Chapter II: optic

2-Optics

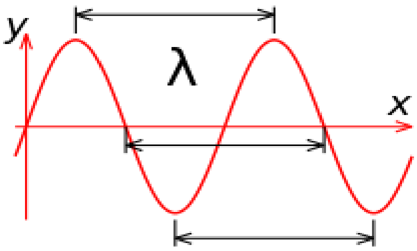
2.1.1. Introduction: Optics is the branch of physics that studies the behaviour and properties of light, including its interactions with matter and the construction of instruments that use or detect it. Optics usually describes the behaviour of visible, ultraviolet, and infrared light. Light is a type of electromagnetic radiation, and other forms of electromagnetic radiation such as X-rays, microwaves, and radio waves exhibit similar properties.

Most optical phenomena can be accounted for by using the classical electromagnetic description of light, however complete electromagnetic descriptions of light are often difficult to apply in practice. Practical optics is usually done using simplified models. The most common of these, geometric optics, treats light as a collection of rays that travel in straight lines and bend when they pass through or reflect from surfaces. Physical optics is a more comprehensive model of light, which includes wave effects such as diffraction and interference that cannot be accounted for in geometric optics. Historically, the ray-based model of light was developed first, followed by the wave model of light. Progress in electromagnetic theory in the 19th century led to the discovery that light waves were in fact electromagnetic radiation.

2.1.2 Nature of light

2.1.2.1 Light is electromagnetic radiation that can be detected by the human eye. Electromagnetic radiation occurs over an extremely wide range of wavelengths, from gamma rays with wavelengths less than about 1×10^{-11} meters to radio waves measured in meters.

2.1.2.2 A wavelength is the distance between one point on a wave to the same point on the next wave, such as a crest or trough. Because it is a distance, it is usually reported in units of nm, mm, cm, or m.



The formula to convert wavelength to frequency is given by:

Speed = Frequency x Wavelength

Therefore, Wavelength = (Speed of the wave)/ (Frequency of the wave)

Symbolically, the formula is represented as:

$$C = f \times \lambda$$

Where,

λ is the wavelength of the wave (measured in meters),

C is the speed of the wave in a given medium (measured in m/s),

f, ν is the frequency of the wave (measured in Hertz or 1/s).

2.1.2.3 The photon: Planck's discoveries paved the way for the discovery of the photon. A photon is the elementary particle, or quantum, of light. Photons can be absorbed or emitted by atoms and molecules. When a photon is absorbed, its energy is transferred to that atom or molecule. Because energy is quantized, the photon's entire energy is transferred (remember that we cannot transfer fractions of quanta, which are the smallest possible individual "energy packets"). The reverse of this process is also true. When an atom or molecule loses energy, it emits a photon that carries an energy exactly equal to the loss in energy of the atom or molecule. This change in energy is directly proportional to the frequency of photon emitted or absorbed. This relationship is given by Planck's famous equation:

$$E = h\nu = \frac{hc}{\lambda}$$

where is the energy of the photon absorbed or emitted (given in Joules,), is frequency of the photon (given in Hertz,), and is Planck's constant,

2.1.2.4 **The Electromagnetic Spectrum**

The **electromagnetic spectrum** is the full range of electromagnetic radiation, organized by frequency or wavelength. The spectrum is divided into separate bands, with different names for the electromagnetic waves within each band. From low to high frequency these are: radio waves, microwaves, infrared, visible light, ultraviolet, X-rays, and gamma rays. The electromagnetic waves in each of these bands have different characteristics, such as how they are produced, how they interact with matter, and their practical applications.

Radio waves, at the low-frequency end of the spectrum, have the lowest photon energy and the longest wavelengths—thousands of kilometers, or more. They can be emitted and received by antennas, and pass through the atmosphere, foliage, and most building materials.

Gamma rays, at the high-frequency end of the spectrum, have the highest photon energies and the shortest wavelengths—much smaller than an atomic nucleus. Gamma rays, X-rays, and extreme ultraviolet rays are called ionizing radiation because their high photon energy is able to ionize atoms, causing chemical reactions.

Visible light and radiation of longer wavelengths are nonionizing; their photons do not have sufficient energy to cause these effects.

Throughout most of the electromagnetic spectrum, spectroscopy can be used to separate waves of different frequencies, so that the intensity of the radiation can be measured as a function of frequency or wavelength. Spectroscopy is used to study the interactions of electromagnetic waves with matter. The types of electromagnetic radiation are broadly classified into the following classes (regions, bands or types):

1. Gamma radiation
2. X-ray radiation
3. Ultraviolet radiation
4. Visible light (light that humans can see)
5. Infrared radiation
6. Microwave radiation
7. Radio waves

This classification goes in the increasing order of wavelength, which is characteristic of the type of radiation.

