Mohamed Khidher University

Faculty of economics, Commercial and Management Sciences

Department of commerce

Specialty: International Commerce & Finance

Module: Advanced Econometric

Academic year 2024/2025

Exercise 1 answers

1. **Which of the following best describes the behavior of the function $y = \alpha e^{\beta x}$ if β>0?**
   a) The function decreases as *x* increases
   b) The function increases at a constant rate
   c) The function increases at an accelerating rate
   d) The function remains unchanged regardless of *x*
2. **Which of the following statements is true about the logarithmic model?**
   a) It grows at a constant rate
   b) It has a horizontal asymptote
   c) It increases rapidly at first and then slows down
   d) It represents exponential decay
3. **If a process follows a logarithmic model, how does its rate of change behave?**
   a) The rate of change remains constant
   b) The rate of change increases exponentially
   c) The rate of change slows down over time
   d) The rate of change is unpredictable
4. **In the power law model $y = \alpha x^{\beta}$, what does the exponent β determine?**
   a) The slope of the linear relationship
   b) How fast y increases or decreases when x changes
   c)The starting value of y
   d) The rate of exponential decay
5. **A power law model is particularly useful for describing:**
   a) Linear relationships
   b) Growth that slows down over time
   c) Relationships with scale-invariance properties
   d) Situations where changes are unpredictable
6. **A population of bacteria follows an exponential growth model given by $P = P_0 e^{0.3t}$, where P_0=100. What is the population after 5 hours? At what point in time the population of bacteria will be doubled?**
   a) 100, 3 hours
   b) 134, 2,30 hours
   c) 448, 2 hours and 18 min
   d) 600, 3 hours

Explantion : $P = 100e^{0.3*5} = 100e^{1.5}$ =100*4.4817 ≈ 448; So, after 5 hours, the population is approximately **448**.

Doubled means $\mathbf{P = 2 \cdot P_0 = 200}$, Using the formula: $200 = 100e^{0.3*t}$ ⇒ 2= $e^{0.3*t}$ Take natural logarithm:

ln(2)=0.3t ⇒ t=ln(2)/0.3 ≈ 2.3103 hours t≈ 0.30.6931≈ 2.31 hours Convert 0.3103 hours to minutes: 0.3103*60 ≈ 18.6 minutes So, the population doubles in approximately **2 hours and 18 minutes**.

7. **What is the primary purpose of Logit and Probit models in statistical analysis?**
   a) To predict continuous outcomes
   b) To model relationships between categorical independent variables
   c) To estimate the probability of a binary dependent variable
   d) To compute linear regression coefficients
8. **What is the key difference between Logit and Probit models?**
   a) Logit assumes a normal distribution, while Probit assumes a logistic distribution
   b) Probit assumes a normal distribution, while Logit assumes a logistic distribution
   c) Logit is used for categorical data, while Probit is used for numerical data
   d) There is no difference; both models yield identical results
9. **In the Logit model, what transformation is applied to the probability of success?**
   a) Square root transformation
   b) Logarithmic transformation
   c) Log-odds
   d) Exponential transformation
10. **Which of the following is a key assumption of the Probit model?**
   a) The errors follow a logistic distribution
   b) The independent variables must be categorical
   c) The errors follow a normal distribution
   d) The dependent variable must be normally distributed
11. **What is the range of predicted probabilities in both Logit and Probit models?**
   a) $-\infty$ to $+\infty$
   b) 0 to 1
   c) -1 to 1
   d) Depends on the independent variables
12. **If the estimated Logit model is: $\ln\left(\frac{P}{1-P}\right) = -2 + 1.5X$, What is the probability of success (P) when X=2?**
    a) 0.50
    b) 0.73
    c) 0.27
    d) 0.90

Explanation : $\ln\left(\frac{P}{1-P}\right) = -2 + 1.5(2)$ =1 take exponential of both side p/(p-1)= e^1 =2.718

P = 2.71(1-P) ; then P ≈ 0.731

13. **If a Probit model estimates: P(Y=1)=Φ(−1+0.8X), What is the probability of success when X=2 using the standard normal CDF (Φ)?**
   a) 0.40
   b) 0.50

c) 0.73
d) 0.80

Explantion: **$P(Y=1)=\Phi(-1+0.8*2) = P(Y=1)=\Phi(0.6) = 0.73$**

14. **If the coefficient of an independent variable in a Logit model is negative, what does it imply?**
a) The probability of success increases as the independent variable increases
b) The probability of success decreases as the independent variable increases
c) The independent variable has no effect on the outcome
d) The variable is not statistically significant
15. **If a Probit model produces a coefficient of 2.5 for an independent variable, how should it be interpreted?**
a) A one-unit increase in the independent variable increases the probability by 2.5
b) A one-unit increase in the independent variable increases the Z-score by 2.5
c) A one-unit increase in the independent variable decreases the probability by 2.5
d) The coefficient does not affect probability predictions

Exercise 2 answers

1. The appropriate model is the **logarithmic model**.
2. The assumed functional form of the relationship between **GDP** and **FDI** is:

$GDP=\beta_0+\beta_1\cdot\ln(FDI)$
$GDP = 1.203 + 0.743 \ln(FDI)$

3. Predict the GDP when FDI = 10

We plug into the model: $GDP=1.203+0.743\cdot\ln(10) \approx 2.913$

4. Interpret the results (coefficients)

Intercept (1.203): is the initial value of GDP; or the value of GDP when the value FDI = 1 unit.

