

Introduction aux probabilités et statistique descriptiv - Dispersion parameter (spread) and Shape parameters

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5. Dispersion parameter (spread)

These parameters aim, in the case of a quantitative nature, to characterize the variability of the data in the sample..

The fundamental dispersion indicators are

- 1 The variance
- 2 The standard deviation.
- 3 The .mean deviation.
- 4 The Coefficient of variation.
- 5 The interquartile, interdecile and interpercentile.

5..Dispersion parameter (spread)

5.1 The variance

- Soit un échantillon de n valeurs observées $x_1, x_2, \dots, x_i, \dots, x_n$ d'un caractère quantitatif X et soit $\bar{x}$ sa moyenne observée. On définit la variance observée notée V ou S^2 comme la moyenne arithmétique des carrés des écarts à la moyenne.

$$V = S^2 = \frac{1}{n} \sum_{i=1}^k n_i (x_i - \bar{X})^2$$

5..Dispersion parameter (spread)

5.1 The variance

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- La formule de la variance qui résulte du théorème de Koenig est donc:

$$V = S^2 = \frac{1}{n} \sum_{i=1}^k n_i x_i^2 - \bar{X}^2$$

5..Dispersion parameter (spread)

5.1 The variance

- Dans le cas de données regroupées en k classes d'effectif n_i (variable continue regroupée en classes), la formule de la variance est la suivante:

$$V = S^2 = \frac{1}{n} \sum_{i=1}^k n_i (c_i - \bar{X})^2 ,$$

ou c_i est le centre de classe

5 Dispersion parameter (spread)

5.2 The standard deviation

- The observed standard deviation corresponds to the square root of the observed variance:

$$\sigma = \sqrt{V} = S = \sqrt{S^2}$$

5. Dispersion parameter (spread)

5.3 The mean deviation

- Let us be a sample of n observed values $x_1, x_2, \dots, x_n$ of a quantitative character X and let $\bar{x}$ be its observed mean. We define the mean deviation denoted $E.M$ as the arithmetic mean of the deviations from the mean

$$E.M = \frac{1}{n} \sum_{i=1}^k n_i |x_i - \bar{X}|$$

5. Dispersion parameter (spread)

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$$E.M = \frac{1}{n} \sum_{i=1}^k n_i |x_i - \bar{X}|$$

- In the case of data grouped into k classes of frequency n_i (continuous variable grouped into classes), the formula is as follows:

$$E.M = \frac{1}{n} \sum_{i=1}^k n_i |c_i - \bar{X}|$$

5. Dispersion parameter (spread)

5.4. The Coefficient of variation

- Observed variance and standard deviation are absolute dispersion parameters that measure the absolute variation of the data independent of the order of magnitude of the data.

$$C.V = \frac{\sigma}{\bar{X}} 100$$

Exprimé en pour cent, il est indépendant du choix des unités de mesure permettant la comparaison des distributions de fréquence d'unité différente.

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5.4. The Coefficient of variation

- Observed variance and standard deviation are absolute dispersion parameters that measure the absolute variation of the data independent of the order of magnitude of the data.
- The coefficient of variation denoted $C.V.$ is a relative dispersion index taking into account this bias and is equal to:

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Exprimé en pour cent, il est indépendant du choix des unités de mesure permettant la comparaison des distributions de fréquence d'unité différente.

5. Dispersion parameter (spread)

5.5. l'interquartile range, interdécile et intercentile range:

- The interquartile range is a measure of variation that is not influenced by extreme values. Its definition is simple: the interquartile range measures the extent of the 50% of values located in the middle of a series of classified data.

$$IQ = Q_3 - Q_1$$

The interpercentile range measures the range of the middle 98% of values in a classified data series.

$$IC = C_{99} - C_1$$

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$$IQ = Q_3 - Q_1$$

- The interdecile interval measures the extent of the 80% of values located in the middle of a series of classified data.

$$ID = D_9 - D_1$$

The interpercentile range measures the range of the middle 98% of values in a classified data series.

$$IC = C_{99} - C_1$$

6.Shape parameters

We define the shape parameters for a quantitative statistical variable, discrete or continuous, with real values.

