Tutorial N°2: Exercises on the propagation of light, plane diopters and the prism.

Exercise 2.1 A photon of red light has a wavelength of $4.50x10^{-7}$ m. Calculate its frequency.

Known/Given:

speed of light, $c\ =\ 3.00\ x\ 10^8 m/s$

wavelength, $\lambda\ =\ 4.50\ x\ 10^{-7} m$

Equation:

$$C\ =\ f\ \times\ \lambda$$

Solution:

To solve for frequency, rearrange the equation so that

$f\ =\ c/\lambda.$

$f\ =\ (3.00\ x\ 10^8 m/s)/(4.50\ x\ 10^{-7} m)\ =\ 6.67\ x\ 10^{14}/s\ =\ 6.67\ x\ 10^{14} Hz$

The frequency of a photon of red light with wavelength $4.50\ x\ 10^{-7} m$ is $6.67\ x\ 10^{14} Hz.$

Exercise 2.2 A photon of green light has a frequency, ν of $5.75\ x\ 10^{14} Hz$. What is its wavelength, λ?

Known/Given:

speed of light, $c\ =\ 3.00\ x\ 10^8 m/s$

frequency, $f\ =\ 5.75\ x\ 10^{14} Hz\ =\ 5.75\ x\ 10^{14} s^{-1}$

Equation:

$$c\ =\ \lambda\nu$$

Solution:

To solve for wavelength, λ, rearrange the equation so that

$\lambda\ =\ c/f$

$\lambda\ =\ (3.00\ x\ 10^8 m/s)/(5.75\ x\ 10^{14} s^{-1})\ =\ 5.22\ x\ 10^{-7} m$

The wavelength of a photon of green light with frequency $5.75\ x\ 10^{14} Hz$ is $5.22\ x\ 10^{-7} m$

Exercise 2.3 Find the energy of an X-ray photon with a wavelength of $1.34x10^{-11} m.$

Known/Given:

speed of light, $c\ =\ 3.00\ x\ 10^8 m/s$

the Planck constant $h = 6.63.10^{-34} J.s$

Equation: $E_{Photon} = h\nu = \frac{hc}{\lambda}$

Solution:

$$E_{Photon} = \frac{hc}{\lambda} = \frac{(6.63.10^{-34} J.s)(3.10^{8} m/s)}{1.34.10^{-11} m} = 1.48.10^{-14} J$$

Exercise 2.4:

One of the rays of a beam of light propagating in air enters a diamond with a refractive index of 2.43.

a. Schematize the situation.

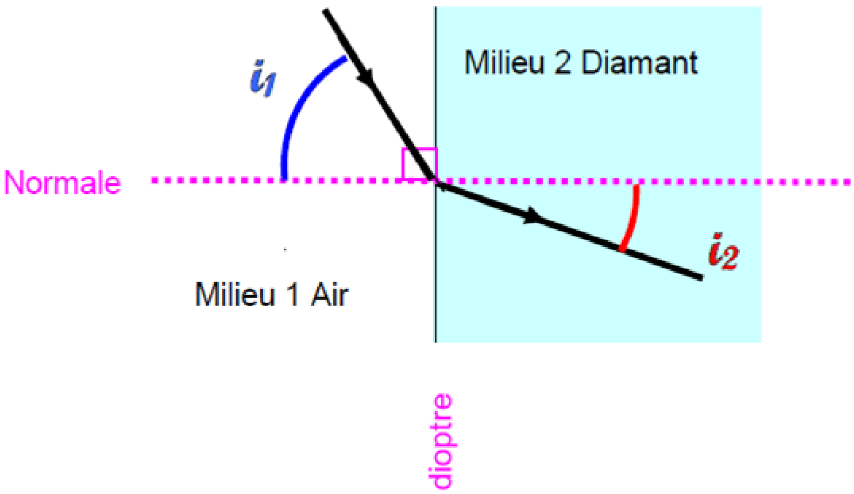
b. Write Descartes' second law.

c- Calculate the angle of incidence to obtain an angle of refraction of 20°.