The Coefficient 0.743:
This means that a **1% increase in FDI** leads to an **approximate increase in GDP by 0.743/FDI** units.
In other words, **GDP increases with diminishing effect as FDI rises**.

5. How does GDP growth respond to an increase in FDI from 15 to 25 and from 25 to 35?

$GDP_{15}=1.203+0.743\cdot\ln(15) = 3.215$
$GDP_{25}=1.203+0.743\cdot\ln(25)\approx 3.594$
$GDP_{35}=1.203+0.743\cdot\ln(35) \approx 3.844$
$\Delta GDP_{15\rightarrow 25}=3.594-3.215=\mathbf{0.379}$
$\Delta GDP_{25\rightarrow 35}=3.844-3.594=\mathbf{0.250}$

- The GDP **increases more when FDI increases from 15 to 25** (ΔGDP = 0.379) compared to when FDI increases from 25 to 35 (ΔGDP = 0.250). This supports the theory: **as FDI increases, its marginal contribution to GDP diminishes**.

Exercise 3 answers

1. The relationship described — *slow growth at first, then faster acceleration* — so, the appropriate model to test this relationship is **Exponential Growth model**:
2. The model that shows the relationship between the two variables is $Innov = \beta_0 e^{\beta_1 R\&D}$

   based on the table: $Innov = 1.752e^{0.312R\&D}$

This model indicates that innovation grows **exponentially** as R&D expenditure increases.

3. The Innovation value when R&D expenditure is 4 units

Plug into the model: $Innov = 1.752e^{0.312*4}$ ≈20.085

4. Interpret the results (coefficients)

- **β_0 = 1.752:** is the initial value of Innovation, or it indicates the value of innovation when R&D = 0.
- **β_1 = 0.312:** is the growth rate, 0.312 indicates a positive exponential growth rate, 1-unit increase in R&D leads to a 32.2% increase in Innovation by a factor of:

5. How does innovation respond to an increase in R&D from 2 to 4 and from 4 to 6?

From R&D = 2 to 4:
$Innov = 1.752e^{0.312*2}$ ≈ 10.77
$Innov = 1.752e^{0.312*4}$ ≈20.085
$\Delta Innovation_{2\to4}$=20.09−10.77 = 9.32
From R&D = 4 to 6:
$Innov = 1.752e^{0.312*4}$ ≈20.085
$Innov = 1.752e^{0.312*6}$ ≈ 37.51
$\Delta innovation_{4\to6}$=37.51−20.058 = 917.43

Exercise 4 answers

1. Perform the model that represents the relationship and convert it into linear form

Power Law Model Form: $EG = \beta_0 Energ^{\beta_1}$

Convert into linear form using logarithms:

Ln(EG)=ln(β_0*)+* β_1 *·ln(Energ)*

So, the regression model becomes:

ln(EG)=C(1)+C(2)·ln(Energ)

From the table:

- C(1)=ln(β_0) = 1.2456 so, β_0=*3.475*
- C(2)= β_1= 0.8723

Thus, the Power law model is: $EG = 3.475 Energ^{0.872}$

2. The estimated power law exponent is: β_1= 0.8723; it shows how fast economic growth responds to changes in energy consumption.
3. Is the relationship statistically significant? Let's look at the **t-statistics** and **p-values**:

- Coefficient C(2) has a P-value = 0.0000 → **significant at all common levels (1%, 5%, 10%)**. So, the relationship is **statistically significant**.

4. What is the economic interpretation of the elasticity coefficient?

The coefficient β_0=*3.475*, is the initial value of economic growth;

The coefficient β_1 =0.8723 indicates that, **1% increase** in energy consumption leads to an approximate **0.87% increase** in economic growth, **on average**. Since the coefficient is **less than 1**, this suggests the energy consumption and economic growth grow at rate **less than proportionally, energy consumption increase faster than economic growth**.

5. How well does the model fit the data?

From the table: R^2=0.8794 ; this means: About **87.94%** of the variation in economic growth is explained by variations in energy consumption. This is a **very good fit**, indicating a **strong explanatory power** of the model.

