<u>Pratical work</u>

<u>Exercise 1</u>

The model that show the relationship between the variable under study is

$$sales = \beta_0 + \beta_1 \ln \text{(Advertising\_expenditure)}$$

Transform the nonlinear model into a linearized form suitable for estimation.

Sales=c(1)+c(2)*log(Advertising_expenditure)

The estimation of β_0 and β_1 is as in the following table

Dependent Variable: SALES
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 02/10/25 Time: 13:41
Sample: 1 10
Included observations: 10
SALES=C(1)+C(2)*LOG(ADVERTISING_EXPENDITURE)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.017230	0.239227	8.432284	0.0000
C(2)	6.093195	0.182665	33.35714	0.0000

R-squared	0.992862	Mean dependent var	9.550000
Adjusted R-squared	0.991969	S.D. dependent var	2.786176
S.E. of regression	0.249681	Akaike info criterion	0.239591
Sum squared resid	0.498725	Schwarz criterion	0.300108
Log likelihood	0.802046	Hannan-Quinn criter.	0.173204
F-statistic	1112.699	Durbin-Watson stat	0.440913
Prob(F-statistic)	0.000000		

$$sales = 2.017 + 6.093 \ln (Advertising\_expenditure)$$

The p-value of 0.0000 indicates that the coefficient c(2) is **statistically significant**. This provides very strong evidence against the null hypothesis (H0:β_1 =0), confirming that there is a strong relationship between advertising and sales.

The coefficient β_1 = 6.093 means that a 1% increase in advertising leads $\frac{6.093}{adv}$. unit increases in sales

Predict the sales when the advertising expenditure is X= 0 or 10.

$sales = 2.017 + 6.093 \ln (0)$ the sales is undefined

$$sales = 2.017 + 6.093 \ln(10) \approx 16.05$$

Exercise 2

1-Which nonlinear model is appropriate to test this relationship.

The problem suggests an **exponential power effect** of inflation on GDP growth, meaning the relationship could be modeled as:

$$Y = \beta_0 e^{\beta_1 X} \qquad Gdp - growth = \beta_0 e^{\beta_1 inflation}$$

2-Estimate the parameters $\boldsymbol{\beta_0 and\ \beta_1}$using nonlinear regression.

To estimate the exponential growth model in E-views we write this equation:

$$gdp_{growth} = c(1) * \exp\,(C(2) * inflation)$$

Dependent Variable: GDP_GROWTH
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 02/10/25 Time: 14:04
Sample: 1980 2023
Included observations: 10
Convergence achieved after 44 iterations
Coefficient covariance computed using outer product of gradients
GDP_GROWTH=C(1)*EXP(C(2)*INFLATION)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.362488	1.350473	1.749379	0.1183
C(2)	-0.042016	0.138598	-0.303148	0.7695

R-squared	0.025647	Mean dependent var	2.010000
Adjusted R-squared	-0.096147	S.D. dependent var	2.297559
S.E. of regression	2.405477	Akaike info criterion	4.770230
Sum squared resid	46.29054	Schwarz criterion	4.830747
Log likelihood	-21.85115	Hannan-Quinn criter.	4.703843
Durbin-Watson stat	2.032607		

$$Gdp - growth = 2.36e^{-0.042 inflation}$$

3-Interpret the results

β_0=2.36: This is the coefficient represent the GDP growth when inflation is zero. In this case, when inflation is 0%, the model predicts that GDP growth will be 2.36%.

β_1=−0.042: This is the coefficient for the inflation variable. The negative sign indicates that **as inflation increases**, GDP growth decreases. Specifically, for every **1% increase in inflation**, GDP growth **decreases by 0.042% from its current value over time**. since the p-value for inflation is greater than 5%, **the effect of inflation on GDP growth** in your model is not statistically significant.

4-How does GDP growth respond to an increase in inflation from 2% to 8%?

$Gdp - growth = 2.36e^{-0.042*(0.02)}$ *=2.358*

$Gdp - growth = 2.36e^{-0.042*(0.08)}$ *=2.352*

When inflation move from 2% to 8% (increase) the GDP grow decreases by 0.006 or 0.6%

Predict **GDP growth** if inflation reaches **10%**.