Solution

a) Schematization

See diagram below



b- Second law of refraction Descartes $n_1 \sin i_1 = n_2 \sin i_2$

c- Calculation of the angle of incidence making it possible to obtain an angle of refraction $i_2 = 20°$ according to the previous law of Descartes

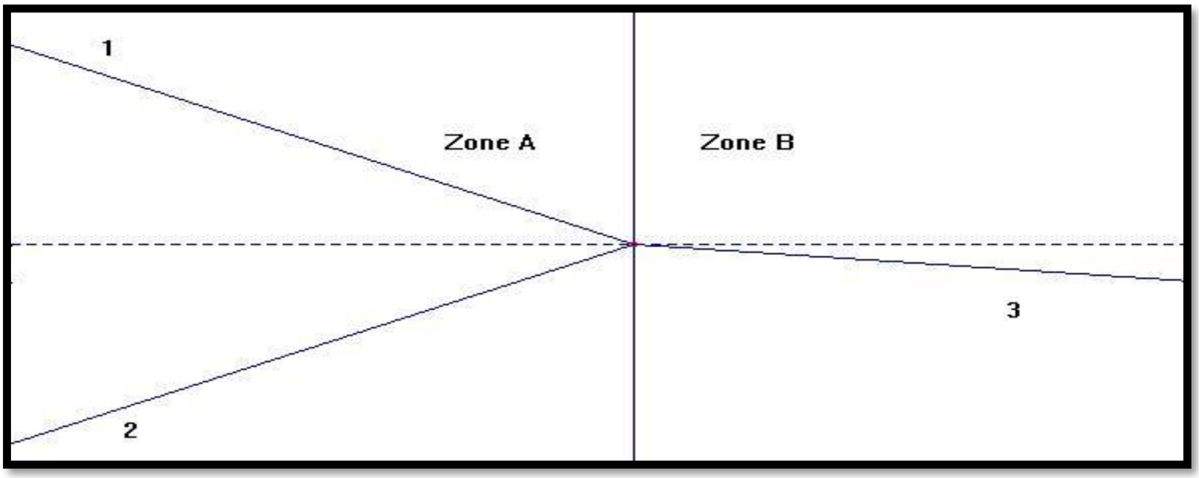
$$\sin \mathrm{i}_1 = \frac{n_2 \sin \mathrm{i}_2}{n_1} = \frac{2.43 \sin 20^0}{1} = 0.83$$

$$\mathrm{i}_1 = \sin^{-1} 0.83 = 56^0$$

We verify that the second medium being more refractive than the first, the deviation of the ray is such that the refracted ray approaches to the normal: $\mathrm{i}_2 = 20^0 < \mathrm{i}_1 = 56^0$.

Exercise 2.5:

A fine luminous brush arrives on a flat diopter separating water from air. We give $n_{water} = 1.33$. We represent the rays observed in the figure below:

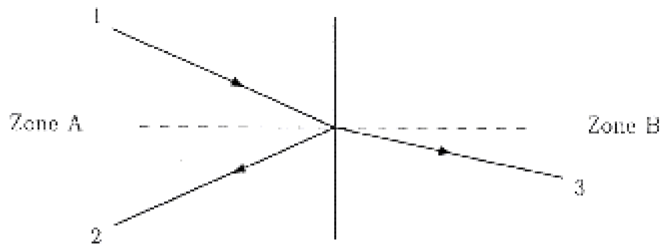


By justifying your answers:

1. Identify the different rays.
2. Indicate the direction of light propagation
3. In what zone is the water located.
4. Calculate the limiting angle of refraction
5. Generalize the result by specifying the zone where the limiting angle is located according to the difference in refraction of the media present and the consequences on the propagation of light from one medium to the other.

Solution

A - To every incident ray, there corresponds a reflected ray on the same side of the diopter, and in the other medium, a refracted ray. The reflected ray and the refracted ray are on the same side of the normal to the diopter.



1. It follows that ray (1) is the incident ray, (2) is the reflected ray and (3) is the refracted ray.
2. From the above, the direction of light is as shown in the figure.
3. The water index $n_{water} = 1.33$ is greater than that of air that is equal to one. The ray (3) approaches to the normal, it therefore propagates in the most refractive medium: water is therefore in zone B.
4. Sin (i_{Bl}) = 1/1.33   so i_{Bl} = 48,75^o
5. The limiting angle of refraction is always found in the most refractive medium (with the greatest index n).

Exercise 2.6:

Rayon light consists of the superposition of three colors: violet, yellow and red. This ray propagates in a glass whose refractive indices for violet, yellow and red radiation are respectively $n_v = 1.530, n_y = 1.517 and\ n_r = 1.513$. The rayon arrives on the diopter plan separating the glass from the air.

1. Calculate the limiting angles of incidence for the color's violet, yellow and red on the diopter separating the glass from the air. The air index being equal to 1.
2. What colors are observed in the air if the ray arrives the diopter at an angle of incidence $\mathrm{i} = 38^0$?
3. Same question if the angle of incidence $\mathrm{i} = 41.38^0$?
4. What can this assembly be used for?

Solution

1. The law of diffraction in the plane diopter: $n \sin \mathrm{i} = \sin r$

The refractive index of glass is greater than that of air; the angle of refraction is greater than the angle of incidence: the limiting angle of incidence i_{lim} corresponds to an angle of refraction $r = 90^0$.

