UNIVERSITY MED KHIDDER BISKRA **08/02/2025**
DEPARTMENT OF COMPUTER SCIENCE
MODULE = Graph Theory.

tutorial session N°I.

Exercise N°I

We have six wagons to sort. In the sorting yard, the wagons enter in the order 2, 5, 3, 6, 1, 4 and must exit in ascending order. Two wagons i and j can be placed on the same track if and only if they enter in the order in which they are supposed to exit.

Draw a graph illustrating the situation, indicating what the vertices and edges of your graph represent. What will be the minimum number of tracks needed for sorting?

Exercise N°II

Three teachers P1, P2, and P3 will have to give a certain number of class hours next Monday to three classes C1, C2, and C3:

- P1 must give 2 hours of class to C1 and 1 hour to C2.
- P2 must give 1 hour of class to C1, 1 hour to C2, and 1 hour to C3.
- P3 must give 1 hour of class to C1, 1 hour to C2, and 2 hours to C3.

How can this situation be represented by a graph? What type of graph do you obtain?

How many time slots will be needed at a minimum?

Exercise N°III

A chess tournament involves 6 people. Each player must face all the others. Construct a graph representing all the possible matches.

- What type of graph do you obtain?
- If each player only plays one match per day, how many days will it take to complete the tournament?
- Use the graph to propose a match schedule.

Exercise N°IV

On a 3x3 chessboard, the two black knights are placed on squares a1 and c1, while the two white knights occupy squares a3 and c3.

Use a graph to determine the alternating moves of the whites and the blacks that will allow the white knights to take the places of the black knights, and vice versa. The whites start.

Exercise N°V

Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of edges.

Show that a simple graph has an even number of vertices with odd degree?

Show that in an assembly of n people, there are always at least 2 people who have the same number of present friends?

Exercise N°VI

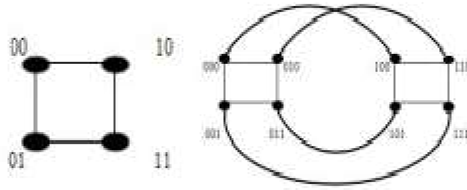
Let Sn = (X, U) be an undirected graph.

An Sn consists of a set of nodes V(Sn) = {(u1, u2,…, un) : ui ∈ {1, 2, …, n} and ui ≠ uj, with i ≠ j}, and there exists a connection ((u1, u2,…, un) (v1, v2,…, vn)) if and only if: u1 = vi, ui = v1, ∀ i ∈ {1, 2, …, n} with ul = vl, ∀ l ∈ {2, 3…, n}. For Sn, we take n copies of Sn-1, and then connect these copies by applying the following rule: A node u = u1, u2,… ui, ui+1…., un is connected to the nodes v = ui, …, u2, u1, ui+1…., un / 2 ≤ i ≤ n.

Provide the graphical representation of S4?

Exercise N°VII

Provide the definition of the graph G represented by the following recursive composition:



Give some relational characteristics of G and justify your statements?