SW 4

Exercise 1:

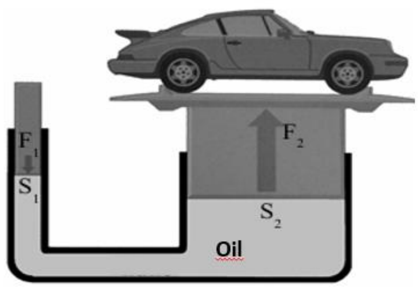
Consider the system described by the image opposite. Neglect the difference in heights of the liquid.

a) Express the force F_2 as a function of the force F_1 and the surfaces S_1 and S_2.

b) How much does the surface S_2 rise when the surface S_1 descends from a height h1?

c) When S_2 is about 100 times greater than S_1, how can we practically make S_2 rise by 2.00 meters?

(S_1 cannot descend more than a few decimeters!)



Solution

a) Neglecting the difference in height of the liquid, the pressure in (1) is the same as in (2).

We therefore have: $P_1=P_2$, therefore $F_1/S_1= F_2/S_2$.

By isolating F_2, we obtain: $F_2 = S_2 \cdot F_1/S_1$. We prefer to write: $F_2=\frac{S2 \cdot F1}{S1}$

b) When the piston in (1) goes down, the one in (2) goes up so that the volume of oil that is driven out in (1) equals that which is pushed in (2). Therefore $S_1 \cdot h_1=S_2 \cdot h_2$

The surface S_2 therefore goes up by $h_2= \frac{S1}{S2} \cdot h_1$

when the surface S_1 goes down by h_1.

Of course, we also have the relation ship: $h_1= \frac{S2}{S1} \cdot h_2$

c) In the case where $S_2/S_1=100$, h1 would have to be lowered by 200 meters to raise the car by 2.00 meters. This is not practical.

What is done in practice is to lower the surface S_1 by h_1 = 20 centimeters, then block the oil duct between sides (1) and (2) and raise the surface S1 by 20 centimeters by injecting oil from an external tank, to open the duct between sides (1) and (2) again.

By performing this sequence of movements 1,000 times, the car is raised by 2.0 meters.

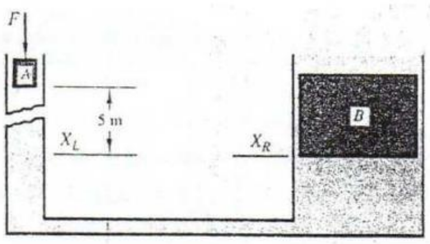
This process requires a volume of oil from the tank equal to $V = S_2 \cdot h_2$.

If S_2 = 3.00 [m^2] and h_2 = 2.00 [m], then the volume of oil injected from the reservoir is:

V =$S_2 \cdot h_2$ =3.00·2.00=6.00 [m^3]. The variation in volume on side (1) is negligible.

Exercise 2:

In the figure opposite, the surfaces of cylinders A and B are respectively 40 and 4000 cm^2 and B has a mass of 4000 kg. The container and the pipes are filled with oil of density d = 0.75. Neglecting the weight of cylinder A, determine the force F that will ensure equilibrium.



Solution:

Let us first determine the pressure acting on piston A:

Since X_L and X_R are at the same level in the same liquid, then Pressure at X_L = pressure at XR.

Pressure at X_L = Pressure under A + pressure due to 5 m of oil = $P_A + d_{oil}.\rho_{water}.g.h$

And Pressure at X_R =$\frac{WEIGHT\ OF\ B}{SURFACE\ OF\ B} = \frac{m_B}{SURFACE\ OF\ B}$

Pressure at X_L = pressure at $X_R \Rightarrow P_A + d_{oil}.\rho_{water}.g.h = \frac{m_B}{SURFACE\ OF\ B}$

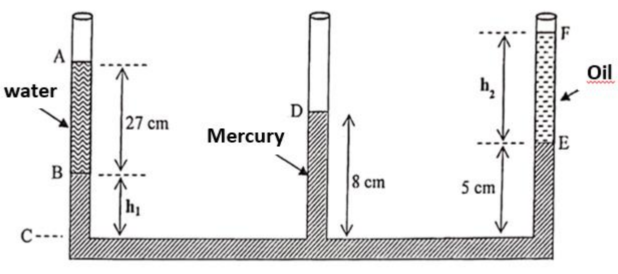
AN : P_A+ 0,75 × 1000 × 9,807 × 5=4000×9,807 / 0,4 ⇒ P_A +36776,25=98070 ⇒ P_A = 61293,75 Pa

Force F = $P_A \times S_A$ = 61293,75 × 4 × 10-3 = 245,175 N

Exercice 3

For the three immiscible liquids in Figure 1, determine the heights at which the ends of the three tubes are open to the atmosphere.

