CHAPTER 3

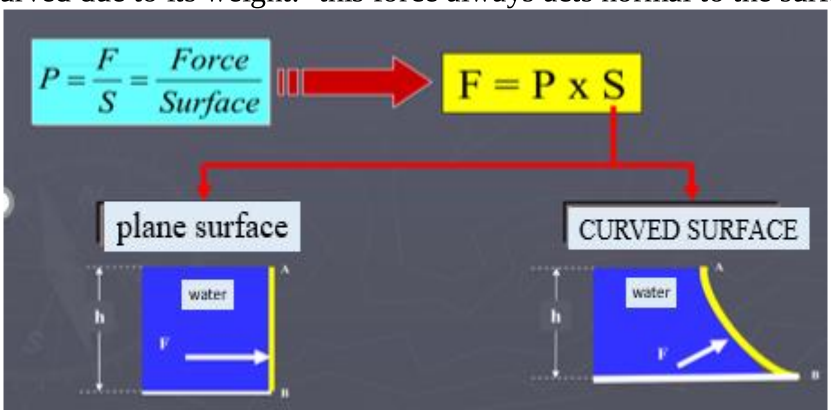
3.1 INTRODUCTION

When a static mass of fluid comes in contact with a plane or curved surface, the fluid exerts a force on the surface. This force is called total pressure. What is the direction in which this total pressure exerted by the fluid acts on the surface? When a fluid is at rest, no tangential force exists on the fluid. That is, when a fluid is at rest, it cannot sustain any shear forces. Hence, the fluid at rest exerts a force in a direction normal to the surface with which it comes in contact.

3.2 Hydrostatic pressure force on surface of any shape

In everyday life, a liquid such as water, for example, needs to be contained in a container (container). This container must then be able to contain this liquid without cracking or dislocating. Knowledge of the pressure forces that are applied to the walls of the container is therefore necessary for its dimensioning.

Total pressure/hydrostatic pressure is defined as the force exerted by a static fluid on a surface either plane or curved due to its weight. -this force always acts normal to the surface.



Expression of hydrostatic force/total pressure

$$dF = PdA = \rho ghdA \longrightarrow F = \int_{AB} dF = \int_{AB} \rho ghdA = \rho g \int_{AB} hdA$$

$$h = y \sin \alpha \Rightarrow F = \rho g \sin \alpha \int_{AB} ydA$$

" *Moment Statique* " de la surface AB par rapport à Ox

$$\int_{AB} ydA = y_c A$$

Ordonnée du centre de gravité de la surface AB

$$\longrightarrow F = \rho g \sin \alpha y_c A = \rho g y_c \sin \alpha A$$

h_c : Profondeur du centre de gravité de AB

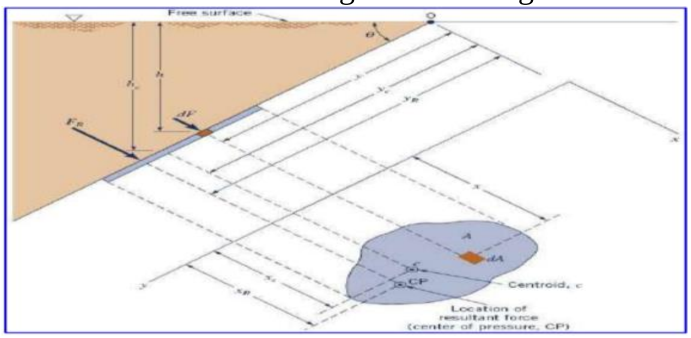
$$\Rightarrow \boxed{F = \rho g h_c A}$$

Eau ($\rho = \rho_\omega$)

$$\boxed{F = \rho_w g h_c A}$$

3.2.1 Hydrostatic pressure force on an inclined plane surface of any shape

Consider a wall of surface A and center of gravity c, immersed in a liquid and inclined at an angle θ to the horizontal. Let us divide surface A into a sufficiently small element dA. The pressure force on the element is determined using the following formula:



$dF = P.dA = \rho.g.h.dA = \rho.g.y.\sin(\theta).dA$

The intensity of the pressure force acting on surface A is:

$FR = \int dF\ A = P.\ dA = \rho.\ g \int h.\ dA = A\ \rho.\ g \int y.\ sin\theta.\ dA$

This integral represents the static moment which is defined as follows:

$\int h.\ dA = hc\ .\ A = A\ yc\ .\ sin\theta.\ A$

From which the equation is written:

$FR = \rho.\ g.\ hc\ .\ A$

The pressure force on a flat surface of arbitrary orientation is equal to the product of the surface area of the wall times the pressure on its center of gravity.

$$\boldsymbol{F = \rho g \bar{h} A}$$

F = Total Pressure, N

ρ = Density of fluid, kg/m^3

A = Area of given surface, m^2

$\bar{h}$= Distance of CG of given surface from free surface, m

3.2.2 Centre of Pressure

The force F_R is not exerted at the center of gravity but at a point called the center of thrust CP = (x_R, y_R).

