PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA
Ministry of Higher Education and Scientific Research

MOHAMED KHIDER UNIVERSITY – BISKRA-
FACULTY OF EXACT SCIENCES AND SCIENCES OF
NATURE AND LIFE
MATTER SCIENCES DEPARTMENT

Physics Handout

Tutorials Reminders
and
Solved exercises

1st year common core Biology

Dr. laznek samira MCB

November 2023

Preface

This handout of tutorials in physics is aimed at first year common core Biology students. Its goal is the application of basic concepts of geometric optics, hydrostatics and hydrodynamics. Geometric optics is an essential tool for understanding the design of optical instruments, their operation and their measurement methods. Hydrostatics and hydrodynamics aim to study the statics and dynamics of fluids.

This handout on Physics consists of six t series of corrected exercises. Each series begins with a lesson reminder followed by application exercises, as following:

Tutorial N°1 : Exercises on dimensional analysis and error calculation**.**

Tutorial N°2 : Exercises on the propagation of light, plane diopters and the prism.

Tutorial N°3 : Exercises on spherical diopters and thin lenses.

Tutorial N°4 : Exercises on plane and spherical mirrors and the reduced eye.

Tutorial N°5 : Exercises on Pascal's law and Archimedes thrust. (Hydrostatic)

Tutorial N°6 : Exercises on Bernoulli's law (hydrodynamics)

Exercise data and solutions will allow students to consolidate their abilities and practice applying the rules provided in the course reminders distributed prior to these exercises. I hope that this handout will be of valuable assistance to Biology students in understanding and mastering the Physics unit and thus successfully completing the first year of Common Core Biology.

TABLE OF CONTENTS

Tutorial N°1

1. Exercises on dimensional analysis and error calculation.

REMINDER

1.1 PHYSICAL DIMENSIONS:

As for any quantity, the value of a fundamental constant can be expressed as the product of a number and a unit. The definitions below specify the exact numerical value of each constant when its value is expressed in the corresponding SI unit. By fixing the exact numerical value, the unit becomes defined, since the product of the numerical value and the unit has to equal the value of the constant, which is postulated to be invariant.

The seven constants are chosen in such a way that any unit of the SI can be written either through a defining constant itself or through products or quotients of defining constants.

The International System of Units, the SI, is the system of units in which

- the unperturbed ground state hyperfine transition frequency of the caesium 133 atom, $\Delta\nu_{Cs}$, is 9 192 631 770 Hz,
- the speed of light in vacuum, c, is 299 792 458 m/s,
- the Planck constant, *h*, is 6.626 070 15 × 10−34 J s,
- the elementary charge, *e*, is 1.602 176 634 × 10^{-19} C,
- the Boltzmann constant, *k*, is 1.380 649 × 10^{-23} J/K,
- the Avogadro constant, N_A, is 6.022 140 76 × 10^{23} mol^{-1},
- the luminous efficacy of monochromatic radiation of frequency 540 × 10^{12} Hz, K_{cd}, is 683 lm/W,

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, meter, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively,

according to Hz = s^{-1} , J = kg m^2 s^{-2} , C = A. s, lm = cd.m^2.m^{-2} = cd.sr and W = kg m^2 s^{-3} .

The numerical values of the seven defining constants have no uncertainty.

Table 1.1. The seven defining constants of the SI and the seven corresponding units they define.

Defining constant	Symbol	Numerical value	Unit
hyperfine transition frequency of Cs	$\Delta\nu_{Cs}$	9 192 631 770	Hz
Speed of light in vacuum	c	299 792 458	m s−1
Planck constant	h	$6.626\ 070\ 15 \times 10^{-34}$	J s
Elementary charge	e	$1.602\ 176\ 634 \times 10^{-19}$	C
Boltzmann constant	k	$1.380\ 649 \times 10^{-23}$	$J\ K^{-1}$
Avogadro constant	N_A	$6.022\ 140\ 76 \times 10^{23}$	mol^{-1}
Luminous efficacy	k_{cd}	683	$lm\ W^{-1}$

1.2 DIMENSIONS OF QUANTITIES:

Physical quantities can be organized in a system of dimensions, where the system used is decided by convention. Each of the seven base quantities used in the SI is regarded as having its own dimension. The symbols used for the base quantities and the symbols used to denote their dimension are shown in Table 1.2.

