

## Review

### I. Basics of Regression Analysis

- **Objective of Regression:** help in estimating relationships between dependent and independent variables.
- **General formula of regression:**  $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$
- **Key Concepts:**
  - Coefficients and their interpretation.
  - $R^2$  and F-statistics are used to check the Goodness of fit of the model.

### 1. Interpretation of Coefficients

#### 1.1 Intercept ( $\beta_0$ ):

- The intercept is the expected value of Y when all X-variables are equal to zero.
- **Example:** If  $\beta_0=5$ , it means that when all X-variables are zero, the predicted Y is 5.

#### 1.2 Coefficients of Independent Variables ( $\beta_1, \beta_2, \dots, \beta_k$ ):

- The coefficient for an independent variable represents the **change in the dependent variable** for a **one-unit increase** in that independent variable, holding all other variables constant.

#### Example 1: Simple Linear Regression $Y=2+3X$

- Here,  $\beta_1=3$ .
- Interpretation: For every **1-unit increase in X**, Y increases by **3 units**.

### 2. Understanding Statistical Significance

- Statistical significance is determined using **hypothesis testing** for each coefficient.
- **Null Hypothesis ( $H_0$ ):** The coefficient ( $\beta$ ) is equal to 0 (X has no effect on Y).
- **Alternative Hypothesis ( $H_1$ ):** The coefficient ( $\beta$ ) different from 0 (X has an effect on Y).

### 3. Key Metrics to Evaluate Significance

#### 3.1 p-Value

- The p-value represents the probability of observing the data if  $H_0$  is true.
- **Decision Rule:**
  - If  $p\text{-value} < \alpha$  (e.g., 0.05), reject  $H_0$ ; the variable is significant.
  - If  $p\text{-value} \geq \alpha$ , fail to reject  $H_0$ ; the variable is not significant.
- **Example:**
  - A p-value of 0.03 indicates that the variable is significant at the 5% significance level.
  - A p-value of 0.15 indicates that the variable is not significant.

#### t-Statistic

- Measures how many standard errors the coefficient is away from 0.

- $t = \frac{\hat{\beta}}{SE\hat{\beta}}$ 
  - $\hat{\beta}$ : Estimated coefficient.
  - $SE\hat{\beta}$ : Standard error of the coefficient.
- **Decision Rule:**
  - Compare t-statistic to a t-statistic critical -critical value (based on the chosen confidence level and degrees of freedom).
- **Rule of Thumb:** If  $|t| > t\text{-critical}$ , the variable is typically significant at the 5% level otherwise the variable has no effect (insignificant).

## II. Linear vs. Non-Linear Models

- **Linear Models:**
  - Assumption: The relationship between variables is linear.
  - Example: OLS regression.
- **Non-Linear Models:**
  - When linear models are inappropriate (e.g., when the dependent variable is binary).
  - Transition into Probit and Logit models (non-linear model).

## III. Types of Variables by Nature

### 1. Dependent Variables (Response Variables):

- **Continuous Variable:**
  - Measured on a continuous scale (e.g., income, temperature, weight).
- **Binary Variable:**
  - Takes two possible values (e.g., 0 or 1, Yes or No).
  - Example: Used in Probit or Logit models to represent decisions like purchase (Yes/No) or loan approval (approved/Denied).
- **Categorical Variable (Multinomial):**
  - Represents multiple categories without a natural order.
  - Example: Choice of a product (Product A, B, or C).
- **Ordinal Variable:**
  - Represents categories with a meaningful order.
  - Example: Customer satisfaction (Low, Medium, High).

### 2. Independent Variables (Explanatory Variables):

- **Continuous Variable:**
  - Example: Age, income, hours of study.
- **Categorical Variable:**
  - Example: Gender (Male/Female), region (North/South/East/West).
- **Dummy Variable (Binary):**
  - A special categorical variable coded as 0 or 1.
  - Example: 1 for "Urban" and 0 for "Rural."
- **Interaction Variable:**

- A product of two variables to study their combined effect.
- Example: Income  $\times$  Education level.

#### IV. Why OLS Fails for Binary Data:

- Predicted probabilities may lie outside [0,1].
- Inefficiency and incorrect assumptions about errors, for that we use probability model (**Probit and Logit Models**)

### 3. Panel Data

- **What is Panel Data?:**

- Combines cross-sectional and time-series data.
- Example: Tracking income and purchasing behavior of individuals over several years.

- **Key Concepts:**

- Fixed effects vs. Random effects.
- Importance of controlling for unobserved heterogeneity.

### 6. Mathematical Foundation

- **Key Equations to Review:**

- Linear regression:  $Y = \beta_0 + \beta_1 X + \epsilon$
- Probit:  $\Pr(Y=1) = \Phi(\beta_0 + \beta_1 X)$ , where  $\Phi$  is the standard normal CDF.
- Logit:  $P(Y = 1) = \frac{e^{\beta_0 + \beta_1 X_1 + \epsilon}}{1 + e^{\beta_0 + \beta_1 X_1 + \epsilon}}$ .
- Panel data regression:
  - Fixed effects:  $Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$ .
  - Random effects:  $Y_{it} = \alpha + \beta X_{it} + v_i + \epsilon_{it}$ .

#### Exercise (1): choose the right answer

- What is the primary goal of a regression analysis
  - To summarize data using descriptive statistics
  - To model the relationship between dependent and independent variables
  - To perform hypothesis testing
  - To measure the central tendency of variables
- In a simple linear regression  $Y = \beta_0 + \beta_1 X + \epsilon$ , what does  $\beta_1$  represent?
  - The predicted value of Y
  - The mean of the dependent variable
  - The slope of the regression line or the effect of X on Y
  - The random error term
- Which of the following is NOT an issue in linear regression?
  - Multicollinearity
  - Heteroscedasticity
  - Model overfitting
  - Non-normal distribution of Y

5. Which of the following is an example of a binary dependent variable?
  - Annual income
  - Temperature (in Celsius)
  - Whether a customer purchased a product (Yes/No)
  - Number of ads clicked by a user
6. Why is Ordinary Least Squares (OLS) not ideal for binary dependent variables?
  - It requires non-linear relationships between variables
  - It assumes the dependent variable is continuous
  - It works only with panel data
  - It cannot handle large datasets
7. Which of the following models is most appropriate for analyzing binary dependent variables?
  - Linear Regression
  - ARIMA Model
  - Probit or Logit Model
  - Fixed Effects Model
8. A researcher is studying whether an individual will apply for a loan (Yes/No) based on income and education level. Which regression model should they use?
  - Probit or Logit
  - Simple Linear Regression
  - Time Series Analysis
  - Panel Data Model
9. What is panel data?
  - Data collected at one point in time for multiple entities
  - Data collected over time for a single entity
  - Data collected for multiple entities over multiple time periods
  - Data used only for time-series models
10. Which of the following is an example of panel data?
  - Monthly sales figures of a company over 5 years
  - Annual GDP of 10 countries in 2020
  - Grades of 100 students across 3 semesters
  - Daily stock prices of a company
11. What is one major advantage of using panel data models?
  - It requires fewer observations
  - It captures both cross-sectional and time-series variations
  - It does not require assumptions about unobserved effects
  - It is easier to estimate than simple regression