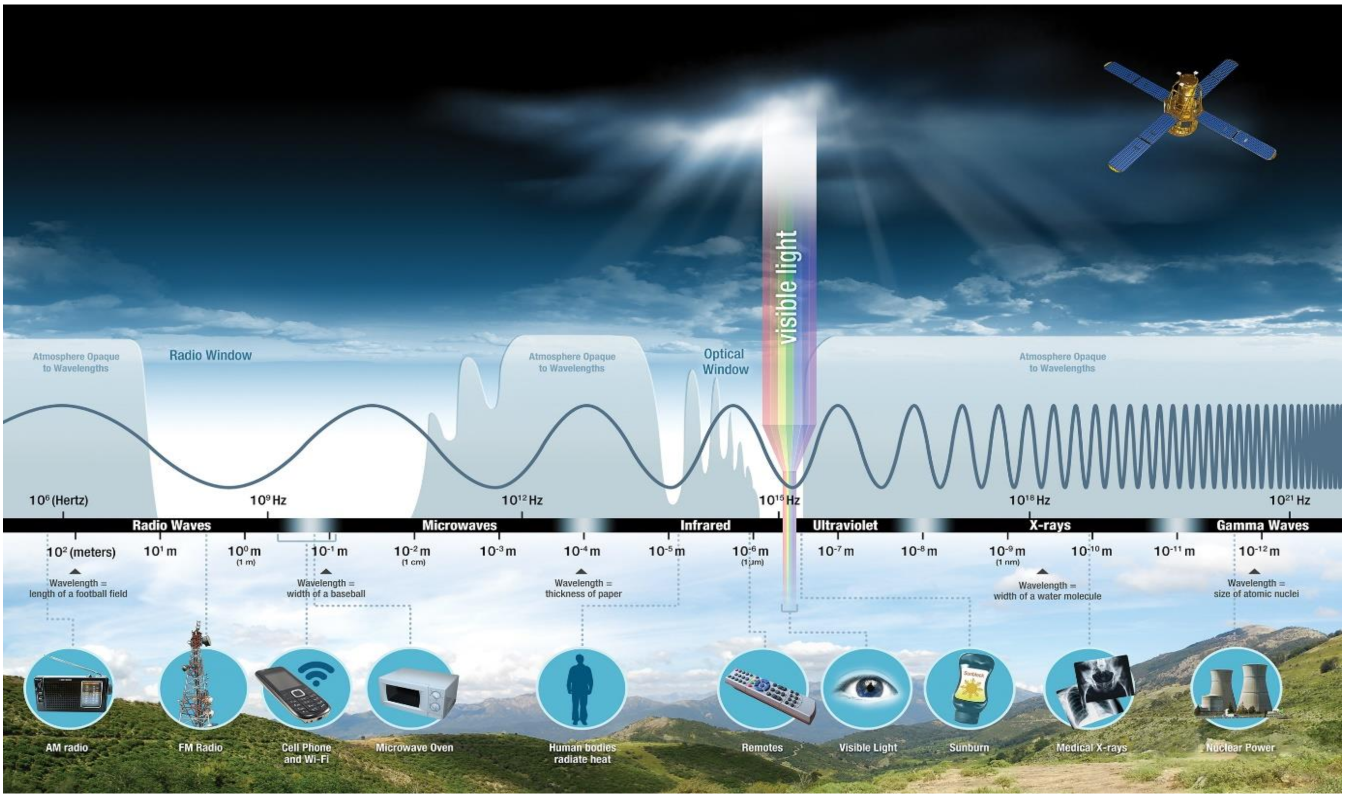


Figure 2.1: The electromagnetic spectrum

2.2. Geometric optics:

2.2.1 Geometrical optics, or **ray optics**, is a model of optics that describes light propagation in terms of *rays*. The ray in geometrical optics is an abstraction useful for approximating the paths along which light propagates under certain circumstances.

The simplifying assumptions of geometrical optics include those light rays:

- propagate in straight-line paths as they travel in a homogeneous medium bend, and in particular circumstances may split in two, at the interface between two dissimilar media
- follow curved paths in a medium in which the refractive index changes may be absorbed or reflected.

Geometrical optics does not account for certain optical effects such as diffraction and interference. This simplification is useful in practice; it is an excellent approximation when the wavelength is small compared to the size of structures with which the light interacts. The techniques are particularly useful in describing geometrical aspects of imaging, including optical aberrations.

2.2.2. Principles of Light Propagation:

The principles of light propagation govern and explain the actions of light as it traverses diverse materials. Here are some key principles to keep in mind:

- ✓ **Rectilinear Propagation**: When light moves through a uniform medium, it progresses in a straight line. This principle is known as rectilinear propagation.
- ✓ **Reflection and Refraction**: When light waves encounter a change in medium, they can be reflected back into the first medium, or refracted (i.e., transmitted and bent) into the new medium.

2.2.2.1 Definition of Optical paramètres

2.2.2.1.1. Refraction index:

The term "refraction of light" refers to how light bends when it travels perpendicularly through transparent media. The refractive index is a measurement of how much a light beam bends as it passes through various materials. The speed of light in a medium reduces as it moves from a rarer to a denser one.

The refractive Index is dimensionless. For a given material, the refractive index is the ratio between the speed of light in a vacuum (c) and the speed of light in the medium (v). If the refractive index for a medium is represented by n, then it is given by the following formula: $n = c/v$

2.2.2.1.2. Diopters:

A diopter is a surface separating two different medium indexes. Apart from those with mirrors or diffracting surfaces, usual optical systems (camera objective lenses, projection lenses, glasses, microscopes) are exclusively made of a number of diopters.