If inflation =10% then, GDP=2.350

Exercise 3

1-Write the model that represent the relationship between the variable under study

$$return = \beta_0 + \beta_1 \ln(Market_{cap})$$

2-Estimate the parameters **β_0** and **β_1**.
Estimate the equation

$$annual\_return = c(1) + c(2)\ln(Market_{cap})$$

Dependent Variable: ANNUAL_RETURN
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 02/10/25 Time: 16:13
Sample: 1994 2023
Included observations: 8
ANNUAL_RETURN=C(1)+C(2)*LOG(MARKET_CAP)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	103.6691	7.999734	12.95907	0.0000
C(2)	-12.26504	1.554146	-7.891818	0.0002

R-squared	0.912128	Mean dependent var	48.87500
Adjusted R-squared	0.897482	S.D. dependent var	35.10050
S.E. of regression	11.23862	Akaike info criterion	7.888907
Sum squared resid	757.8396	Schwarz criterion	7.908768
Log likelihood	-29.55563	Hannan-Quinn criter.	7.754957
F-statistic	62.28080	Durbin-Watson stat	0.887975
Prob(F-statistic)	0.000219		

$$return = 103.67 - 12.27\ln(Market_{cap})$$

3-Interpret the results

Intercept (103.67): This represents the estimated return when **Market Cap = 1**.

The negative coefficient (-12.27) means that as **market capitalization increases**, the return **decreases**. And, since the **p-value is less than 5%**, the coefficient **-12.27 is statistically significant**. This means:There is **strong evidence** that market capitalization has a significant effect on returns. Specifically, for a **1% increase in market capitalization**, the return decreases by approximately **12.27/market cap unit.** The absolute decrease gets smaller relative to the increase in market capitalization.

4-How does Apple's market cap affect its annual stock returns?

The Apple's market cap affect annual stock return negatively.

Predict Apple's **stock return** if its **market cap** reaches **$4 trillion**.

$$return = 103.67 - 12.27 \ln(4) = 86.66$$

Does the **logarithmic model** explain why Apple's **growth rate slowed over time**?

Yes it does, **β_1<0** return decreases as market capitalisation increases (decrease @decreasing rate).

A **logarithmic model** assumes that as a company grows larger, its growth rate slows down. This fits well with **Apple's trajectory**:

Exercise 4

1-Write the model that represent the relationship between the variables.

$$annual\ revenue = \beta_0 customers^{\beta_1}$$

2-Use the logarithmic transformation to perform a linear regression

$$\ln(annual\ revenue) = \ln(\beta_0) + \beta_1 \ln\ (customer)$$

3- Estimate the parameters β, Interpret the estimated β (discuss what the value of β indicates about the relationship between customer count and revenue)

Based on the value in the table below, we conclude that: $\ln(\beta_0) = 0.17$ means $\beta_0 = 1.187$

$$annual\ revenue = 1.187 customers^{1.014}$$

Interpretation:

The coefficient **1.187** represents the **baseline level of revenue** when the number of customers is **one**. That is: If there is **only 1thousand customer**, the annual revenue would be **1.187 units** (million dollars).

The exponent **1.014** indicates the **elasticity of revenue with respect to customers**: Since the exponent is **slightly above 1**, it suggests a **more-than proportional** relationship: If the number of customers **doubles**, annual revenue increases by slightly **more than double**. If the number of customers increases by **1%**, revenue increases by **about 1.014%** (showing mild increasing returns to scale).

Dependent Variable: LOG(ANNUAL_REVENUE)
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 02/10/25 Time: 17:02
Sample: 1 10
Included observations: 10
LOG(ANNUAL_REVENUE)=C(1)+C(2)*LOG(CUSTOMERS)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.171500	0.028717	5.972155	0.0003
C(2)	1.014752	0.017270	58.75918	0.0000

R-squared	0.997688	Mean dependent var	1.704224
Adjusted R-squared	0.997399	S.D. dependent var	0.744699
S.E. of regression	0.037977	Akaike info criterion	-3.526802
Sum squared resid	0.011538	Schwarz criterion	-3.466284
Log likelihood	19.63401	Hannan-Quinn criter.	-3.593189
F-statistic	3452.642	Durbin-Watson stat	2.343292
Prob(F-statistic)	0.000000		

4-Use the estimated model, to predict the annual revenue for a branch serving 12,000 customers Show your calculation and interpret the result in practical terms

$$annual\ revenue = 1.187 * 12000^{1.014} = 16{,}245.78$$

5. Based on the model findings, discuss potential strategies for the company:

- If β<1 (diminishing returns), how should the company plan for new branch openings or expansions?
- $annual\ revenue = 1.187 * 12000^{0.14}$ =3.725, if the elasticity is less than 1, the revenue doesn't grow with the opening of new branch, the revenue grows at decreasing rate so, its better to don't open a new branch.