the lateral strain to **longitudinal strain** is known as the Poison's ratio and is denoted by v.

Poisson's ratio (v) = **-** lateral strain / longitudinal strain



$$v = -\varepsilon y/\ \varepsilon x = -\varepsilon z\ /\varepsilon x\ ;\ \varepsilon y = \varepsilon z = -v\sigma x/\ E.$$

Where ε_x is strain in the x-direction and ε_y and ε_z are the strains in the perpendicular direction. The negative sign indicates a decrease in the transverse dimension when ε_x is positive.

For most engineering materials, the value of (ν) is between 0.15 and 0.33. For most steel, it lies in the range of 0.25 to 0.3, and 0.20 for concrete.

- **Biaxial deformation:**

If an element is subjected simultaneously by Tensile stresses, σ_x and σ_y, in the x and y directions, the strain in the x-direction is σ_x / E and the strain in the y direction is σ_y / E. Simultaneously, the stress in the y direction will produce a lateral contraction on the x-x direction of the amount ($-\nu\, \varepsilon_y$ or $-\nu\, \sigma_y/E$). The resulting strain in the x direction will be :

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \quad \text{or} \quad \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1-\nu^2}$$

and

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \quad \text{or} \quad \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1-\nu^2}$$

- **Triaxial deformation :**

If an element is subjected simultaneously by three mutually perpendicular normal stresses σ_x, σ_y, and σ_z, which are accompanied by strains ε_x, ε_y, and ε_z, respectively,

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

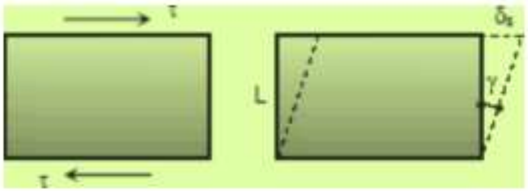
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Tensile stresses and elongation are taken as positive. Compressive stresses and contraction are taken as negative.

II- 5 Shear Strain:

Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape.



The change in angle at the corner of an original rectangular element is called the **Shear Strain** and is expressed as:

$$\gamma = \frac{\delta s}{L}$$

The ratio of the shear stress **τ** and the shear strain **γ** is called the **modulus of elasticity** in shear **or modulus of rigidity** and is denoted as G, in MPa.

$$G = \frac{\tau}{\gamma}$$

The relationship between the shearing deformation and the applied shearing force is :

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

Where **V** is the shearing force acting over an area $\mathbf{A_s}$.

- **Relationship Between E, G, and ν**

The relationship between modulus of elasticity E, shear modulus G and Poisson's ratio ν is given as :

$$G = \frac{E}{2(1+\nu)}$$

➢ Bulk Modulus of Elasticity or Modulus of Volume Expansion, K

The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as :

$$K = \frac{E}{3(1-2\nu)} = \frac{\sigma}{\Delta V / V}$$

Where V is the volume and ΔV is change in volume. The ratio ΔV / V is called **Volumetric Strain** and can be expressed as:

$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1-2\nu)}{E}$$

II-6 Resistance condition:

For security reasons related to the use of the devices, it is required that:

$$\sigma_{max} \leq \sigma_p$$

Where: $\sigma_p = \frac{\sigma_e}{s}$

σ_p, σ_e, s : are applied stress, limit stress and factor of safety respectively.

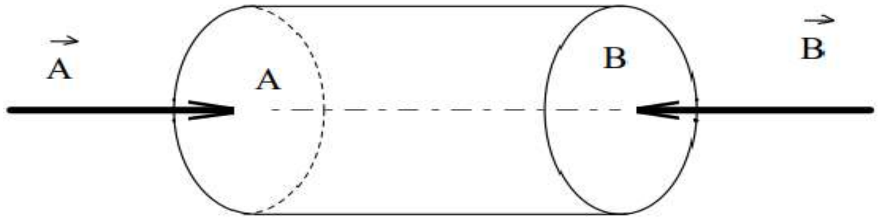
$s = \frac{\sigma_e}{\sigma_p}$ is ratio of this strength (ultimate or yield strength) to allowable strength is called the factor of safety, it is takes the values according for different materials and varies between $2 \leq s \leq 10$

- Elongation, Z % = $\frac{L_f - L_0}{L_0} \times 100$, is chiefly influenced by uniform elongation, which is dependent on the strain-hardening capacity of the material.

II.2 COMPRESSION

II.2.1 Definition

A beam is subjected to simple compression when it is subjected to two forces directly opposite, applied to the surface center of the extreme sections and which tend to shorten it.



The elements of reduction in G of the cohesion efforts are expressed by:

$$\{Cohesion\} = \begin{Bmatrix} N & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{G\,(\vec{x},\vec{y},\vec{z})} \quad \text{avec } N < 0$$

II.2.2 Compressive

The test piece does not regain its initial length, so the compute of the reduction of Area by relationship following $A\% = \frac{A_0 - A_f}{A_0} x100$

II.2.3. Elastic deformations

The property noted above made it possible for different materials to establish the relationship:

$$\frac{F}{S} = -E\frac{\Delta l}{l} \quad \text{with} \quad \Delta l < 0$$

- Normal stress

We define the normal stress σ in the straight section (S) by the relation:

$$\sigma = \frac{N}{S} \quad \text{with : } \sigma < 0 \text{ because } N < 0$$

- **HookE's law**

  We have already seen

$$\sigma = \frac{N}{S} \quad \text{and} \quad \frac{F}{S} = E\frac{\Delta l}{l}$$

We can conclude that:

$$\sigma = E\frac{\Delta l}{l} = E.\mathcal{E}$$

II.2.4 Resistance condition

For safety reasons, the normal stress σ must remain lower than a limit value called practical extension stress:

$$\sigma_{pe} = \frac{\sigma_e}{s}$$

s is a safety coefficient which varies from 1.1 to 10 depending on the of domains application.

The resistance condition simply reflects the fact that the real stress must not exceed the previous threshold, i.e.:

$$|\sigma_{real}| = \frac{|N|}{S} < \sigma_{pe}$$