Optical systems generally have a revolution axis and diopters used are generally spherical or plane. The system axis is the line goring through the diopters centers of curvature; it is perpendicular to the diopters planes.

2.2.2.1.3. The law of Refraction:

When light travels from one medium to another, it generally bends, or *refracts*. The law of refraction gives us a way of predicting the amount of bend. This law is more complicated than that for reflection, but an understanding of refraction will be necessary for our future discussion of lenses and their applications. The law of refraction is also known as Snell's Law, named for Willobrord Snell, who discovered the law in 1621.

2.2.2.1.4. Snell's LAW

Like with reflection, refraction also involves the angles that the incident ray and the refracted ray make with the normal to the surface at the point of refraction. Unlike reflection, refraction also depends on the media through which the light rays are travelling. This dependence is made explicit in Snell's Law via *refractive indices,* numbers which are constant for given media.

Snell's Law is given in the following diagram.

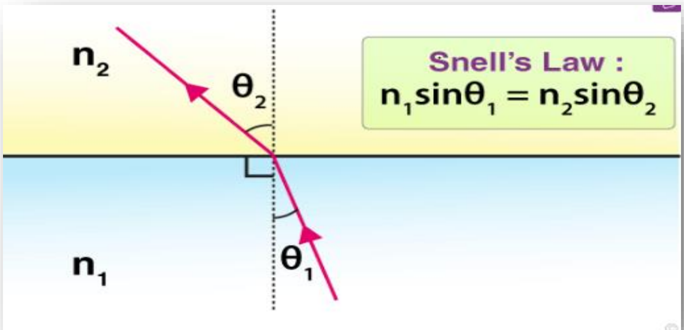


Figure 2.2: Snell's Law.

The normal on the surface is used to gauge the angles that the refracted ray creates at the contact point.

2.2.2.1.5. Complex Snell's Law Diagram:

A complex diagram of Snell's Law displays something that is not directly obvious. A ray of light passes through the glass and standing behind it the viewer experiences refraction through three media. The situation is represented in the following diagram :

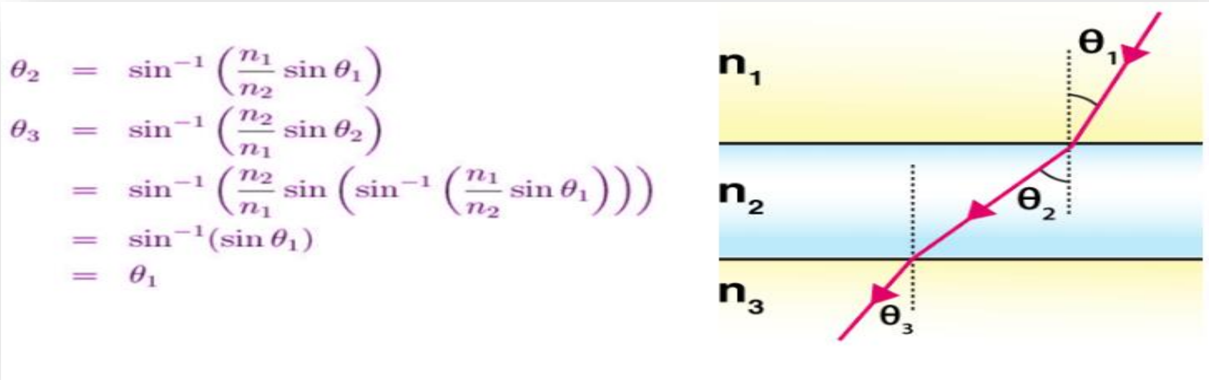


Figure 2.3: A complex diagram of Snell's Law.

2.2.2.1.6. Critical angle:

The critical angle is the angle of incidence where the angle of refraction is 90 degrees. Light must travel from an optically denser medium to an optically less dense medium.

$$\theta_c = \sin^{-1}\left(\frac{n1}{n2}\right)$$

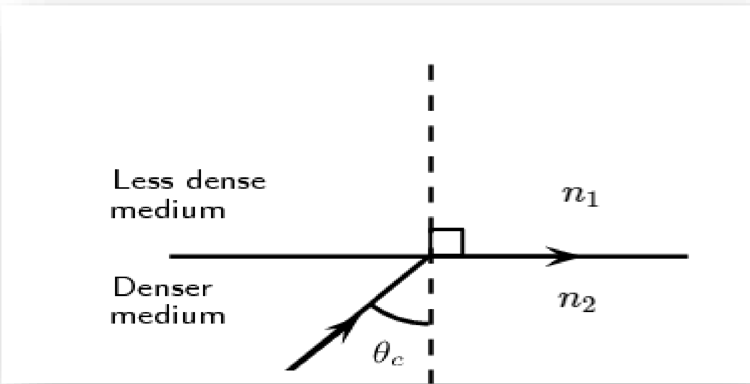


Figure 2.4: When the angle of incidence is equal to the critical angle, the angle of refraction is equal to**90**°.

If the angle of incidence is bigger than this critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called **total internal reflection**.

The conditions for total internal reflection are:

1. Light is travelling from an optically denser medium (higher refractive index) to an optically less dense medium (lower refractive index).
2. The angle of incidence is greater than the critical angle.

