CHAPTER 2

HYDROSTATIC PRESSURE

2.1 INTRODUCTION

The air around us at sea level presses down on us at 1 bar. We do not feel this pressure. But if you swim down into the ocean just a few meters and you will start to notice a change. You will start to feel an increase of pressure on your eardrums. The deeper you go under the sea, the greater the pressure pushing on you will be.

2.2 Hydrostatic pressure

Hydrostatic pressure is the pressure that is exerted by a fluid at equilibrium at a given point within the fluid, due to the force of gravity. Hydrostatic pressure increases in proportion to depth measured from the surface because of the increasing weight of fluid exerting downward force from above.

Pressure = normal force exerted by a fluid per unit area

Pressure = Force/Area

Force = mass x acceleration = m x g (acceleration in gravity)

So: Pressure = F/A = mg/A

- Density = Mass/Volume ; Mass= Density x Volume
- We now have Pressure = (density x volume x acceleration)/area.

The pressure due to the liquid alone (i.e. the gauge pressure) at a given depth depends only upon the density of the liquid, the acceleration of gravity and the distance below the surface of the liquid.

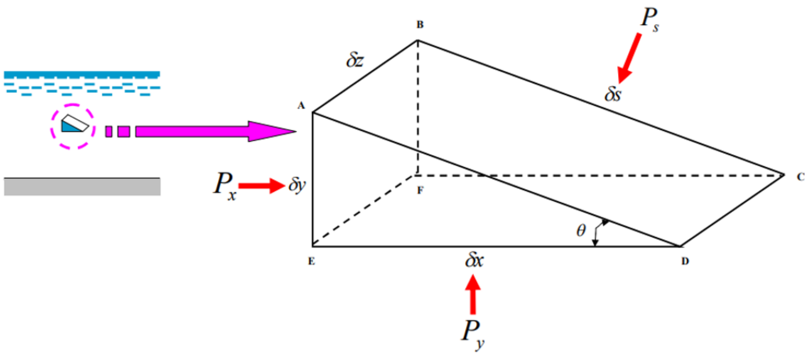
The formula that gives the P pressure on an object submerged in a fluid is therefore:

$P = \rho * g * h$

ρ (rho) is the density of the fluid,
g is the acceleration of gravity
h is the height of the fluid above the object

2.2.1 Pressure at a point of a fluid

Let us consider an element of a fluid ABCDEF(triangular prism) and let P_x, P_y and P_s be the pressures in the 3 directions x, yand s:Let us establish the relationship between P_x, P_y and P_s



* According to the axis of x :

- Force due to P_x : $F_{xx} = P_x . (ABFE) = P_x \partial y \partial z$

- Force due to Py : $F_{yx} = 0$

- Component due to P_s : $F_{sx} = - P_s.(ABCD.\sin\theta) = - Ps. \partial s \partial z \, \partial y / \partial s$

Because $\sin\theta = \partial y / \partial s$

SO : $F_{sx} = - Ps. \, \partial z \, \partial y$

and since the fluid is in equilibrium : $Fxx + Fyx + Fsx = 0$

whence : $P_x . \partial y \, \partial z - P_S. \partial y \, \partial z = 0$

and so : $P_x = P_s$

* According to the axis of y :

- Force due to P_x : $F_{xy} = 0$

- Force due to Py : $F_{yy} = P_y . (CDEF) = P_y \, \partial x \partial z$

- Component due to P_s : $F_{sy} = - P_s.(ABCD.\cos\theta) = - Ps. \partial s \partial z \, \partial x / \partial s$

Because $\cos\theta = \partial x / \partial s$

SO : $F_{sy} = - Ps. \, \partial z \, \partial x$

and since the fluid is in equilibrium : $Fxy + Fyy + Fsy = 0$

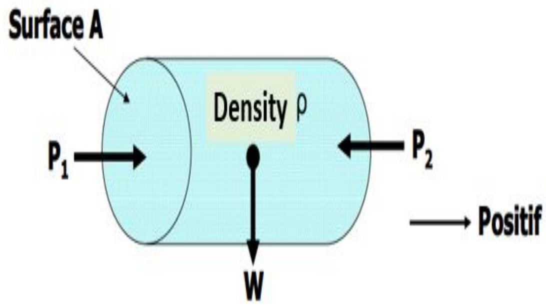
whence : $P_x . \partial x \, \partial z - P_S. \partial x \, \partial z = 0$

and so : $P_y = P_s$

$$\left.\begin{array}{l} P_x = Ps \\ P_y = Ps \end{array}\right] \rightarrow P_x = Py = Ps$$

