University of Biskra
Mathematics Department
Module: Analysis 2

First year license
2024/2025

Catch-up Exam

Exercise 1 (./08pts)

Calculate the following integrals:

1. $\int(-2x^2+5)\cos(x)dx$.
2. $\int \frac{\sin(x)}{6-3\sin^2(x)}dx.$
3. $\int \frac{3}{(x-1)(x^2-4)}dx.$
4. $\int \frac{5x-2}{x^2+4x+5}dx.$

Exercise 2 (./06pts)

Solve the following first order differential equations:

$$-7y' + 7\frac{y}{x} = 5x^2.$$

$$y' + \frac{y}{x} = y^2x^2.$$

Exercise 3 (./06pts)

Solve the following second order differential equations:

$$y'' + y = x - 5.$$

$$y'' + y' = 3\sin(x).$$

EXO1:

① $I = \int (-2x^2+5)\cos(x)\,dx = ?$

$$\begin{cases} u = (-2x^2+5) \\ v' = \cos(x) \end{cases} \Rightarrow \begin{cases} u' = -4x \\ v = \sin(x) \end{cases}$$ 0,5

$$\Rightarrow I = (-2x^2+5)\sin(x) + 4\int x\sin(x)\,dx$$

$$\begin{cases} u = x \\ v' = \sin(x) \end{cases} \Rightarrow \begin{cases} u' = 1 \\ v = -\cos(x) \end{cases}$$ 0,5

$$\Rightarrow I = (-2x^2+5)\sin(x) - 4x\cos(x) + 4\int \cos(x)\,dx$$ 0,5

$$= (-2x^2+5)\sin(x) - 4\cos(x) + 4\sin(x) + c \,/\, c \in \mathbb{R}.$$ 0,5

② $J = \int \frac{\sin(x)\,dx}{3(2-\sin^2(x))}\,dx = ?$

we put $t = \cos(x) \Rightarrow dt = -\sin(x)\,dx$. 0,5

$$\Rightarrow J = \frac{1}{3}\int \frac{\sin(x)\,dx}{2-(1-\cos^2(x))}\,dx = \frac{1}{3}\int \frac{\sin(x)}{1+\cos^2(x)}\,dx.$$ 0,5

$$= -\frac{1}{3}\int \frac{1}{1+t^2}\,dt = -\frac{1}{3}\arctan(t) + C.$$ 0,5

$$= -\frac{1}{3}\arctan(\cos(x)) + C \,/\, c \in \mathbb{R}.$$ 0,5

③ $\int \frac{3}{(x-1)(x^2-4)}\,dx = 3\int \frac{1}{(x-1)(x-2)(x+2)}\,dx.$

0,5 ×3

$$= \int \frac{\alpha}{x-1} + \frac{\beta}{x-2} + \frac{\gamma}{x+2}\,dx$$

$$= \int \frac{-1}{x-1}\,dx + \int \frac{3/4}{x-2}\,dx + \int \frac{1/4}{x+2}\,dx$$

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$= -\ln(|x-1|) + \frac{3}{4}\ln(|x-2|) + \frac{1}{4}\ln(|x+2|) + c.$ 0.5

(4) $\int \frac{5x-2}{x^2+4x+5}dx = \frac{5}{2}\int \frac{2x+4}{x^2+4x+5}dx - \int \frac{12}{(x+2)^2+1^2}dx$ 0.5 0.5

$= \frac{5}{2}\ln(x^2+4x+5) - 12\arctan(x+2) + c.$ 1 $/c \in \mathbb{R}.$

EXO2:

$-7y' + 7\frac{y}{x} = 5x^2$ —— (*)

$y_H = ?$

$-7y' + 7\frac{y}{x} = 0 \Rightarrow \frac{y'}{y} = \frac{1}{x}.$

$\Rightarrow \ln(y) = \ln(x) + c$

$\Rightarrow \boxed{y_H = kx}$ 1

$y_G = ?$ $y_G = k(x)x \Rightarrow y'_G = k'(x)x + k(x).$

$(*) \Rightarrow -7k'(x)x \cancel{-7k(x)} + \cancel{\frac{7k(x)x}{x}} = 5x^2$

$\Rightarrow k'(x) = \frac{5}{7}x \Rightarrow k(x) = \int \frac{5}{7}x\,dx = \boxed{\frac{5}{14}x^2 + c}$ 1

$\Rightarrow \boxed{y_G = \left(\frac{5}{14}x^2 + c\right)x}$ 1

$y' + \frac{y}{x} = y^2 x^2 \Rightarrow$ is a Bernoulli equation with $n = 2$.