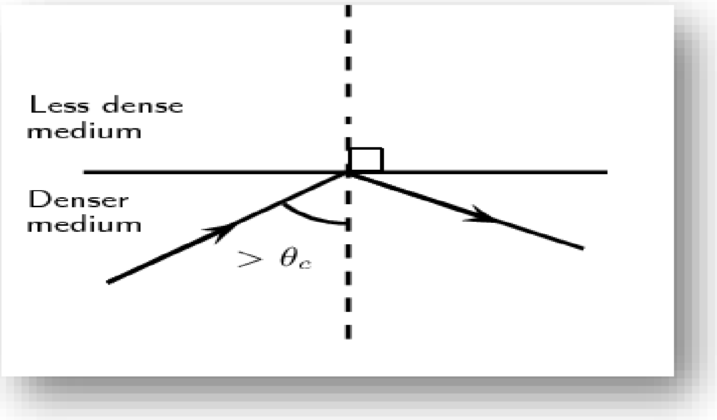


Figure 2.5: When the angle of incidence is greater than the critical angle, the light ray is reflected at the boundary of the two media and total internal reflection occurs.

2.2.2.1.7. Optical systems

An optical system is a set of surfaces which reflect (mirrors) or refract (diopter) light rays. A centered system has a symmetrical axis. Systems dotted with diopters only are called dioptrics (lenses, spectacles, microscopes). Systems composed of diopters and mirrors are said to be catadioptric (telescopes)

Stigmatic image of a luminous point in an optical system

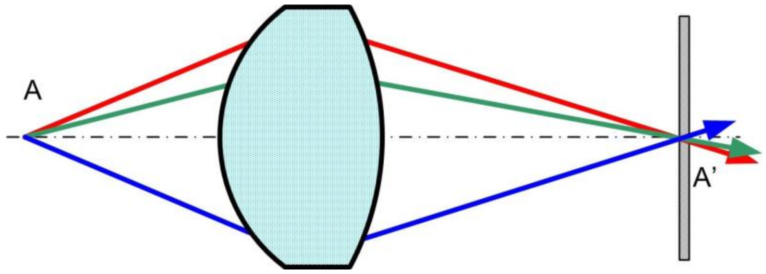
Let us consider a point A in a first space called "object space". From A let us run a set of luminous rays going through the system. If all these rays converge to the same point A' of the image space, we can write:

- ✓ A' is the image of A through the system. It is also said that A' is the conjugate of A
- ✓ The system is said to be stigmatic for AA' conjugation

We demonstrate that stigmatism implies a constant value for the optical path (AA')

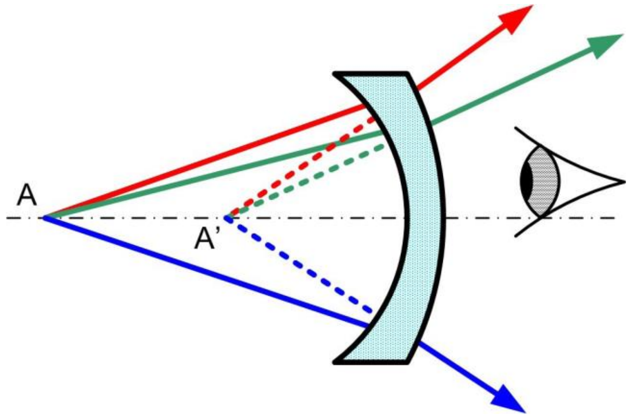
The case of a **real image:**

The image may be observed on a screen in the image space.