The diffraction law is then: $n \sin i_{lim} = \sin 90^0$

From where: $\sin i_{lim} = \frac{1}{n}$

$$i_{lim} = \sin^{-1}\frac{1}{n}$$

For violet radiation: $i_{lim} = \sin^{-1}\frac{1}{1.530} = 40.81^0$

For yellow radiation: $i_{lim} = \sin^{-1}\frac{1}{1.517} = 41.24^0$

For red radiation: $i_{lim} = \sin^{-1}\frac{1}{1.513} = 41.37^0$

2- The angle of incidence $i = 38^0$ is less than the limit angles of the three colors, the three colors will emerge in the air. The refraction angles are different for the three radiations; hence, they will be separated after refraction by diopters in air.
3- The angle of incidence $i = 41.30^0$ is greater than the limiting angles of incidence of the color's violet and yellow; these two colors will not emerge in the air and will be reflected in the glass on the surface of the diopter. On the other hand, $i = 41.30^0$is less than the limiting angle of red radiation, only this radiation will be refracted.
4- This assembly can be used as a disperser in spectroscopy to separate colors from a light beam or as a chromatic filter to eliminate certain radiation by reflection on the surface of the diopter.

Exercise 2.7:

We want to determine the refractive index n of a glass. To do this, we use this glass to make a prism whose base is an equilateral triangle. We place it at the minimum deviation. The minimum deviation angle D measured is42^0.

Calculate its refractive index.

Solution

The deviation is minimal for $i = i' = \sin^{-1}\left(n \sin\frac{A}{2}\right)$

$$D = 2 \times \sin^{-1}\left(n \sin\frac{A}{2}\right) - A$$

$$2 \times \sin^{-1}\left(n \sin\frac{A}{2}\right) = D + A$$

$$\sin^{-1}\left(n \sin\frac{A}{2}\right) = \frac{D + A}{2}$$

$$n \sin\frac{A}{2} = \sin\left(\frac{D + A}{2}\right)$$

$$n = \sin\left(\frac{D + A}{2}\right) \Big/ \sin\frac{A}{2}$$

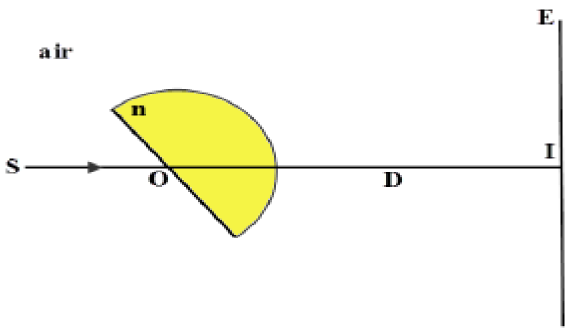
$$n = \sin(51^0)/\sin(30^0)$$

$$n \approx 1.55$$

Exercise 2.8:

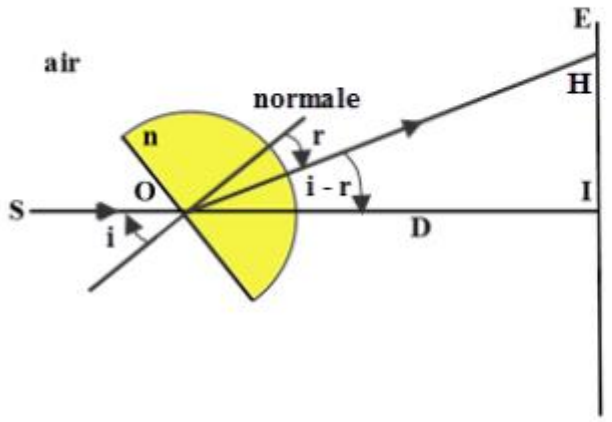
A point source S emits a monochromatic light ray, which arrives on the flat face of a half-cylindrical block, of index n, at its center O with an angle of incidence $i = 40^0$(figure below). We place a screen E at a distance D=1m from the center O. Consider H the point of impact of the ray emerging from the block on the screen E. The deviation observed is IH=0.24m.

1- Trace the path of the light ray from S until H .
2- Calculate the angle of refraction r at the point O.
3- Calculate the index n of this block



Solution

1. Walk of the ray coming from S.



2. In the triangle OIH: $\tan(i-r) = \frac{IH}{D} = \frac{0.24}{1} = 0.24$ $(i-r) = 13.5^0$ $r = i - 13.5^0 = 40^0 - 13.5^0 = 26.5^0$
3. The law of refraction at the point A is written:

$\sin i = n \sin r$ $n = \frac{\sin i}{\sin r} = \frac{\sin 40^0}{\sin 26.5^0} = 1.44$