Mohamed Khidher University

Faculty of economics, Commercial and Management Sciences
Department of commerce

Specialty: International Commerce & Finance
Module: Advanced Econometric
Academic year 2024/2025

Exercise series N°1

Exercise 1

1. **Which of the following statements best describes a nonlinear model in statistical analysis?**
   a) A model in which the dependent variable is linearly related to the independent variables.
   b) A model that cannot be expressed as a straight line when plotted on a graph, often involving transformations or interactions of variables.
   c) A model in which all the independent variables are categorical.
   d) A model that exclusively uses polynomial functions to describe relationships between variables.
2. **A nonlinear model can be transformed into a linear model by:**
   a) Adding interaction terms.
   b) Applying mathematical transformations such as logarithms.
   c) Increasing the sample size.
   d) Removing independent variables.
3. **Which of the following is an example of a nonlinear model?**
   a) $Y=\beta_0+\beta_1X_1+\beta_2X_2$
   b) $Y=\beta_0e^{\beta_1X}$
   c) $Y=\beta_0+\beta_1X+\beta_2X^2$
   d) $Y=\ln(\beta_0)+\beta_1X$
4. **A key advantage of nonlinear models is:**
   a) They are always easier to interpret than linear models.
   b) They can capture complex relationships that linear models cannot.
   c) They do not require initial guesses for parameter estimation.
   d) They always have higher predictive accuracy than linear models.
5. Which of the following statements correctly differentiates between the **logarithmic model, exponential growth model, and power law model**?
   a) In a **logarithmic model**, the dependent variable increases at a decreasing rate as the independent variable increases, and the model takes the form $Y=\beta_0+\beta_1\ln X$.
   b) In an **exponential growth model**, the dependent variable grows at a constant percentage rate relative to its current value, and the model takes the form $Y=\beta_1e^{X}$.
   c) In a **power law model**, the relationship follows a proportional scaling rule, meaning a percentage increase in X leads to a fixed percentage change in Y, and the model takes the form $Y=\beta_0X^{\beta_1}$.
   d) All of the above are correct.

6. **Which model would be most appropriate for studying the relationship between years of education and income growth over time?**
   a. Logarithmic Model
   b. Exponential Growth Model
   c. Power Law Model
   d. None of the above
7. **Which model is best suited for analyzing a situation where the population of a city doubles every 10 years?**
   a. Logarithmic Model
   b. Exponential Growth Model
   c. Power Law Model
   d. None of the above
8. **Which of the following models represents a logarithmic relationship between X and Y?**
   a) $Y = \ln(\beta_0) + \ln(\beta_1)\ln(X)$
   b) $Y = \beta_1 e^X$
   c) $Y = \beta_0 X^{\beta_1}$
   d) $Y = \beta_0 + \beta_1 \ln X$
9. **Which of the following statements best describes an exponential growth model?**
   a) A 1% increase in X leads to a **fixed percentage increase** in Y.
   b) A 1% increase in X leads to a **decreasing increase** in Y over time.
   c) A **fixed increase** in X leads to a **fixed increase** in Y.
   d) A **fixed increase** in X leads to a **decreasing percentage increase** in Y.
10. **You are analyzing the relationship between advertising expenditure and sales revenue. Initially, a small increase in advertising leads to a large increase in sales, but at higher levels of advertising, the increase in sales slows down. Which model is most appropriate?**
   a) $sales = \ln(\beta_0) + \ln(\beta_1)\ln(adv)$
   b) $sales = \beta_1 e^{adv}$
   c) $sales = \beta_0 adv^{\beta_1}$
   d) $sales = \beta_0 + \beta_1 \ln adv$
11. **In a logarithmic model of the form $Y = \beta_0 + \beta_1 \ln X$, what does the coefficient β_1 represent?**
   a) The absolute change in Y for a one-unit increase in X.
   b) The percentage change in Y for a one-unit increase in X.
   c) The change in Y associated with a **1% increase** in X.
   d) The exponential growth rate of Y.
12. **Which of the following real-world examples is best modeled using an exponential growth model $Y = \beta_1 e^X$?**
   a) The relationship between the length of a side of a square and its area.
   b) The population of a bacteria culture that doubles every hour.
   c) The relationship between advertising spending and sales, where initial spending has a large impact but diminishes over time.
   d) The relationship between the speed of a car and the
13. In economics, the **Cobb-Douglas production function** is often used to model output as a function of labor and capital. This function is given by: $Y = \alpha K^{\beta} L^{\gamma}$, which type of model does this equation represent?
   a) Logarithmic Model

b) Exponential Growth Model
c) Power Law Model
d) None of the above

14. **You are analyzing data on social media followers of a brand over time. Initially, the number of followers grows very quickly, but over time, the rate of growth slows down. Which model would best describe this trend?**
    a) Logarithmic Model
    b) Exponential Growth Model
    c) Power Law Model
    d) Linear Model
15. **The value of an investment grows continuously at a constant percentage rate over time. Which model best describes this scenario?**
    a) Logarithmic Model
    b) Exponential Growth Model
    c) Power Law Model
    d) Quadratic Model
16. **A company notices that each additional $1,000 spent on advertising results in a smaller increase in sales than the previous $1,000. Which model best describes this relationship?**
    a) Linear Model
    b) Logarithmic Model
    c) Exponential Growth Model
    d) Power Law Model
17. **Researchers found that as a city's population increases, the number of gas stations follows a power law. If the equation is: $G = \beta_0 P^{\beta_1}$ where *G* is the number of gas stations and *P* is the population, what does β_1 represent?**
    a) The **percentage change** in G when P increases by 1%.
    b) The **absolute change** in G when P increases by 1 unit.
    c) The **base number of gas stations** in the city.
    d) The **rate of decline** in gas stations over time.
18. **A researcher studies the relationship between employee experience (years) and productivity using a logarithmic model: Y=2+1.5ln(X) where:**
    Y is productivity (in efficiency units),
    - X is years of experience.