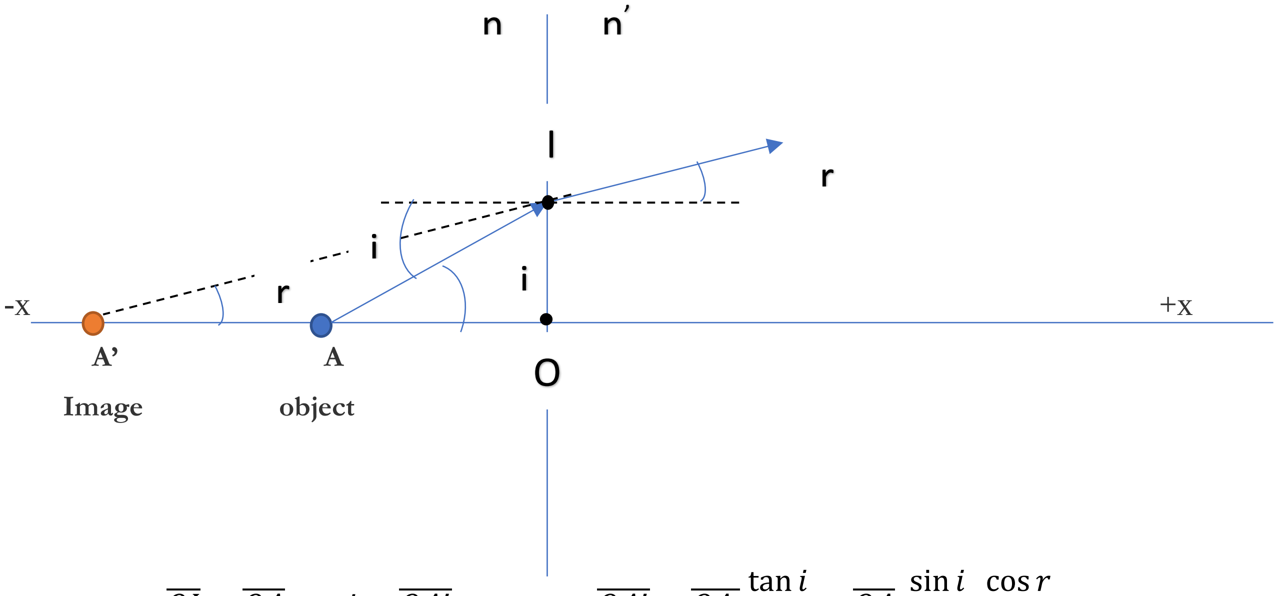


The case of a **virtual image** :

The image cannot be observed on a screen. It is nevertheless visible by an observer situated in the image space.



2.2.2.1. conjugate formula of diopter plane: see the figure below



$$\overline{OI} = \overline{OA}.\tan i = \overline{OA'}\,.\tan r \Longrightarrow \overline{OA'} = \overline{OA}.\frac{\tan i}{\tan r} = \overline{OA}.\frac{\sin i}{\cos i}.\frac{\cos r}{\sin r}$$

$$n \sin i = n' \sin r \Rightarrow \sin i = \frac{n'}{n} . \sin r \Rightarrow \sin r = \frac{n}{n'} \sin i$$

$$\Rightarrow \begin{cases} \sin^2 r + \cos^2 r = 1, \quad \cos r = \sqrt{1 - \sin^2 r} \\ \cos r = \sqrt{1 - \left(\frac{n}{n'}\right)^2 \sin^2 i} \\ \sin^2 i + \cos^2 i = 1, \quad \cos i = \sqrt{1 - \sin^2 i} \end{cases}$$

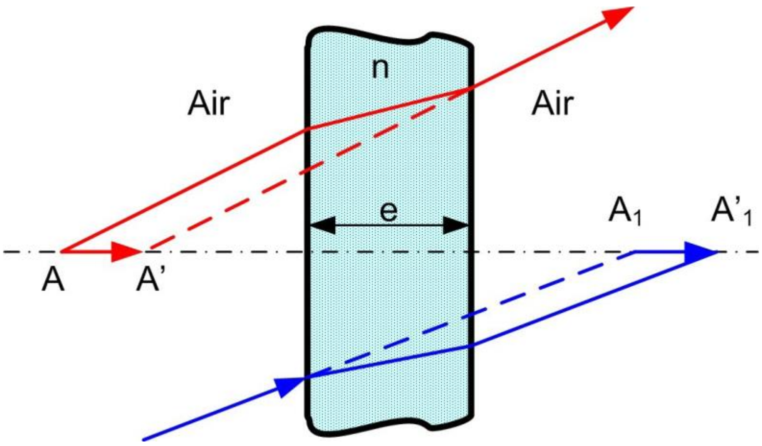
$$\overline{OA'} = \overline{OA} . \frac{n'}{n} . \sqrt{\frac{1 - \left(\frac{n}{n'}\right)^2 \sin^2 i}{1 - \sin^2 i}}$$

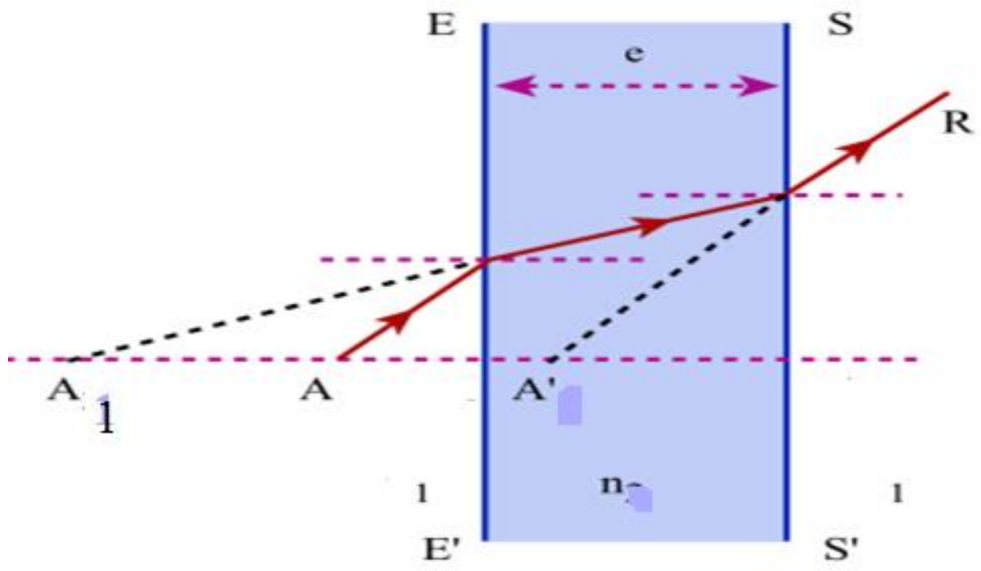
The plane diopter is therefore not perfectly stigmatic. However, if the angle i is sufficiently small, to be able to neglect $\sin^2 i$, we can write to the second order at i :

$$\frac{\overline{OA'}}{n'} = \frac{\overline{OA}}{n}$$

parallel plates

The parallel plate is composed of two distant dioptric planes of e, n is the medium index. An object point A has, as an image, a point A' situated on the perpendicular led from A to the plates sides.