- 1 **Skewness coefficient.(Asymmetry)**
- 2 **Kurtosis coefficient (heaviness of tail)**

6. Shape parameters

6.1 Skewness coefficient.

- 1 The Pearson Skewness coefficient involves the *Mo* mode: when it exists, it is defined by:

$$S_k = \frac{\bar{X} - Mo}{\sigma}$$

- Si S_k est égal à 0, the reduced variable has the same flattening as a bell curve, we say that the variable is normal.

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- If $S_k < 0$, When Skewness coefficient is negative, the distribution is more spread out to the left: we say that there is skew to the right

6. Shape parameters

6.1 Skewness coefficient.

- 1 Le coefficient d'asymétrie de **Yule** fait intervenir la médiane et les quartiles, il est défini par:

$$Y = \frac{Q_1 + Q_3 - 2Me}{Q_3 - Q_1}$$

- Si Y est égal à 0, the reduced variable has the same flattening as a bell curve, we say that the variable is normal.

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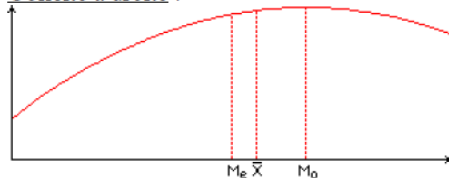
Shape parameters

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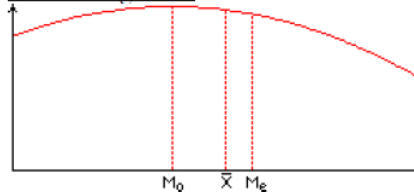
Lorsque le coefficient d'asymétrie est positif, la distribution est plus étalée à droite : on dit qu'il y a obliquité à gauche.

Lorsque le coefficient d'asymétrie est négatif, la distribution est plus étalée à gauche : on dit qu'il y a obliquité à droite.

Obliquité à droite :



Obliquité à gauche :



Shape parameters

6.2 Kurtosis coefficient

Here again several definitions are possible.

- The **Pearson** kurtosis coefficient is:

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$F_2 = \frac{\mu_4}{\mu_2^2} - 3$$

Shape parameters

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- The **Yule** kurtosis coefficient is:

$$F_2 = \frac{\mu_4}{\mu_2^2} - 3$$

Shape parameters

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- If $F_2 = 0$, the statistical polygon of the reduced variable has the same flattening as a bell curve, we say that the variable is **mesokurtic**. (normal)

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- If $F_2 > 0$, the statistical polygon of the reduced variable is less flattened than a bell curve, we say that the variable is **leptokurtic**. (less flattened)

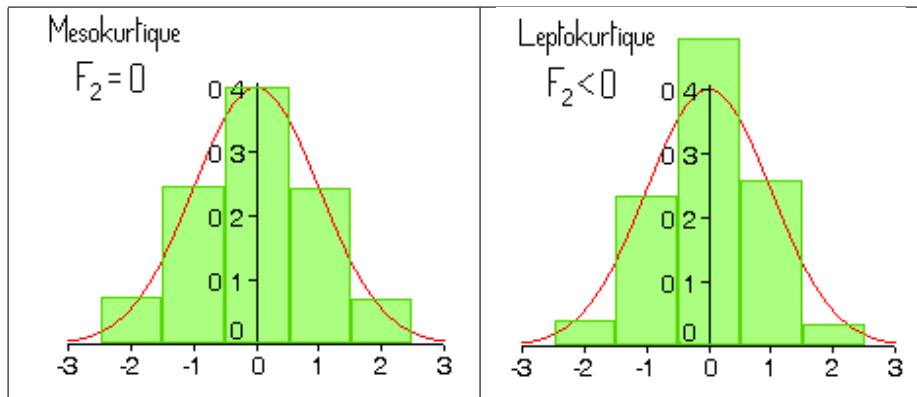
Shape parameters

6.2 Kurtosis coefficient:

- If $F_2 = 0$, the statistical polygon of the reduced variable has the same flattening as a bell curve, we say that the variable is **mesokurtic**. (normal)
- If $F_2 > 0$, the statistical polygon of the reduced variable is less flattened than a bell curve, we say that the variable is **leptokurtic**. (less flattened)
- If $F_2 < 0$, the statistical polygon of the reduced variable is flatter than a bell curve, we say that the variable is **platikurtic**. (more flattened)

Shape parameters

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