Exercise 5 answers

1. This is a **logistic regression model** (logit), appropriate for binary outcomes like **default = 1** or **no default = 0**. The model is:

$\ln P/(1-P) = \beta_0+\beta_1 \cdot \text{Income}+\beta_2 \cdot \text{DTI}+\beta_3 \cdot \text{Credit Score}+\beta_4 \cdot \text{Loan Amount}$

2. The logit regression, based on the table

$\ln P/(1-P) = -3.2154 - 0.0028\text{Income} + 0.0735\text{DTI} - 0.0146\text{Credit Score} + 0.0012 \cdot \text{Loan Amount}$

3. Interpretation of each coefficient

All coefficients represent the effect on the **log-odds** of default. Here's the sign and interpretation:

- **Income (-0.0028): translate this into odds, we exponentiate the coefficient: $e^{-0.0028} \approx$ 0.9972; odds=0.9972-1=0.002796 = - 0.2796%** This means 1 unit increase in **income** decreases the probability of default by 0.2796%.
- **Debt-to-Income Ratio (0.0735):let's translate this into odds, we exponentiate the coefficient: $e^{0.0735} \approx$ 1.07627; odds = 1.07627 – 1 = 0.007627 = 0.7627%.** This means **1** unit increase in **debt-to-income ratio** increases the probability of default by 0.7627%.
- **Credit Score (-0.0146): let's translate this into odds, we exponentiate the coefficient: $e^{-0.0146} \approx$ 0.9855; odds = 0.9855 – 1 = 0.0145= 1.45%.** This means 1 unit increase in **credit score** decreases the probability of default by 1.45%.
- **Loan Amount (0.0012): let's translate this into odds, we exponentiate the coefficient: $e^{0.0012} \approx$ 1; odds = 1.0012 – 1 = 0.0012 = 0.12%.** This means 1 unit increase **in Loan amount** increases the probability of default by 0.12%.

4. Estimate the probability of default for a borrower : Income = 50,000, DTI = 40, Credit Score = 650, Loan Amount = 15,000, **Plug into the logit equation**

$$\widehat{P} = \frac{e^{-3.2154-0.0028(50)+0.0735(40)-0.0146(650)+0.0012\,(15)}}{1+e^{-3.2154-0.0028(50)+0.0735(40)-0.0146(650)+0.0012\,(15)}} \approx 0.0000508 \approx 0$$

Interpretation: With this data, the model sees the borrower as **very low risk** — the probability that this borrower defaults is close to zero.

5. Impact of increasing credit score from 650 to 700 Recalculate logit using Credit Score = 700:

- Old credit score term: −0.0146·650=−9.49−0.0146·650=−9.49
- New credit score term: −0.0146·700=−10.22−0.0146·700=−10.22

The probability becomes slightly **lower**, making the default probability **even smaller**. The absolute change in probability is very small because we were already near 0. **Increasing credit score reduces the risk of default**

6. Recommendations to reduce loan default risk

Based on the regression results:

1. **Favor borrowers with higher income**
2. A high DTI significantly increases default risk. Discard the applicant with high **DTI limits** or require financial counseling for high-DTI borrowers.
3. **Be cautious with large loan amounts**
   Though the effect is smaller, larger loans increase risk. Introduce tighter controls on large loan approvals or require stronger credit profiles.

Exercise 6 answers

1. Since the dependent variable is binary (adopt or not adopt) and the error term is **normally distributed**, the appropriate model is a **Probit regression model**.

The Model is:

P(adoption) = $\Phi(\beta 0+\beta 1\cdot \text{Income}+\beta 2\cdot \text{Education}+\beta 3\cdot \text{Awareness}+\beta 4\cdot \text{Costs})$
P(adoption) = $\Phi(-2.5 + 0.05\text{Income} + 0.01\text{Education} + 0.2\text{Awareness} + 0.15\text{Costs})$

2. Interpretation of coefficients:
   - **Intercept (-2.5):** is the probability of adoption when all other variables are 0.
   - **Income (0.05):** Higher income **increases** the probability of adoption.
   - **Education (0.10):** More years of education **increase** the probability of adoption.
   - **Awareness (0.20):** Greater environmental awareness **significantly increases** the probability.
   - **Costs (0.15): Higher electricity costs increase** the probability of adoption renewable energy
3. Key determinants of adoption: is **Awareness and costs**
4. Does higher household income increase adoption likelihood? **Yes.** The coefficient for income is positive and statistically significant (0.05, $p = 0.012$), indicating that higher income **increases** the probability of adoption.
5. Do rising electricity costs encourage or discourage adoption? **Encourage.** The coefficient is **positive** (0.15) and significant, meaning that higher electricity costs **increase** the probability of adoption —
6. Calculate the probability of adoption for given household: **Income = $70,000, Education = 10 years, Awareness = 5, Electricity Costs = $140**

$P = \Phi(-2.5+0.05\cdot 70+0.10\cdot 10+0.20\cdot 5+ 0.15\cdot 140) = \Phi(24) \approx 1$

This gives a **very high probability of adoption**, This household has a near-zero probability of adopting renewable energy.

7. Strongest influence & policy suggestion:
   - **Awareness (coefficient = 0.20)** has the **strongest positive effect**.

Recommendation:

- **Invest in awareness campaigns** and **educational programs** about environmental impact and renewable energy benefits.