The pressure of a fluid at a point is the same in all directions

2.2.2 Equality of pressures on the same horizontal plane

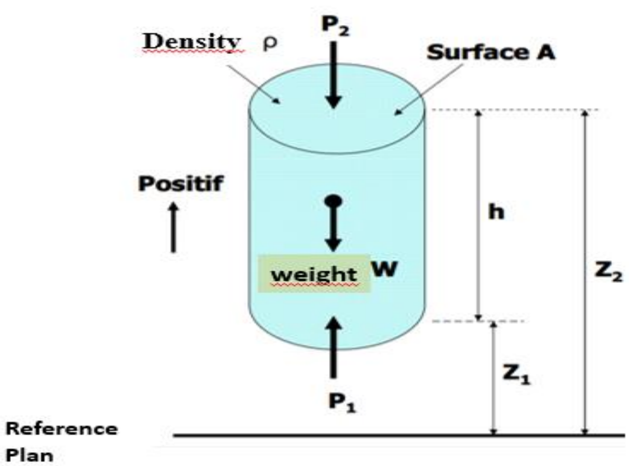


P_1A-P_2A + 0 =0 P_1 = P_2

(because the component of the weight W along the horizontal is zero) ρSurface AP1P2WPositive

On the same horizontal plane, all pressures are equal(Isobar Pressures)

2.3 Fundamental Equation of Hydrostatics



$$P_1 - P_2 = \rho g(Z_2 - Z_1) \Rightarrow P_1 + \rho g Z_1 = P_2 + \rho g Z_2$$

$$P_1 + \rho g Z_1 = P_2 + \rho g Z_2 \Rightarrow \frac{P_1}{\rho g} + Z_1 = \frac{P_2}{\rho g} + Z_2$$

$$\boxed{Z + \frac{P}{\rho g} = C^{ste}}$$

By setting Z_2 -Z_1 = h and $P_2 = P_0$,

we will have: $P_1 = P_0 + \rho gh$

and if $P_0 = 0$: $P_1 = \rho gh$

The pressure therefore increases linearly as a functionof depth

2.4 types of pressure

Four main types of pressure:

a. Gage Pressure : Difference between atmospheric and absolute pressure

b. Vacuum Pressure : Occasionally used for a negative gage pressure

c. Atmospheric Pressure = 101.325 kPa

d. Absolute Pressure =

i. Gage Pressure + Atmospheric Pressure

ii. Atmospheric Pressure - Vacuum Pressure

2.4.1 Effective pressure (gauge)and absolute pressure

At point M, the pressure is equal to: $P_M = Po + \rho gh$

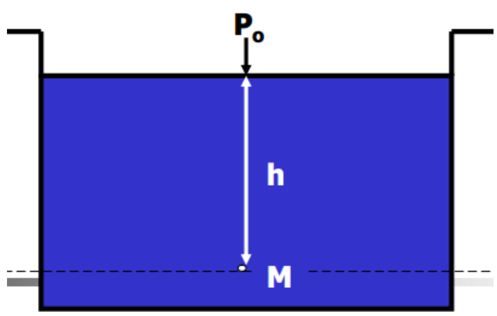
At the free surface of the fluid, the pressure is generally

represented by the atmospheric pressure Patm, hence:

$P_M = P_o + \rho gh$:absolute pressure

if we neglect the influence of atmospheric pressure ($P_{atm} = 0$):

$P_M = \rho gh$:gauge pressure (Effective pressure)



2.4.2 VOID HEIGHT

In some cases, absolute pressure is lower than atmospheric pressure:

$P_M = P_o + \rho gh$

A depression is then created whose corresponding height, called “Void Height”, is equal to:

$$h_{void} = \frac{P_{atm} - P_{abs}}{\rho g}$$

2.5 Load and piezometric height

$Z + \frac{P}{\rho g}$ = Constant

with:

Z [L] : position height or geometric dimension

$\frac{P}{\rho g}$ [L] : Piezometric height

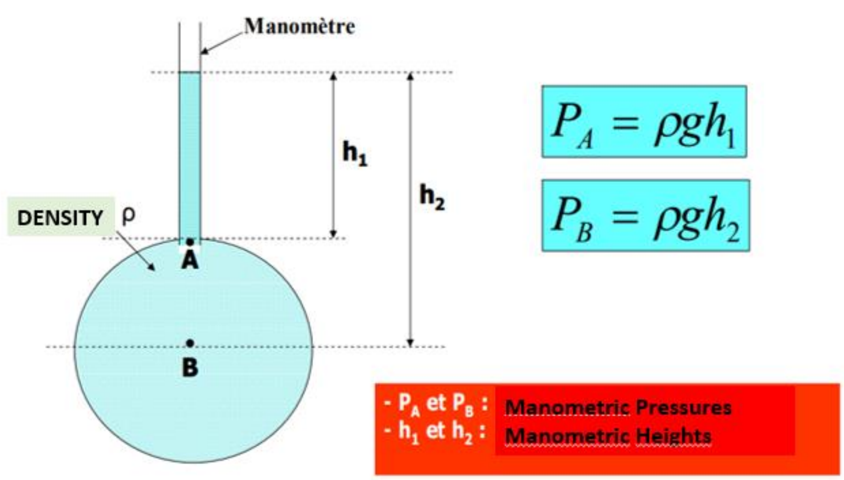
$Z + \frac{P}{\rho g}$ [L] : Total height or load

2.6 Pressure measuring devices

The device used depends on the importance of the pressures to be measured.There are 2 types of pressure measuring devices:

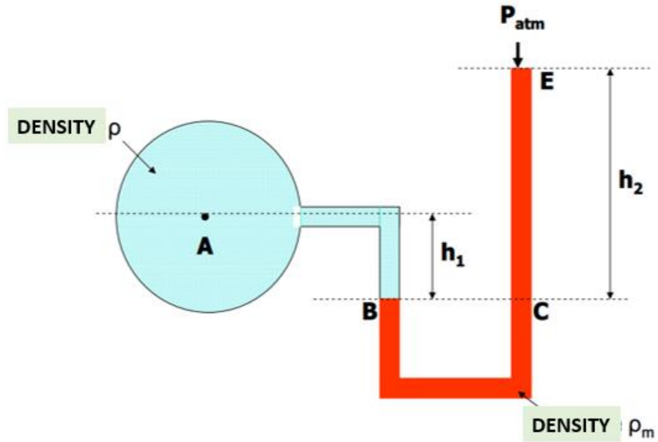
- Manometric tubes: used for measuring relatively low pressures (… in laboratories)
- Mechanical pressure gauges: used for measuring relatively higher pressures (1 to 2 kg/cm2)

2.6.1 Pressure measurement by manometric tubes

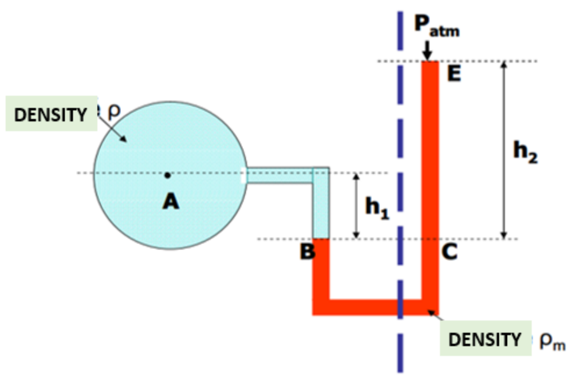


It is a device used only for measuring the pressures of liquids and not gases.

2.6.2 The manometric tube in the shape of '' U ''



According to the law of hydrostatics, we can write that: $P_B = P_C$



- **Left Part:** $P_B = P_A + \rho g h_1$
- **Right Part:** $P_C = P_E + \rho_m g h_2 = P_{atm} + \rho_m g h_2$