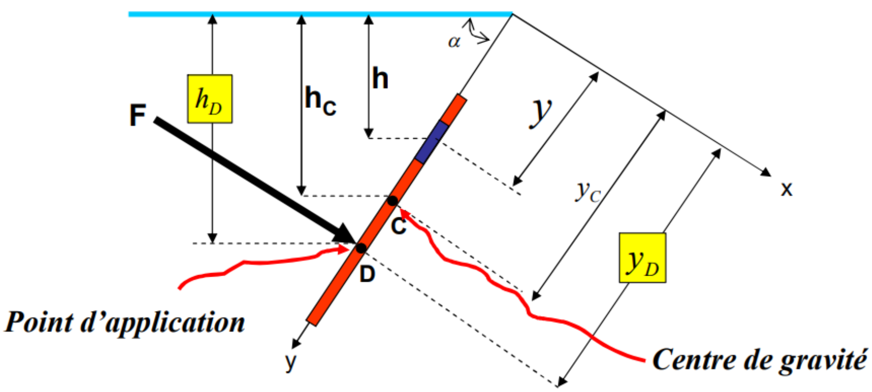
The point of application of total pressure on the surface is called centre of pressure.

To determine the coordinates of the center of pressure, we consider the moment of the force relative to the x and then y axis and we write as follows:

$\boldsymbol{F_R.\ y_R = \int_A y.\ dF}$

$\boldsymbol{\rho.g.\ y_c\ .\sin(\theta).\ A.\ y_R = \rho.g.\sin(\theta).\ \int_A y^2\ .\ dA}$

$\boldsymbol{y_R = \frac{\int_A y^2.dA}{A.yc}}$



Pour déterminer h_D, la profondeur du point d'application de la force résultante F , il suffit d'utiliser le principe des moments : " *Le moment , par rapport au point O , de la force résultante est égal à la somme des moments élémentaires* " : $\boxed{M_o F = \sum_{AB} M_i}$

avec : $M_o F = F.y_D = \rho g h_c A y_D = \rho g y_D y_c \sin\alpha A$

et $\sum_{AB} M_i = \int_{AB} y dF = \int_{AB} y \rho g y \sin\alpha dA = \int_{AB} \rho g y^2 \sin\alpha dA = \rho g \sin\alpha \int_{AB} y^2 dA$?

le terme $\boxed{\int_{AB} y^2 dA}$ représente le " ***Moment d'Inertie*** " de la surface AB par rapport à l'axe Ox = **Iox**

On aura donc : $\boxed{M_o F = \rho g y_D y_c \sin\alpha . A}$ et $\boxed{\sum_{AB} M_i = \rho g \sin\alpha . I_{OX}}$

et $M_o F = \sum_{AB} M_i \Rightarrow \rho g y_D y_C \sin\alpha . A = \rho g \sin\alpha . I_{ox}$

Et donc : $$\boxed{y_D = \frac{I_{ox}}{y_c A}}$$

Remarque : Utilisation du théorème de Huygens : $\boxed{I_{ox} = I_{cc} + y_c^2 A}$

avec **I_{cc}** : Moment d'inertie de la surface AB par rapport à un axe passant par son **centre de gravité C .**

La formule précédente devient alors : $$\boxed{y_D = \frac{I_{cc} + y_c^2 A}{y_c A} = y_c + \frac{I_{cc}}{y_c A}}$$

ou bien selon la verticale

$$\boxed{h_D = h_c + \frac{I'_{cc}}{h_c A'}}$$

avec : - **A'** : Projection verticale de la surface AB
- **I'_{cc}** : Moment d'inertie de la surface A' par rapport à l'axe passant par son centre de gravité .

h* is defined as the point of application of the total pressure on the surface.

$h^* = \bar{h} + \frac{I_G \sin^2\theta}{A\bar{h}}$

I_G = MI about centroidal axis parallel to free surface of water i.e X-X, m^4

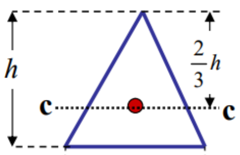
$\bar{h}$ = Distance of CG of given surface from free surface, m

A = Area of given surface, m^2

θ=Angle of inclination $=90^0$ (For vertical plane)

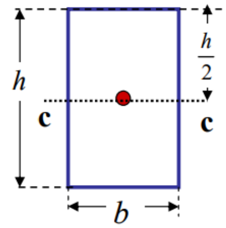
$=0^0$ (For horizontal plane)

