Tutorial N°3: Exercises on spherical diopters and thin lenses.

Exercise 3.1:

A spherical diopter with top S and center C separating 2 media with indices n =1 and n'=4/3 has a radius of curvature |r|=4 cm.

1) Write the formulas of the spherical diopter without demonstration: conjugate formula, transverse magnification and focal lengths.

2) This diopter gives an image A'B' (p' = $\overline{SA'}$) of a real object AB (p = $\overline{SA}$) such that the magnification γ is equal to +2.

a- Calculate the distances p and p' and on a scale figure, place the points S, C, A and A'.

b- Calculate the focal lengths f and f'.

c- Is the diopter convergent or divergent; convex or concave? Place S, C, A , A', f and f' in axe xx'.

Solution

1- Conjugate formula: $\frac{n'}{p'} - \frac{n}{p} = \frac{(n'-n)}{r}$

Transverse magnification:$\gamma = \frac{n}{n'}\frac{p'}{p}$

Image focal length: $f' = \frac{n'r}{n'-n}$

Object focal length: $f = \frac{-nr}{n'-n}$

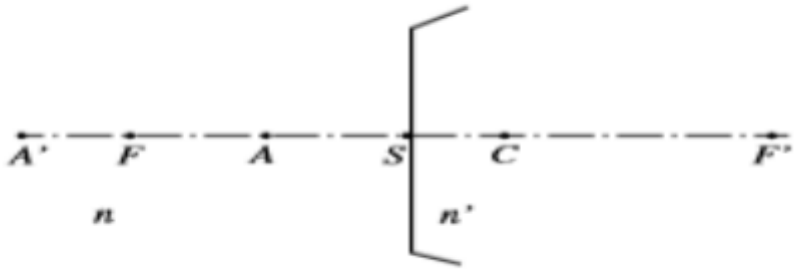
a) We obtain: $\gamma = 2 = \frac{3p'}{4p} \Longrightarrow p' = \frac{8}{3}p$

We replace: $p' = \frac{8}{3}p$ in the Conjugate formula, we find:

$$\frac{4}{3p'} - \frac{1}{p} = \frac{1}{3r} = \frac{1}{2p} - \frac{1}{p} = -\frac{1}{2p} \Longrightarrow p = -\frac{3r}{2}$$

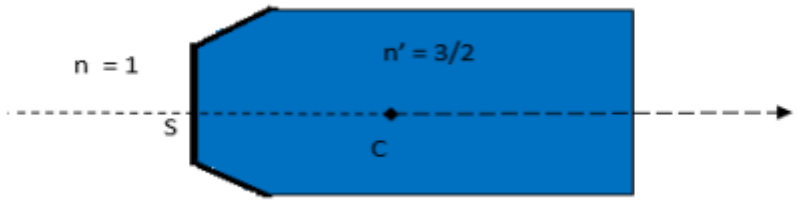
p is negative, the object is real and the image is virtual. $So\ r > 0, r = 4\ cm, p = -6\ cm\ and\ p' = -16cm.$

c) $\boldsymbol{f' = 16cm\ and\ f = -12cm}$ **.** The diopter is convergent and convex.



Exercise 3.2:

A spherical diopter with a radius of curvature of 10 cm separates two media with indices n =1 and n'=3/2.



Determine the position of the focal lengths, Calculate and draw the position of the image of an object AB.

Place a:

a) 60 cm from the top and real;

b) 10 cm from the top and real;

c) 5 cm behind the diopter (virtual object).

Same questions if we reverse the indices

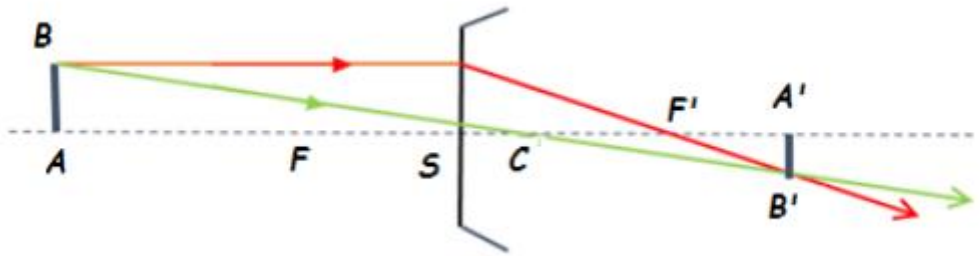
Solution

We suppose (p' =$\overline{SA'}$) and (p =$\overline{SA}$)