$\Rightarrow y^{-2} y' + y^{-1}/x = x^2$ ——— $(**)$

we put $z = y^{-1} \Rightarrow z' = -y' y^{-2}$ 0,5

$(**) \Rightarrow -z' + z/x = x^2$ ——— $(***)$ 0,5

$z_H = ?$

$-z' + z/x = 0 \Rightarrow z'/z = \frac{1}{x} \Rightarrow \boxed{z_H = KX}$ 0,5

$z_G = ?$ $\quad z_G = xK(x) \Rightarrow z'_G = xK'(x) + K(x)$

$(***) \Rightarrow -xK'(x) + \cancel{K(x)} + \cancel{\frac{K(x)x}{x}} = x^2$

$\Rightarrow K'(x) = -x \Rightarrow \boxed{K(x) = -\frac{1}{2}x^2 + C}$ 0,5

$\Rightarrow \boxed{z_G = -\frac{1}{2}x^3 + CX}$ 0,5

as $z = y^{-1} \Rightarrow \boxed{y_G = \frac{1}{-\frac{1}{2}x^3 + CX}}$ with $C \in \mathbb{R}$. 0,5

Ex 3:

$y'' + y = x - 5$ / $R^2 \equiv y''$, $R \equiv y'$ and $1 \equiv y$.

$y_H = ?$ $\quad R^2 + 1 = 0 \Rightarrow R^2 = -1 \Rightarrow R = 0 \pm 1i$ 0,5

$\Rightarrow \boxed{y_H = C_1 \cos(x) + C_2 \sin(x)}$ 0,5

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$y_p = ?$: $y_p = ax + b$

$\Rightarrow y_p' = a$

$y_p'' = 0$

$\Rightarrow 0 + ax + b = x - 5 \Rightarrow \begin{cases} a = 1 \quad 0,5 \\ b = -5 \quad 0,5 \end{cases} \Rightarrow \boxed{y_p = x - 5}$ 0,5

$\Rightarrow \boxed{y_G = c_1 \cos(x) + c_2 \sin(x) + x - 5}$ / c_1 and $c_2 \in \mathbb{R}$. 0,5

$y'' + y' = 3\sin(x)$ / $R^2 \equiv y''$, $R \equiv y'$ and $1 \equiv y$.

$y_H = ?$ $R^2 + R = 0 \Rightarrow R(1 + R) = 0 \Rightarrow \begin{cases} R = -1 \\ R = 0 \end{cases}$ 0,5

$\Rightarrow \boxed{y_H = c_1 e^{-x} + c_2}$ 0,5 / c_1 and $c_2 \in \mathbb{R}$.

$y_p = ?$ $y_p = a\sin(x) + b\cos(x)$.

$\Rightarrow y_p' = -b\sin(x) + a\cos(x)$.

$y_p'' = -a\sin(x) - b\cos(x)$.

$\Rightarrow -(a+b)\sin(x) + (a-b)\cos(x) = 3\sin(x) + 0\cos(x)$

$\Rightarrow \begin{cases} a + b = -3 \\ a - b = 0 \end{cases} \Rightarrow \boxed{a = b = -\frac{3}{2}}$ 0,5 x2

$\Rightarrow y_p = -\frac{3}{2}\sin(x) - \frac{3}{2}\cos(x)$ 0,5

$\Rightarrow \boxed{y_G = (c_1 e^{-x} + c_2) - \frac{3}{2}\left(\cos(x) + \sin(x)\right)}$ 0,5

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