The parallel plate is composed of two distant dioptric planes of e, n is the medium index. An object point A has, as an image, a point A' situated on the perpendicular led from A to the plate's sides. We show that :

The dis placement of the image relative to the Object is $AA' = e\left(1 - \frac{1}{n}\right)$

<u>Demonstration</u>

Diopter D1 :

Air	D1	Milieu 1
n=1		n
A objet	$\frac{\overline{OA_1}}{n} = \frac{\overline{OA}}{1}$	A1 the image from a diopter D1

Diopter D2 :

Milieu 1	D2	Air
n		1
A1 objet	$\frac{\overline{O'A'}}{1} = \frac{\overline{O'A_1}}{n}$	A' the image from a diopter D2

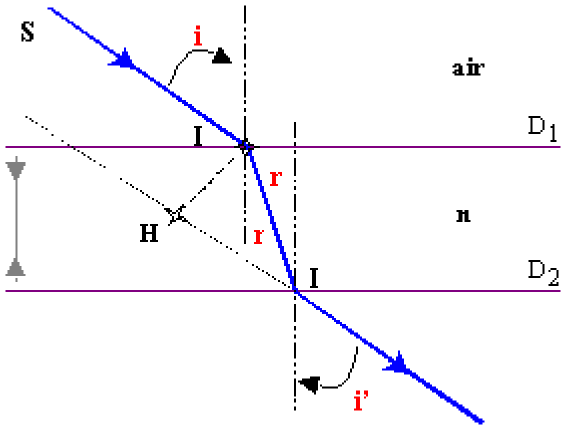
$$\overline{AA'} = \overline{AO} + \overline{OO'} + \overline{O'A'} =$$

$$= \frac{\overline{A_1O}}{n} + e + \frac{\overline{O'A_1}}{n} = e + \frac{\overline{O'A_1} + \overline{A_1O}}{n} = e + \frac{\overline{O'O}}{n} = e - \frac{e}{n}$$

$$AA' = e\left(1 - \frac{1}{n}\right)$$

The lateral displacement d is $d = e.\frac{\sin(i-r)}{\cos r}$

Demonstration

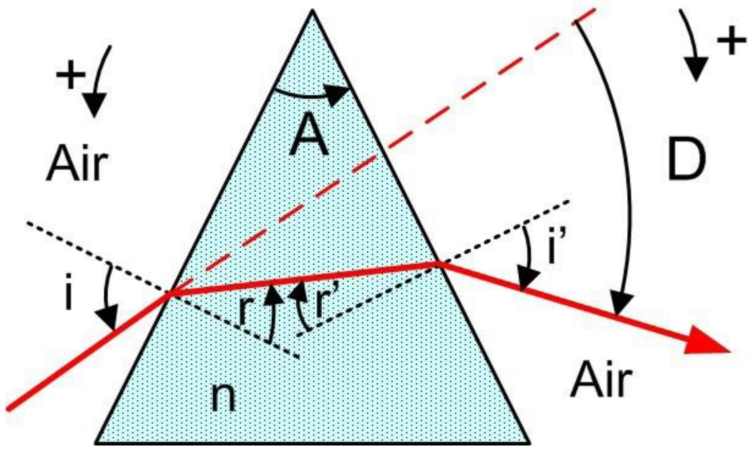


$$\begin{cases} \sin(i-r) = \frac{IH}{II'} = \frac{d}{II'} \\ \cos r = \frac{IN}{II'} = \frac{e}{II'} \end{cases}$$

$$d = e.\frac{\sin(i-r)}{\cos r}$$

Prisms

A prism of index n is composed of two dioptric planes forming an angle A. Following figure bellow, a luminous ray enters from side 1 under incidence i and comes out of side 2 under incidence i', the corresponding refraction angles in the prism are r and r', D is the deviation from the ray provoqued by the prism. The angular sign convention is normal for side 1 and inversed for side 2.



$$\begin{cases} \sin i = n \sin r \\ n \sin r' = \sin i' \\ A = r + r' \\ D = i + i' - A \end{cases}$$

At the minimum of deviation: i = i' et r = r', we obtain a relationship between n, A and D, allowing index measures of optical material:

$$\begin{cases} \sin i = n \sin r \\ n \sin r' = \sin i' \\ A = 2r \\ D = 2i \end{cases}$$