And if we do not take P_{atm} into account $P_C = \rho_m g h_2$

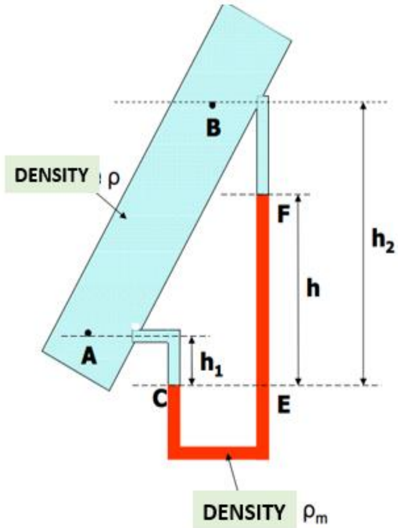
$$P_B = P_C \longrightarrow P_A + \rho g h_1 = \rho_m g h_2$$

And finally: : $P_A = \rho_m g h_2 - \rho g h_1$

Note:-
If the fluid of density ρ is a gas, its density is negligiblecompared to that of the manometric liquid:

$$P_A = \rho_m g\left(h_2 - \frac{\rho}{\rho_m} h_1\right) \rightarrow 0 \quad \Longrightarrow \quad \rho \langle\langle \rho_m \Rightarrow P_A = \rho_m g h_2$$

Measuring the pressure difference with a U-shaped manometer:



- **Right Branch:**

$$P_C = P_A + \rho g h_1$$

- **Left Branch:**

$$P_E = P_B + \rho g (h_2 - h) + \rho_m g h$$

and as $P_C = P_E$

$$P_A + \rho g h_1 = P_B + \rho g (h_2 - h) + \rho_m g h$$

And so: $P_A - P_B = \rho g (h_2 - h_1) + (\rho_m - \rho) g h$

and if the fluid is a gas $(\rho_m >> \rho)$: $P_A - P_B = \rho_m g h$

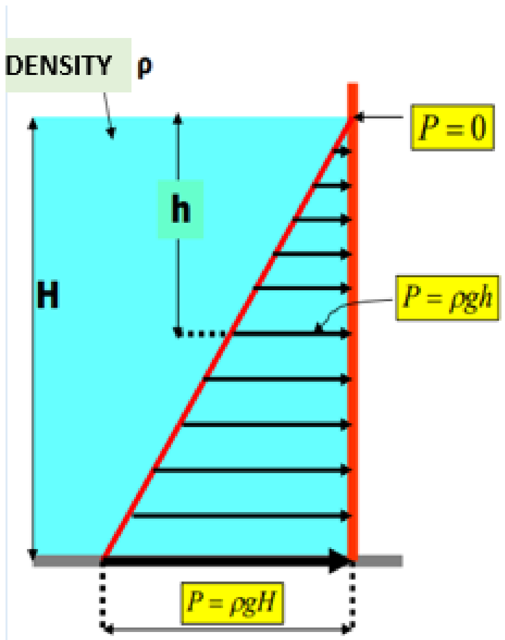
2.6.3 Water Manometer and Mercury Manometer

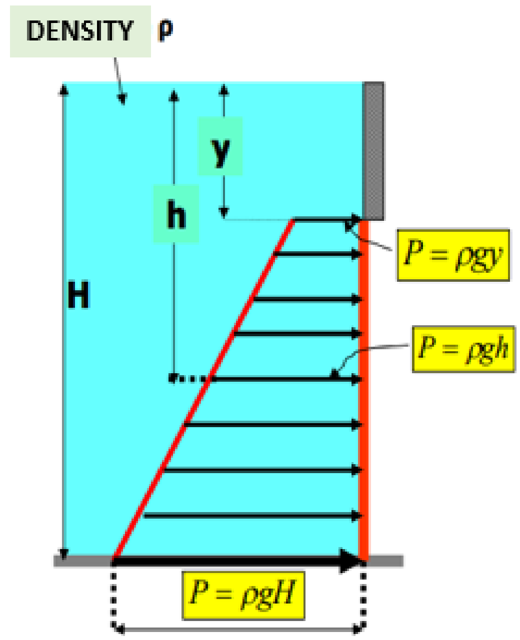
Water manometers are used to measure relatively low pressures because their use for high pressures would lead to the development of tubes with exaggerated dimensions. This is why, and given its high density, we prefer to use Mercury as a manometric liquid.

2.7 Pressure diagram

This involves plotting the pressure evolution graph on a surface, taking into accountthe fact that the pressure varies linearly with depth according to the law

$P = \rho g h$


DENSITY ρ
h
H
P = 0
P = ρgh
P = ρgH


DENSITY ρ
y
h
H
P = ρgy
P = ρgh
P = ρgH