

Directed Work 3

Objective:

The objective is to explore functional dependencies, Armstrong's axioms, elementary functional dependencies, transitive closure, attribute closure, keys, covers, and minimal covers.

Exercise 1

1. Consider the relational schema $R(A, B, C, D, E)$ and the following set of functional dependencies:

1. $A \rightarrow BC$
2. $B \rightarrow C$
3. $AC \rightarrow D$
4. $D \rightarrow E$
5. $A \rightarrow D$

Rewrite these functional dependencies to elementary functional dependencies

2. Consider the relational schema $R(A, B, C, D, E)$ with the following functional dependencies:
 1. $A \rightarrow BC$
 2. $B \rightarrow D$
 3. $CD \rightarrow E$

Verify whether $A \rightarrow E$ holds using Armstrong's axioms. Justify your answer.

3. Let F be the following set of functional dependencies:

$$F = \{ AB \rightarrow C; B \rightarrow D; CD \rightarrow E; CE \rightarrow GH; G \rightarrow A \}.$$

Using **Armstrong's axioms** and the **attribute closure** method, demonstrate that the following functional dependencies can be deduced from F :

1. $AB \rightarrow E$
2. $BG \rightarrow C$
3. $AB \rightarrow G$

Exercise 2

Let us consider the schema of the following relation: $R(A, B, C, D, E)$.

This relation is defined extensionally by the following tuples:

A	B	C	D	E
a1	b2	c2	d3	e2
a1	b2	c2	d1	e4
a2	b3	c2	d1	e5
a2	b4	c5	d1	e5

1. Among the following functional dependencies, which ones apply to the extension of R ?

$E \rightarrow D$,	$E \rightarrow A$,	$\{A, E\} \rightarrow C$
$D \rightarrow E$,	$B \rightarrow C$,	$\{A, E\} \rightarrow D$
$E \rightarrow B$,	$B \rightarrow A$,	$\{A, B\} \rightarrow A$

2. Determine, among the following attribute groups, which ones are candidate keys or superkeys in R based on the given extension.

A	C	E	$\{A, B, E\}$	$\{B, E\}$
B	D	$\{A, D\}$	$\{C, D\}$	$\{A, B, C, D, E\}$

Exercise 3

Let's denote the set of functional dependencies as:

$$F = \{ A \rightarrow B, B \rightarrow C, AC \rightarrow D, D \rightarrow E \}$$

1. Compute A^+ , B^+ , AC^+ , D^+
2. Compute F^+ (transitive closure)

Exercise 4:

Let $R(A, B, C, D, E, F)$ be a relation with the following set of functional dependencies:

$$F = \{ A \rightarrow B, B \rightarrow C, AC \rightarrow D, D \rightarrow E, CE \rightarrow F \}$$

Questions:

1. Compute the closure of attribute A, A^+ .
Compute the closure of the set $\{A, C\}$, $(AC)^+$.
2. By applying Armstrong's axioms—particularly transitivity—derive all non-trivial functional dependencies from F, thereby identifying any additional dependencies that form part of F^+ .
3. Determine whether the set AC is a key for relation R (i.e., whether $(AC)^+ = \{A, B, C, D, E, F\}$).
4. Based on your results, identify all possible minimal keys for R.

Exercise 5

Let $R(A, B, C, D, E)$ be a relational schema, and let the following sets of functional dependencies be given:

$$F1 = \{A \rightarrow B, B \rightarrow C, A \rightarrow C, C \rightarrow D, AD \rightarrow E\}$$

$$F2 = \{A \rightarrow B, B \rightarrow C, A \rightarrow D, A \rightarrow E\}$$

Tasks:

1. Determine whether $F1$ is a cover of $F2$
2. Determine whether $F2$ is a cover of $F1$
3. Check if $F1$ and $F2$ are equivalent:

Exercise 6

Let the relation $R(A, B, C, D, E, F)$ be given with the set of functional dependencies:

$$F = \{AC \rightarrow D, B \rightarrow AF, C \rightarrow BE, F \rightarrow EC\}$$

Find an irreducible equivalent (Minimal Cover) of this set of functional dependencies.