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

2.2.2.2. spherical Diopters

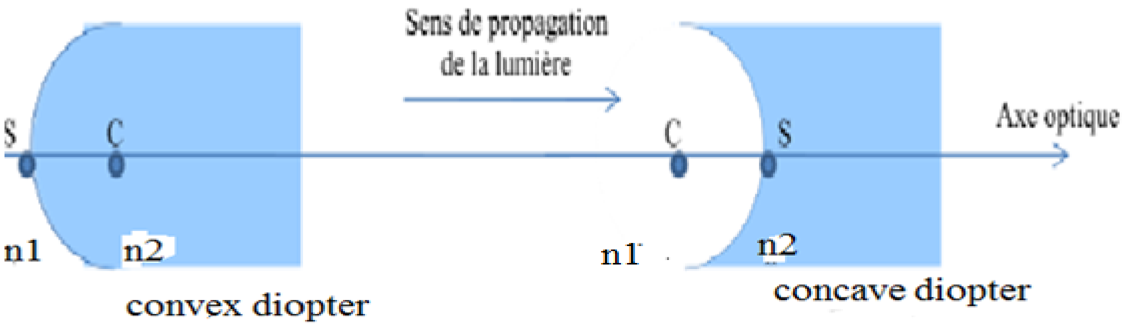
A spherical diopter is a portion of refractive spherical surface separating two homogeneous and transparent media of different indices. It is characterized by:

- The center C of the sphere called the diopter center

- The point S called the top of the diopter.

- The optical axis, the axis of symmetry of revolution of the diopter, passing through points C and S.

- The radius of the sphere $R = \overline{SC}$, called the radius of curvature, an algebraic quantity which is negative for a concave spherical diopter $\overline{SC} < 0$ and positive for a convex spherical diopter $\overline{SC} > 0$.



Note: in geometric optics, the measurement of distances is algebraized. Along the optical axis, we choose as positive direction the direction of propagation of the light (generally from left to right).

Four possible cases for the spherical diopter:

- $\overline{SC} < 0$ and $n1 < n2 \rightarrow$ Diverging concave spherical diopter.

- $\overline{SC} < 0$ and $n1 > n2 \rightarrow$ Spherical concave convergent diopter.

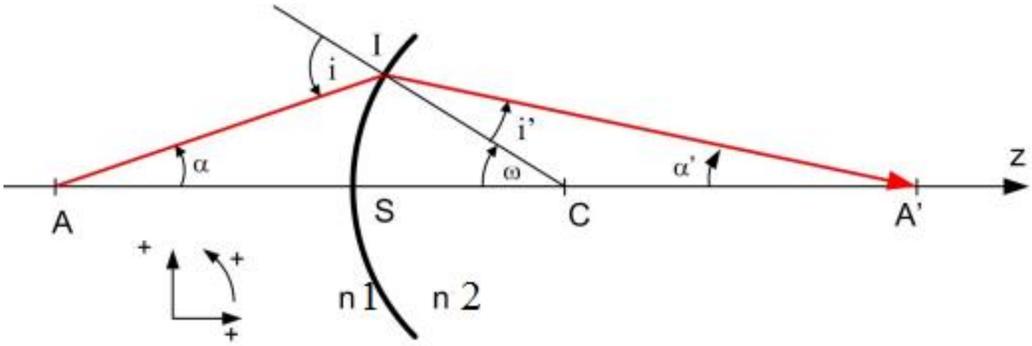
- $\overline{SC} > 0$ and $n1 < n2 \rightarrow$ Convergent convex spherical diopter.

- $\overline{SC} > 0$ and $n1 > n2 \rightarrow$ Divergent convex spherical diopter.

Conjugation relationships

The image of a luminous point in a diopter

Let us consider a spherical diopter separating two mediums of indexes n and n', defined by its curvature center as C, its vertex as S, its curvature ray $R = \overline{SC}$. All lengths and angles are orientated in accordance with the trigonometry convention A point A is situated on the object space on line SC. The ray arising from A through S is perpendicular to the diopter, it is not deviated. Another ray arising from A going through any point I from the diopter is subject to refraction, the ray arising cuts SC at a point A'.. According to figure bellow:



We consider n1=n and $n2 = n'$

i is the incidence angle of the ray on the dioptic.

i' is the refraction angle ,

$n.sin(i) = n'.sin(i')$.

An usual formula in the triangle CAI gives:

$$\frac{\overline{CA}}{\sin(\pi - i)} = \frac{\overline{CA}}{\sin(i)} = -\frac{\overline{IA}}{\sin(\omega)}$$

$$\overline{CA} < 0, \overline{IA} < 0, \omega < 0, i < 0$$

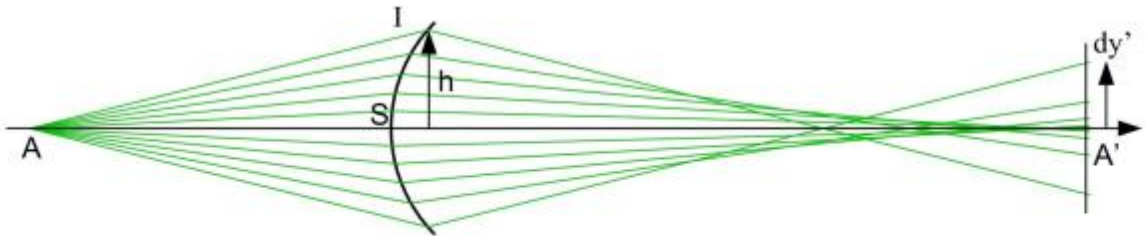
One can also write :

$$\frac{\overline{CA'}}{\sin(i')} = -\frac{\overline{IA'}}{\sin(\omega)}$$

Therefore :

$$\frac{\overline{CA}}{\overline{CA'}} = \frac{\overline{IA}.\sin(i)}{\overline{IA'}.\sin(i')} = \frac{n}{n'}.\frac{\overline{IA}}{\overline{IA'}}$$

Conjugate stigmatism would mean that A' does not depend on the position of I. It is thus necessary that CA' remains fixed, similarly for the IA/IA' ratio. This is obtained only in a particular position of A and is not achieved in general cases.