What is the estimated productivity for an employee with **8 years of experience**?

a) 2.5
b) 3.5
c) 4.7
d) 5.2

19. **The number of COVID-19 cases in a city follows an exponential growth model: $C_t = 100e^{0.1t}$ where:**

- Ct is the number of cases at time t (in days),
- t=0 corresponds to **Day 0** with **100 cases**.

Estimate the number of cases after **10 days**.

a) 200
b) 271
c) 500
d) 1,000

Case Study 1: E-Commerce Growth

An **e-commerce company** tracks the number of users visiting its website. The data shows **rapid initial growth**, which later slows down as the market reaches saturation.

Discussion Questions:

1. Based on this trend, which model (logarithmic, exponential, or power law) would best describe website traffic?
2. Suppose the company estimates a model V=200+50ln(T), where V is the number of visitors and T is time in months. Predict the number of visitors in **month 12**.
3. How would the interpretation change if the model was **exponential instead of logarithmic**?

Case Study 2: Energy Consumption & Population

Researchers are studying how **energy consumption** (E) scales with **population size (P)** in different countries. They estimate the following **power law model**: $E = 0.5P^{0.75}$.

Discussion Questions:

1. What does the exponent **0.75** tell us about the relationship between energy and population?
2. If Country A has **50 million** people and Country B has **100 million**, does Country B use **twice as much** energy?
3. Why might a **power law** be more appropriate than a **linear model** for this relationship?

<u>Pratical Work</u>

<u>Exercise 2 :</u>

Problem Statement:
A company tracks the sales *Y* (in thousands of dollars) of a product based on advertising expenditure *X* (in thousands of dollars). The relationship between sales and advertising is believed to follow the nonlinear model,The company collects the following data:

Advertising Expenditure (X)	Sales (Y)
1.5	4.8
2.0	6.3
2.5	7.5
3.0	8.5
3.5	9.4
4.0	10.2
4.5	11.0
5.0	11.9
5.5	12.6
6.0	13.3

Tasks:

1. Perform the model that show the relationship between these two variables?
2. Transform the nonlinear model into a linearized form suitable for estimation.
3. Estimate β_0 and β_1 ?
4. Predict the sales when the advertising expenditure is X= 0 or 10.

Exercise 3

The **Federal Reserve** is studying the relationship between **inflation** and **GDP growth** in the **United States**. Some economists believe that inflation has an **exponential power effect** on economic growth:

Year	Inflation (%)	GDP Growth (%)
1980	13.5	-0.3
1985	3.6	4.2
1990	5.4	1.9
1995	2.8	2.7
2000	3.4	4.1
2005	3.2	3.5
2010	1.6	2.5
2015	0.1	2.9
2020	1.2	-3.4
2023	4.5	2.0

1. Which nonlinear model is appropriate to test this relationship.
2. Estimate the parameters $\beta_0 and\ \beta_1$ using nonlinear regression.
3. Interpret the results .
4. How does **GDP growth** respond to an **increase in inflation** from 2% to 8%?
5. Predict **GDP growth** if inflation reaches **10%**.

Exercise 4: An investment analyst is studying the relationship between **Apple Inc.'s market capitalization** and its **stock market return** over the past **30 years**. The hypothesis is that as Apple grows, its returns **diminish**, following a **logarithmic model.** Dataset: Apple Inc. (1994-2023, 30 Years of Data) is as follow :

Year	Market Cap ($B)	Annual Return (%)
1994	3.0	110.2
1995	5.2	82.5
2000	8.5	65.7
2005	35.0	49.8
2010	190.0	34.2
2015	650.0	20.3
2020	2,000.0	15.8
2023	2,900.0	12.5

- Write the model that represent the relationship between the variable under study
- Estimate the parameters **β_0** and **β_1**.
  How does **Apple's market cap** affect **its annual stock returns**?
  Predict Apple's **stock return** if its **market cap** reaches **$4 trillion**.
  Does the **logarithmic model** explain why Apple's **growth rate slowed over time**?
  Compare Apple's **early growth phase (1994-2010)** to its **mature phase (2010-2023)**—has the relationship changed?

<u>Exercise 5 :</u>

A national telecommunications company wants to understand whether larger branch customer bases yield proportionally higher revenue or if the relationship follows a scaling law. Preliminary analyses suggest that revenue and customer count follow a **power law relationship**. The company has collected data from 10 of its branches over the past fiscal year.

Dataset (10 Branches of the Company)

Branch ID	Customers (in thousands)	Annual Revenue (million USD)
1	1	1.2
2	2	2.5
3	3	3.3
4	4	4.8
5	5	6.2
6	6	7.5
7	7	8.6
8	8	9.8
9	9	11.0
10	10	12.3

1. Write the model that represent the relationship between the variables.

2. Use the logarithmic transformation to perform a linear regression

3. Estimate the parameters β, Interpret the estimated β (discuss what the value of β indicates about the relationship between customer count and revenue)

4. Use the estimated model, to predict the annual revenue for a branch serving 12,000 customers Show your calculation and interpret the result in practical terms

5. Based on the model findings, discuss potential strategies for the company:

- If $\beta<1$ (diminishing returns), how should the company plan for new branch openings or expansions?