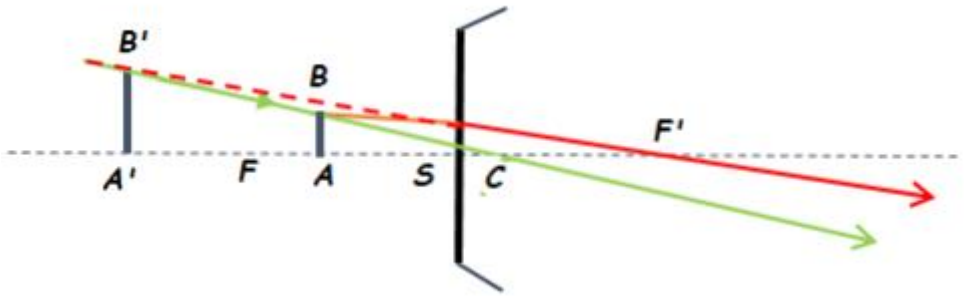
a) r is positive, the diopter is convergent. We then have $\overline{SF} = f = -2r = -20\ cm\ and$

$\overline{SF'} = f' = 3r = 30\ cm.$

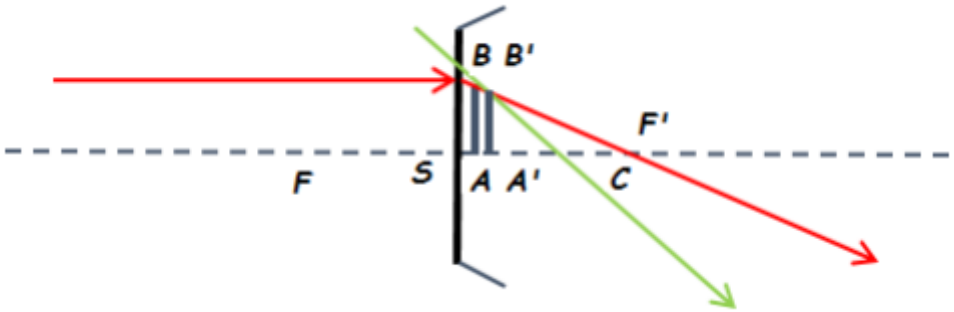
If $\overline{SA}$= −60 cm, $\overline{SA'}$ = 45 cm. The image is real and reversed



a) If $\overline{SA} = -10\ cm, \overline{SA'} = -30\ cm$. The image is virtual in the same side as the object.

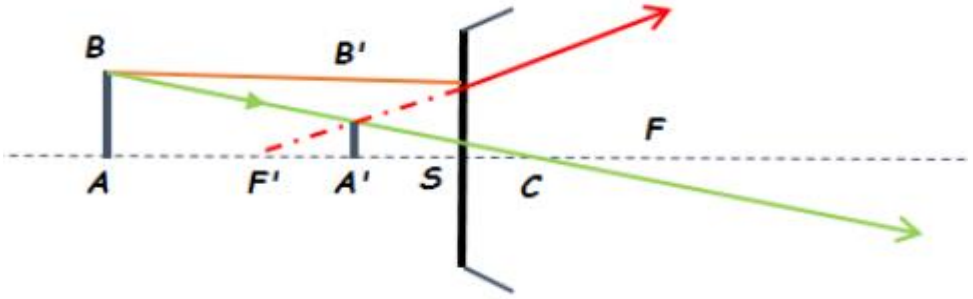


c) If p=5 cm, p' = 6 cm. The object is virtual and the image is real

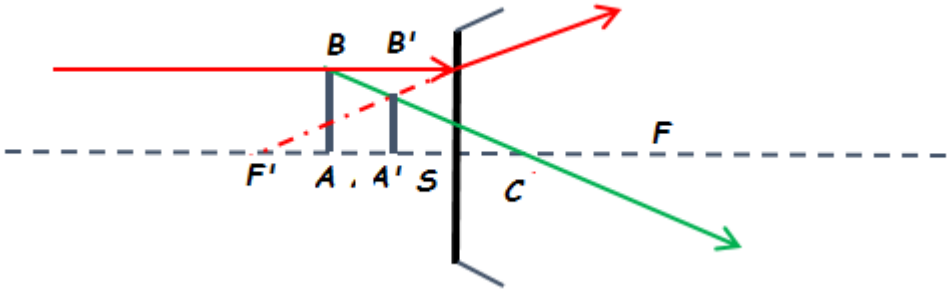


If we reverse the indices, f' =−20 cm and f =30 cm. The diopter is divergent.

a) If p= − 60 cm, p' =−13.33 cm. The object is real and the image is virtual in the same side as the object.



b) If p=−10 cm, p=−5 cm. The object is real and the image is virtual in the same side as the object.



c) If p=5 cm, p' =4 cm. The object is virtual and the image is real.