3.2.3 Surface area and moment of inertia of some particular figures



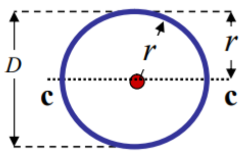
Triangle

$$A = \frac{bh}{2} \qquad I_{cc} = \frac{bh^3}{36}$$



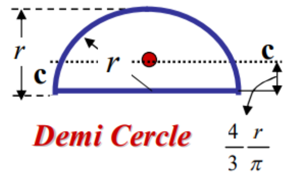
Rectangle

$$A = bh \qquad I_{cc} = \frac{bh^3}{12}$$



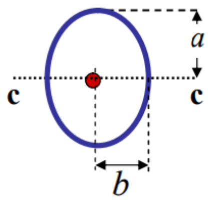
Cercle

$$A = \pi r^2 = \pi \frac{D^2}{4} \qquad I_{cc} = \frac{\pi r^4}{4} = \frac{\pi D^4}{64}$$



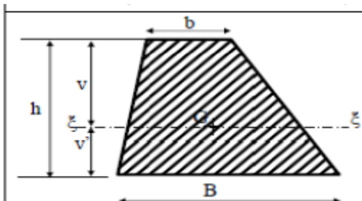
Demi Cercle

$$A = \pi \frac{r^2}{2} = \pi \frac{D^2}{8} \qquad I_{cc} = \frac{\pi r^4}{8} = \frac{\pi D^4}{128}$$



Ellipse

$$A = \pi ab \qquad I_{cc} = \frac{\pi a^4 b}{4}$$



$$v = \frac{h}{3}\left(\frac{2B+b}{B+b}\right) \; ; \; v' = \frac{h}{3}\left(\frac{B+2b}{B+b}\right)$$

$$S = \frac{h}{2}(B+b) \; ; \; I_{\xi\xi} = \frac{h^3(B^2+4Bb+b^2)}{36(B+b)}$$

3.3 Case of vertical submerged surfaces (vertical plane rectangular wall)

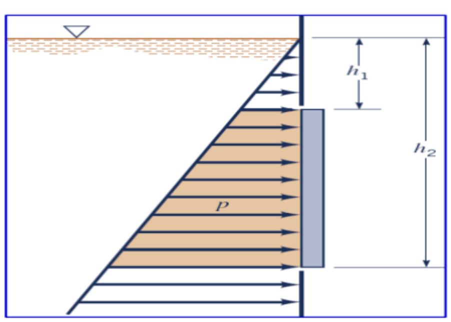
It will be the same reasoning as for the Inclined surface but with

$\Theta = 90° \rightarrow \sin\alpha = 1$

Let us therefore consider a vertical plane rectangular wall immersed vertically SO :

$$F_R = \rho . g . h_c . A = \rho . g . \frac{h_1+h_2}{2} . A$$

$$F_R = \frac{\rho . g . h_1 + \rho . g . h_2}{2} . A$$



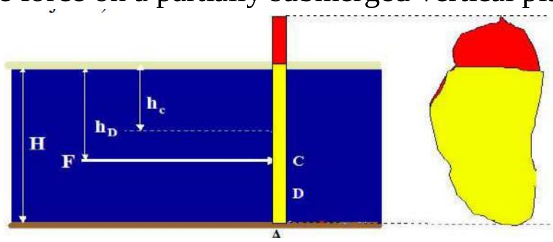
3.3.1 Hydrostatic pressure force on a vertical plane surface

Pressure force on a partially submerged vertical plane surface$FR = \rho.\ g.\ hc.\ A$ (A is the yellow surface)

$F_R = \rho.\ g.\ h_c.\ A$

(A is the yellow surface)

Pressure force on a partially submerged vertical plane surface



3.4 Case of horizontal submerged surfaces (horizontal plane wall)

Consider a wall of unit width and surface A immersed horizontally at a depth h.

The force of the hydrostatic pressure on the horizontal wall A is:

$F = P.S = \rho g h_c A$ with $h_c = h \rightarrow F = \rho.g.h.A$

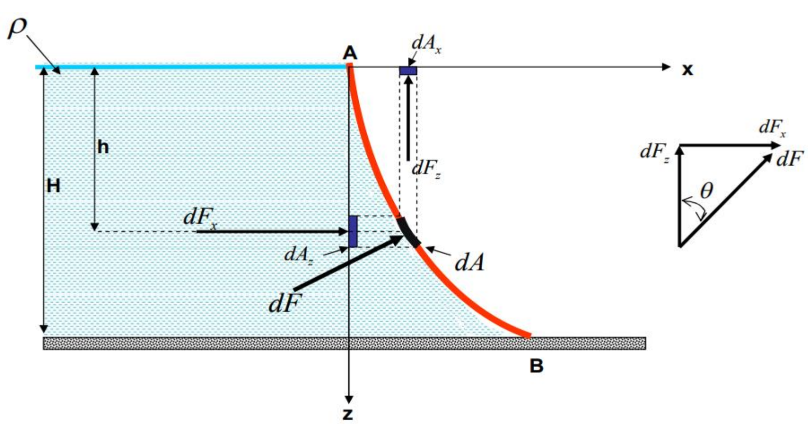
This means that the pressure force on a horizontal wall corresponds to the weight of the column of liquid of height h.