We are given: the density of mercury ρ_{Hg} = 13600 kg/m3. the density of water ρ_{H20}= 1000 kg/m3, the density of oil ρ_{oil}= 780 kg/m3



Solution de l'Exercice 1 : [6/6 points] Corrigé de l'Examen 12 Févr 2022.

Déterminer les hauteurs h_1 et h_2 ?

① Appliquer la Relation Fondamentale de l'Hydrostatique (RFH) entre A et B, on obtient :

$$P_A + \rho_{H_2O} \cdot g \cdot z_A = cst \,, \qquad P_B + \rho_{H_2O} \cdot g \cdot z_B = cst.$$

$$P_A + \rho_{H_2O} \cdot g \cdot z_A = P_B + \rho_{H_2O} \cdot g \cdot z_B \text{ ..... [1]} \quad 0,5$$

② Appliquer R.F.H. entre B et C, on obtient :

$$P_B + \rho_{Hg} \cdot g \cdot z_B = P_C + \rho_{Hg} \cdot g \cdot z_C \text{ ..... [2]} \quad 0,5$$

③ Appliquer R.F.H. entre C et D, on obtient :

$$P_C + \rho_{Hg} \cdot g \cdot z_C = P_D + \rho_{Hg} \cdot g \cdot z_D \text{ ..... [3]} \quad 0,5$$

On somme [1] + [2] + [3], on obtient : 0,5

$$P_A + \rho_{H_2O} \cdot g \cdot z_A + \rho_{Hg} \cdot g \cdot z_B + \rho_{Hg} \cdot g \cdot z_C = P_D + \rho_{Hg} \cdot g \cdot z_D + \rho_{Hg} \cdot g \cdot z_C + \rho_{H_2O} \cdot g \cdot z_B$$

Comme : $P_A = P_D = P_{atm}$ {Les tubes sont ouverts à l'Atmosphère).

$$\rho_{H_2O} \cdot g \cdot (z_A - z_B) + \rho_{Hg} \cdot g (z_B - z_C) = \rho_{Hg} \cdot g \cdot (z_D - z_C)$$

$$(z_B - z_C) = \frac{\rho_{Hg} \cdot (z_D - z_C) - \rho_{H_2O} \cdot (z_A - z_B)}{\rho_{Hg}} \quad 0,5$$

AN: $z_B - z_C = \frac{13600 \cdot 0,08 - 1000 \cdot 0,27}{13600} = 0,0601\ m$

$\boxed{h_1 = 6,01\ cm}$ 0,5

Pour (h_2) on procède de la même manière :

Ⓐ Appliquer R.F.H entre F et E, on obtient :

1/5

$P_F + \rho_{huile} \cdot g \cdot z_F = P_E + \rho_{huile} \cdot g \cdot z_E \ \ldots\ [4]$ 0,5

Ⓑ Appliquer R.F.H. entre E et C ; on obtient :

$P_E + \rho_{Hg} \cdot g \cdot z_E = P_C + \rho_{Hg} \cdot g \cdot z_C \ \ldots\ [5]$ 0,5

Ⓒ Appliquer R.F.H. entre C et D, on obtient :

$P_C + \rho_{Hg} \cdot g \cdot z_C = P_D + \rho_{Hg} \cdot g \cdot z_D \ \ldots\ [6]$ 0,5

On somme $[4] + [5] + [6]$; on obtient :

$P_F + \rho_{huile} \cdot g z_F + \rho_{Hg} \cdot g \cdot z_E + \rho_{Hg} \cdot g \cdot z_C = \rho_{huile} \cdot g \cdot z_E + \rho_{Hg} \cdot g \cdot z_C + \rho_{Hg} \cdot g \cdot z_D + P_D$

Comme : $P_F = P_D = P_{atm}$. 0,5

$\rho_{huile}(z_F - z_E) = \rho_{Hg} \cdot (z_D - z_C) + \rho_{Hg} \cdot (z_C - z_E)$.

AN: $(z_F - z_E) = \frac{\rho_{Hg} \cdot (z_D - z_C) + \rho_{Hg} \cdot (z_C - z_E)}{\rho_{huile}}$ 0,5

$z_F - z_E = \frac{13600 \cdot \{0,08 - 0,05\}}{780}$; $\boxed{h_2 = 0,523\ m = 52,3\ cm}$

5/5 Points 0,5