Exercise 3.3:

A lens forms an image of an object 20 cm away from it. The image is at 6 cm from the lens and on the same side as the object.

a) What is the focal length of the lens?

b) Determine the nature of the lens.

c) If the object is 0.4 cm in size, what is the size of the image?

d) Determine the nature of the image.

e) Make the diagram

Solution

a) $\overline{OA} = -20\text{cm}$ and $\overline{OA'} = -6\text{cm}$

Conjugate formula: $\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}} = \frac{1}{f'} given\ f' = -8.57cm$

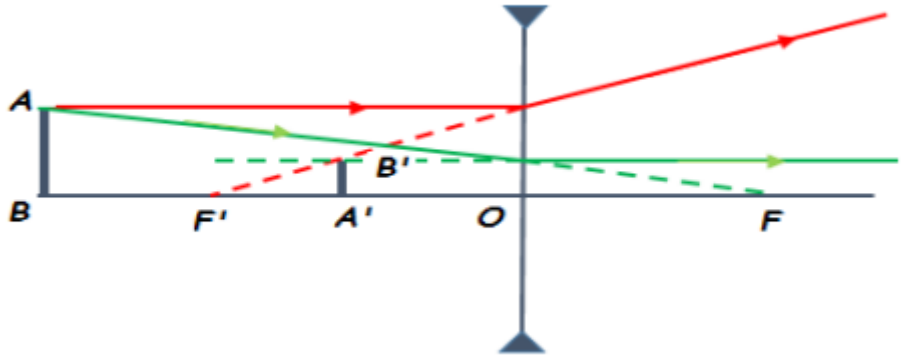
b) It is a divergent lens, $\overline{OF'} < 0$

c) The magnification is given by:

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}} = \frac{p'}{p} = 0.3, \overline{A'B'} = 0.12cm$$

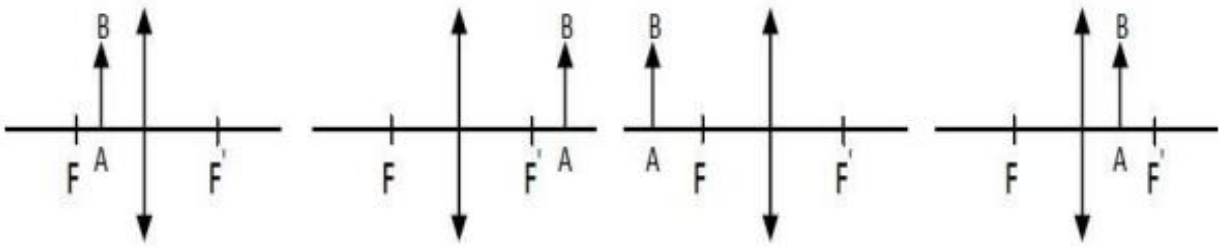
d) It is a virtual image, straight and reduced 0.3 times.

e) Diagram

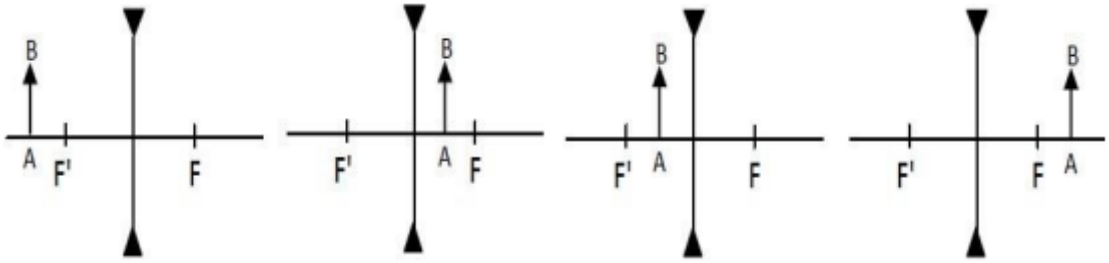


Exercise 3.4:

1. When the lens is convergent, complete the following constructions:



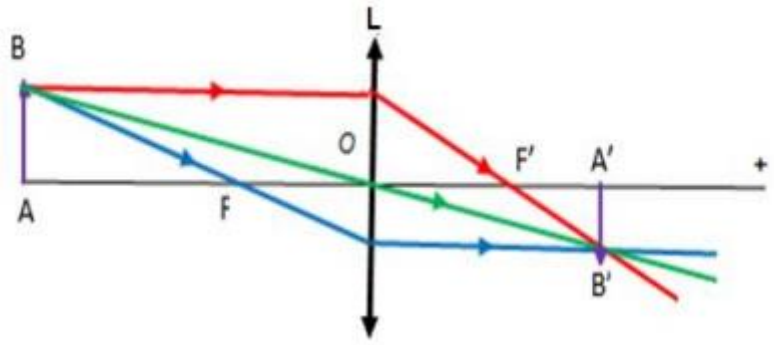
1. When the lens is divergent, complete the following constructions:



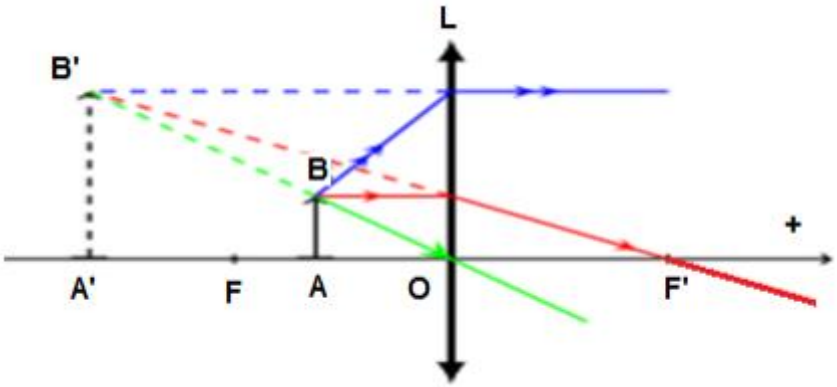
Solution

1. Construction of the image by a converging lens of an object

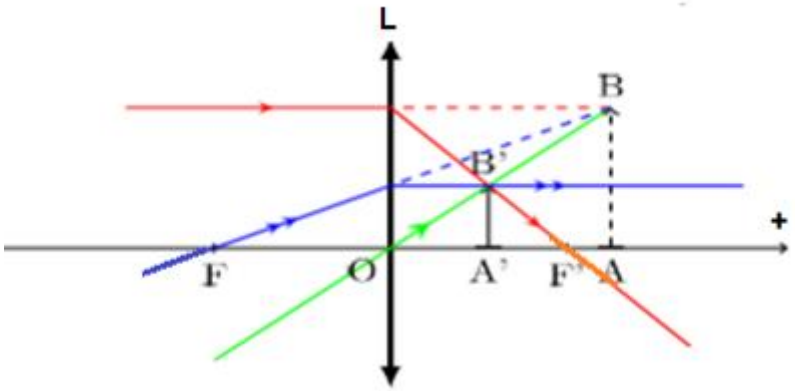
a- A real object, right reversed image



b- Real object, virtual image

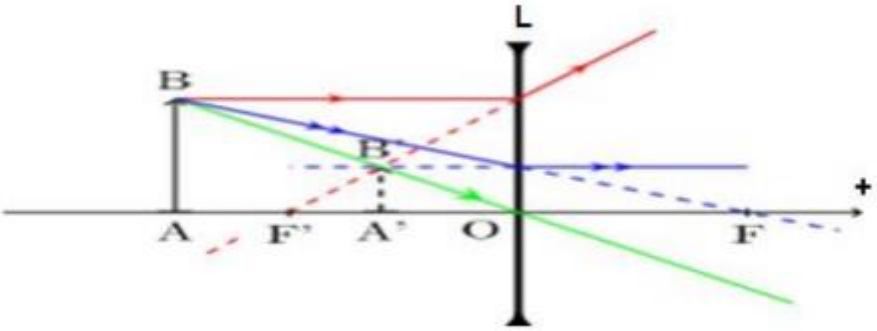


c- Virtual object, real image

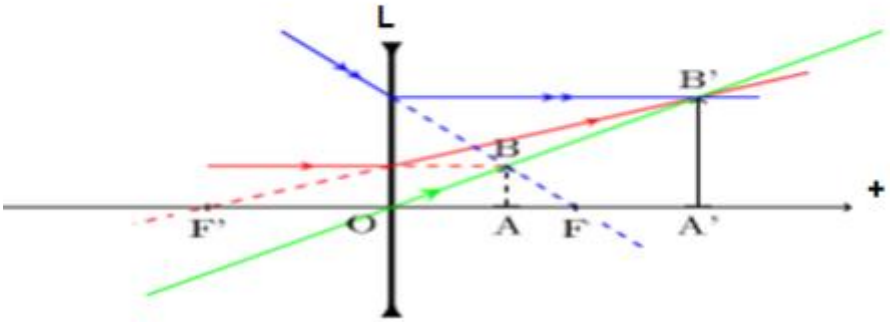


2. Construction of the image by a lens diverging from an object

a- A real object, right virtual image



b- A virtual object, real right image



c- A virtual object, reversed virtual image