The depth of the point of application is: $h_D = h_c = h$

3.5 Case of pressure forces exerted by fluids on curved surfaces

The resultant of the pressure forces on a curved surface is easier to calculate by dividing the force into its vertical and horizontal components.

Consider the diagram in Figure representing a block of liquid and subjected to various forces:



3.5.1 Hydrostatic pressure force

Horizontal component

$dF_x = dF \sin\theta = \rho.g.h.d.A \sin\theta = \rho.g.h.d.A_z$

$\int dF_x = F_H = \rho.\ g \int A\ h.\ dA_z$

From where : $F_H = \rho.\ g h_c A_z$

with A_z: Vertical projection of the curved surface AB

h_c: Depth of the center of gravity of Az

The calculation of the horizontal component F_H is reduced to the calculation of a pressure force on a vertical plane surface.

Vertical component

$dF_z = dF\cos\theta = \rho.g.h.d.A\cos\theta$

$= \rho.g.h.d.A_x$

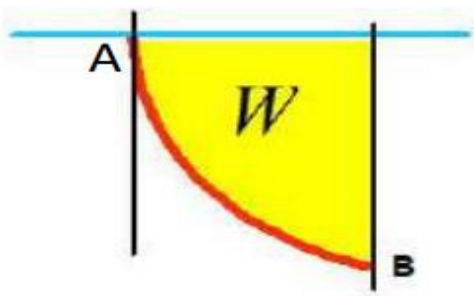
$\int dF_z = F_V = \rho.\, g \int_W dW = \rho.\, g.\, W$

From where : $F_V = \rho.\, g.\, W$

With W: Volume delimited by:

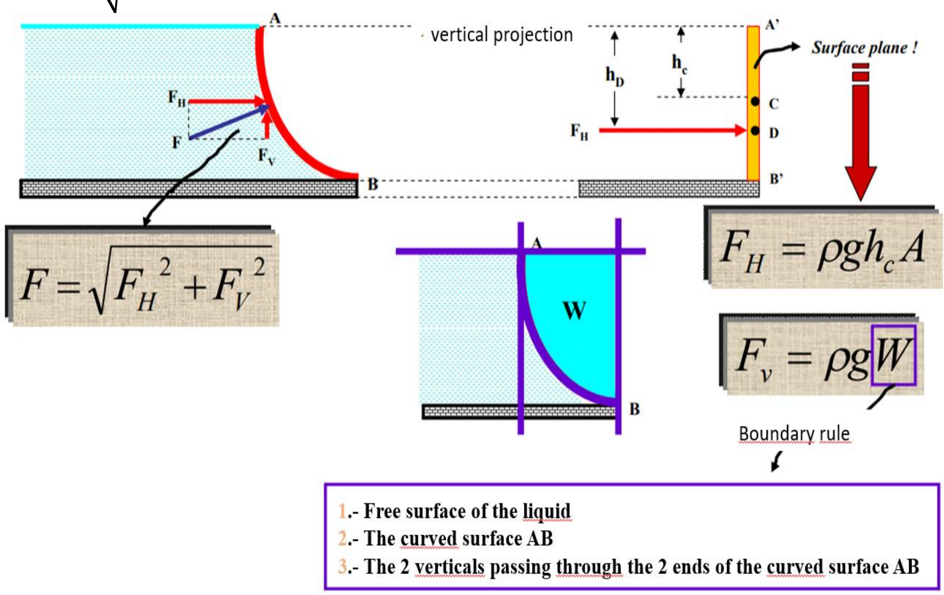
- The curved surface AB
- The free surface of the fluid
- The 2 verticals drawn from the 2 ends A and B of the surface.

The calculation of the vertical component F_V is therefore summed up by the calculation of the Weight of the fluid represented by the volume displaced by the surface AB.



Calculation of the resulting pressure force

The calculation of the 2 components FH and FV then allows the resultant F to be determined by the expression: $F = \sqrt{F_H^2 + F_V^2}$



3.5.2 Position of the point of application of the Pressure Force

The point of application of the resultant F is obtained if the components F_H and F_V are known. In the general case, it will be necessary to establish the equation of the curve AB and that of the segment representing the force F (equation of a straight line) taking into account that the angle of inclination of the resulting force F relative to the horizontal is obtained by the following formula:

$$\theta = arctg\,\frac{F_V}{F_H}$$