University of Biskra
Mathematics Department
Module: Analysis 2

First year license
2023/2024

Worksheet N° 3

Remark: Treat only the ***first four examples*** of each exercise and leave the rest to the students.

Exercise 1 *(Separated variables equations)* Find the solution of the following equations and, if possible, express your solution in the form $y = f(x)$.

$y' = \frac{e^{x-y}}{1+e^x}$ *with* $y(1) = 0$ and $y' = \frac{x^2y-y}{1+y}$ *with* $y(3) = 1$.

- $y' - 2xy = 2x$
- $y' = yx^2 - 2x^2$
- $xy + y' = y^2\ln(x)$
- $y' = xy^2 - x - y^2 + 1$
- $y' = xe^{x+y}$
- $y' = \frac{1+y^2}{1+x^2}$
- $y' = \frac{y^2+1}{xy+y}$
- $y\ln(x)y' = \frac{y^2+1}{x}$
- $\ln(x)y' = \frac{y}{x}$
- $(2xy^3 + 4x)y' = xy^2 + y^2$
- $(xy + y)y' = x - xy$
- $y' = \frac{\sin(1/x)}{x^2\cos(y)}$

Exercise 2 *(Linear differential equations)* Solve the following first order differential equations

- $xy' - 2y = -x$
- $xy' - 2y = x^3e^x$ with $y(1) = 0$
- $xy' - y = 2x\ln(x)$
- $xy' + 2y = \frac{cos(x)}{x}$
- $(x+1)y' + 2y = (x+1)^{5/2}$
- $y' + y\ tan(x) = \cos^2(x)$
- $y' + 2\frac{y}{x} = \frac{4}{x}$ with $y(1) = 6$
- $xy' - 2y = x^4e^x$
- $xy' + y = (1+x)e^x$

Exercise 3 *(Homogeneous equations)* Show that each of the following differential equations is homogeneous and find the general solution of the equation.

- $y' = \frac{x^2e^{y/x}+y^2}{xy}$
- $y' = \frac{\sqrt{x^2-y^2}+y}{x}$
- $y' = \frac{x^4+2y^4}{xy^3}$

Exercise 4 *(Bernoulli's equations)* Solve the following Bernoulli's differential equations:

- $y' + \frac{y}{x} = 3x^2y^2$
- $y' - 4y = 2e^x\sqrt{y}$
- $y' - \frac{3}{4}y = (9x-3)y^5$
- $3y' + \frac{3}{x}y = 2x^2y^4$

Exercise 5 *(Riccati's equations)* Solve the following differential equations:

- $x^3y' + y^2 + yx^2 + 2x^4 = 0$
- $y' + \frac{y}{x} = 3x^2y^2 + xe^x$
- $(x^2+1)y' = y^2 - 1$
- $(y' - y^2)\cos x + y(2\cos^2 x + \sin x) = \cos^3 x$
- $2xyy' = 1 + y^2 + cos(x)$

Exercise 6 *(Second order differential equations)* Solve the following differential equations :

- $y'' - 3y' + 2y = 0$
- $y'' - 2y' + y = 0$
- $y'' + y = 2\cos(x)$
- $y'' - 3y' + 2y = xe^x$
- $y'' + yy' + y^2 = xy^3$
- $y'' + 2y' + 2y = 0$
- $y'' + y = 2\cos(x)$
- $y'' - 3y' + 2y = \sin(x) + 2e^{-x} + xe^{-2x} + x^2 + x + 1$
- $y'' - 2y' + y = x^2e^x$
- $y'' - 6y' + 9y = x^3e^{3x}$*(by variation of the constant)*