

Worksheet N° 1 & 2

Exercise 1

1. Calculate the following:

$$\begin{array}{lll} 1) \int 2dx & 2) \int x^2 + 2x - 1dx; & 3) \int \frac{\theta^2+3}{\sqrt{\theta}}d\theta; \\ 4) \int xy^2 + y^2\sqrt[3]{y}dy; & 5) \int x^2 + \sqrt{x} + \frac{1}{x^4\sqrt{x}}dx; & 6) \int \sin(\theta) - \cos(\theta)d\theta. \end{array}$$

2. Find the real function f that satisfying the following conditions

$$\begin{array}{ll} 1) f'(x) = 3x^2 + e^{-x} \quad \text{and } f(0) = 1; & 3) f''(x) = \sin(x) - e^{-2x}, \quad f'(0) = \frac{5}{2} \text{ and } f(0) = 0; \\ 2) f'(x) = \sqrt[3]{x^2} - \frac{1}{x^2} \quad \text{and } f(1) = 3; & 4) f''(x) = \sin(x) - \cos(x), \quad f'(0) = 0 \text{ and } f(\frac{\pi}{2}) = 0. \end{array}$$

Exercise 2 (by substitution) Find the antiderivatives of the following functions:

$$f(x) = \tan(x), \quad f(x) = \sin^3(x) \cos^2(x), \quad f(x) = \frac{\sin(x)\cos(x)}{(1+\cos(2x))^2}, \quad f(x) = \frac{\arcsin(x)}{\sqrt{1-x^2}},$$

$$f(x) = \frac{x}{x^4+a^2}, \quad f(x) = \frac{1}{x \ln(x)}, \quad f(x) = xa^{x^2}, \quad f(x) = \frac{(a^x-b^x)^2}{a^xb^x} \text{ with } a, b > 0.$$

$$\int \sin^{2p+1}(x) \cos^q(x)dx \text{ and } \int \sin^p(x) \cos^{2q+1}(x)dx, \text{ with } p, q \in \mathbb{N}$$

$$\int \frac{1}{x \ln^n(x)} \text{ with } n \in \mathbb{Z}^*$$

Exercise 3 (by parts) Let $n \in \mathbb{N}^*$. Calculate the following using integration by parts.

$$\begin{array}{llll} 1. \bullet \int \arctan(x)dx, & \bullet \int \arcsin(x)dx, & \bullet \int \frac{\arcsin(\sqrt{x})}{\sqrt{x}}dx, & \bullet \int \frac{x \arctan(x)}{(1+x^2)^2}dx \\ 2. \bullet \int \ln(x)dx, & \bullet \int x \ln(x)dx, & \bullet \int (\ln(x))^2 dx, & \bullet \int x^n \ln(x)dx \\ 3. \bullet \int x^2 e^{-x}dx, & \bullet \int x^n e^{-x}dx, & & \\ 4. \bullet \int \sin(x)e^x dx, & \bullet \int \cos(\beta x)e^{\alpha x}dx, & \bullet \int \sin(\beta x)e^{\alpha x}dx, & \\ 5. \bullet \int \sqrt{a^2 - x^2}dx, & \bullet \int \cos^2(x)dx, & \bullet \int \sin^4(x)dx & \end{array}$$

Exercise 4 (Rational and irrational functions) Calculate the following integrals

$$\begin{array}{llll} \bullet \int \frac{x+1}{x^2+x+2}dx, & \bullet \int \frac{x+1}{x^2+x-2}dx, & \bullet \int \frac{x+1}{(x-1)(x-2)}dx, & \bullet \int \frac{x+1}{(x-1)^2(x-2)}dx, \\ \bullet \int \frac{x^3+2x^2+3x-1}{(x^2+1)(x^2-4)}dx, & \bullet \int \frac{x^5+x^4-8}{x^3-4x}dx, & \bullet \int \frac{x+1}{\sqrt{(x-1)(x-2)}}dx, & \bullet \int \frac{x+1}{\sqrt{(1-x)(x-2)}}dx, \\ \bullet \int \frac{x^3+2x^2+3x-1}{x^2+2x+1}dx, & \bullet \int \frac{1}{2 \cos^2(x) + \cos(x) \sin(x) + \sin^2(x)}dx & \bullet \int \ln(x^2 + 2x - 3)dx, & \\ \bullet \int \frac{1}{(x^2-4)^2}dx, & \bullet \int \frac{1}{(x^2+4)^2}dx, & \bullet \int \frac{1}{(x^2-4)^4}dx, & \bullet \int \frac{1}{(x^2+4)^4}dx, \end{array}$$

$$\int \frac{2x^{1/2} + 3x^{1/4}}{1 + x^{1/4}} dx, \quad \bullet \int \frac{2x^{1/2} + 3x^{1/3}}{1 + x^{1/3}} dx, \quad \bullet \int \sqrt[3]{5x-1} dx, \quad \bullet \int \frac{dx}{\sqrt{\sqrt{x}-2}}, \quad \bullet \int \sqrt[3]{\frac{x+1}{x-1}} dx.$$

Exercise 5 Calculate the following integrals:

1. $\int_0^{+\infty} e^{-x} dx$ and $\int_0^{+\infty} x^2 e^{-x} dx$.

2. $\int_1^e \ln(x) dx$ and $\int_e^{2e} x^2 \ln(x) dx$.

3. $\int_0^{2\pi} \sin(x) \cos^2(x) dx$ and $\int_0^{\frac{\pi}{2}} \sin(x) e^x dx$.

Exercise 6 Let's consider the function f defined by:

$$f(x) = \begin{cases} \alpha \left(\frac{1}{x}\right)^{\alpha+1} & \text{if } x > 1 \\ 0 & \text{else.} \end{cases}$$

With α is a positive real number.

Question: Discuss the existence of the integral $\int_a^{+\infty} x^n f(x) dx$ according to n with $n \in \mathbb{N}^*$.

Exercise 7 (*Leave the exercise to the students*)

Using the definite integration, show that the surface delimited by a circle of radius R and center (x_0, y_0) is πR^2 .