Paraxial approximation

Figure precedent shows that the ray tranversal aberration increases with the height of incidence h of I on the diopter. Let us seek the limit A' of the intersection of the refracted ray when h leads to 0. When I leads towards S, the relationship (7) becomes :

$$\frac{\overline{CA}}{\overline{CA'}} = \frac{n_1}{n_2}.\frac{\overline{SA}}{\overline{SA'}}$$

The study of aberrations shows that the distance gap dy' between the refracted ray and A' on a plane going through A' and perpendicular to the axis is approximately proportional to h 3 . For small values of h, dy' is very short, there is an approximate stigmatism. In this case, rays incidences i and i' on the surface of the diopter are low, sinus and radian angles are very close, the relationship becoming:

$$n_1.i = n_2.i'$$

$$\overline{SC} = R$$

$$\overline{CA} = \overline{SA} - \overline{SC}$$

$$\overline{CA'} = \overline{SA'} - \overline{SC}$$

$$\frac{\overline{SA} - \overline{SC}}{\overline{SA'} - \overline{SC}} = \frac{n_1}{n_2}.\frac{\overline{SA}}{\overline{SA'}}$$

$$\boldsymbol{\frac{n_2}{\overline{SA'}} - \frac{n_1}{\overline{SA}} = \frac{n_2 - n_1}{\overline{SC}}} = V$$

V: vergence or power of the diopter (unit: Diopter = m-1).

- If V > 0: Converging diopter

- If V < 0: Divergent diopter

Focus, focal distance, refracting power

The image focus is the image of the point towards infinity on the axis: $\frac{1}{\overline{SA}} = 0$

$$\boldsymbol{F' = \overline{SF'} = \frac{n_2}{n_2 - n_1}.\overline{SC} = \frac{n_2}{V}}$$

F' is the image focus, SF' = f ' is the image focal distance of the diopter.

The object focus is such that its image is to infinity on the axis:

$$\boldsymbol{F = \overline{SF} = \frac{-n_1}{n_2 - n_1}.\overline{SC} = -\frac{n_1}{V}}$$

F is the object focus SF= f is the object focal distance of the diopter.

We notice that:

$$\frac{\overline{SF'}}{\overline{SF}} = -\frac{n_2}{n_1} < 0$$

$\overline{SF}$and $\overline{SF'}$ have opposite signs, F and F' belong to two different medium.

And so:

$$\overline{SF} + \overline{SF'} = \overline{SC}$$

Axial magnification

$$\boldsymbol{\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_1}{n_2}.\frac{\overline{SA}}{\overline{SA'}}}$$

• If $\gamma > 0$ (+) the image is straight (it has the same direction as the object).

• If $\gamma < 0$ (-) the image is reversed (reverse direction).

• If $|\gamma| > 1$ the image is larger than the object.

• If $|\gamma| < 1$ the image is smaller than the object.

• If $\gamma = 1$ the image and the object have the same size

The characteristics of the image:

- The position of the image: $\overline{SA'}$

- If $\overline{SA'} > 0$ the image is real

- If $\overline{SA'} < 0$ the image is virtual

The nature of the image:

⟶ Say if it is real or virtual.

⟶ Say if it is straight or upside down:

⟶ Say if it is enlarged, reduced or the same size as the object

Geometric construction of the image

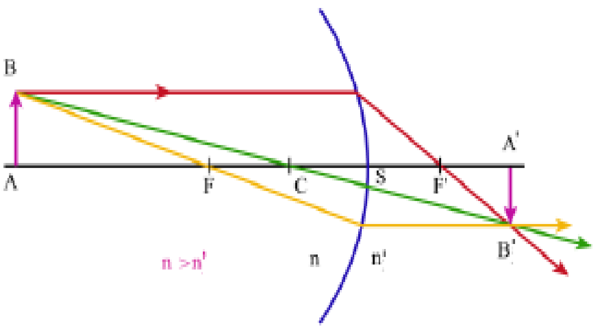
- You must place the object AB: real or virtual

- Construct the image B' of point B: simply consider two rays coming from this point:

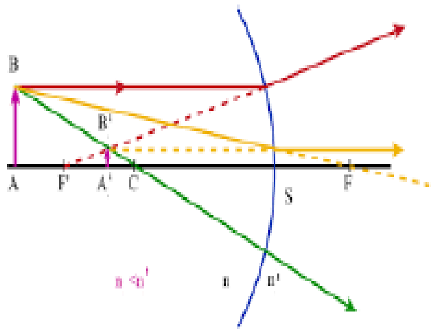
the incident ray parallel to the optical axis passes through F' the incident ray which passes through F leaves the diopter parallel to the optical axis

The ray which passes through the center C of the diopter is not deviated

- A’ is the orthogonal projection of B’ on the optical axis.



Concave convergent



Concave divergent