Table 1.2. Base quantities and dimensions used in the SI

Base quantity	Typical symbol for quantity	Symbol for dimension
Time	t	T
Length	l, x, r, etc	L
Mass	m	M
Electric current	I, i	I
Thermodynamic temperature	T	Ɵ
amount of substance	n	N
luminous intensity	I_V	J

All other quantities, with the exception of counts, are derived quantities, which may be written in terms of base quantities according to the equations of physics. The dimensions of the derived quantities are written as products of powers of the dimensions of the base quantities using the equations that relate the derived quantities to the base quantities. In general, the dimension of any quantity Q is written in the form of a dimensional product,

$$\textit{\textbf{dim Q}} = T^{\alpha} L^{\beta} M^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta}$$

where the exponents α, β, γ, δ, ε, ζ and η, which are generally small integers, which can be positive, negative, or zero, are called the dimensional exponents.

1.3 ABSOLUTE UNCERTAINTY:

The absolute uncertainty in a quantity is the actual amount by which the quantity is uncertain, e.g.if L = 6.0 ± 0.1 cm, the absolute uncertainty in L is 0.1 cm. Note that the absolute uncertainty of a quantity has the same units as the quantity itself.

1.4 RELATIVE UNCERTAINTY:

This is the simple ratio of uncertainty to the value reported. As a ratio of similar quantities, the relative uncertainty has no units.

1.5 CALCULATION OF ABSOLUTE UNCERTAINTY:

To calculate the absolute uncertainty of a physical quantity$f(x; y; z; \dots)$, which depends on the measurable variables x; y; z…, we use the differential method by following the following steps:

0We calculate the partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \dots.$

- We write the differential according to these partial derivatives:

$$df = \left|\frac{\partial f}{\partial x}\right| dx + \left|\frac{\partial f}{\partial y}\right| dy + \left|\frac{\partial f}{\partial z}\right| dz + \cdots ..$$

- We approximate the differential df to its absolute uncertaintyΔf: $df = \Delta f$. Likewise for the differentials of the variables $dx = \Delta x, dy = \Delta y, \dots$
- We calculate the absolute uncertainty from the expression:

$$\Delta f = \left|\frac{\partial f}{\partial x}\right| \Delta x + \left|\frac{\partial f}{\partial y}\right| \Delta y + \left|\frac{\partial f}{\partial z}\right| \Delta z + \cdots ..$$

- We can then use the absolute uncertainty Δf to calculate the relative uncertainty$\frac{\Delta f}{f}$.

1.6 CALCULATION OF RELATIVE UNCERTAINTY:

To calculate the relative uncertainty of a physical quantity $f(x; y; z; \dots)$we use the logarithmic differential method by following the following steps:

- We write the logarithm lnfaccording to the logarithms of the variables $lnx, lny, lnz \dots$

- We calculate the differential $d(lnf)$ in functions of the differentials $d(lnx), d(lny)$, $d(lnz)$ ...
- Knowing that for example: $d(lnf) = \frac{df}{f}$, we replace the logarithmic differentials by the differentials: $\frac{df}{f}, \frac{dx}{x}, \frac{dy}{y}, \frac{dz}{z}, \ldots$.
- The relative uncertainty $\frac{\Delta f}{f}$ is obtained by replacing the differentials $\frac{df}{f}, \frac{dx}{x}, \frac{dy}{y}, \frac{dz}{z}$ .., with the relative uncertainties $\frac{\Delta f}{f}, \frac{\Delta x}{x}, \frac{\Delta y}{y}, \frac{\Delta z}{z}, \ldots$
- We can then use the relative uncertainty $\frac{\Delta f}{f}$ to calculate the absolute uncertainty Δf.

Tutorial N°1: Exercises on dimensional analysis and error calculation.

Exercise 1.1:

Establish from simple formulas, the dimensions and fundamental units of quantities following speed v, acceleration a, force F, surface S, volume V, density p, energy E, pressure P.

Solution

The dimension of the physical quantity speed v is

$$\dim v = \frac{length}{time} = \frac{L}{T} = LT^{-1} \text{ unit } m.s^{-1}$$

The dimension of the physical quantity acceleration a is

$$\dim a = \frac{length}{T^2} = \frac{L}{T^2} = T^{-2}L \text{ unit } m.s^{-2}$$

The dimension of the physical quantity force F is

$$\dim F = mass\ x\ acceleration = mass\ x\ \frac{length}{T^2} = \frac{LM}{T^2} = T^{-2}LM \text{ unit kg}.m.s^{-2}$$

The dimension of the physical quantity pressure P is

$$\dim P = \frac{force}{area} = \frac{LMT^{-2}}{L^2} = T^{-2}L^{-1}M \text{ unit kg}.m^{-1}.s^{-2}$$

Exercise 1.2:

Experience shows that the force with which a liquid acts on a ball immersed in it is proportional to the radius of the ball r as well as its speed v. We write its expression:

$F = 6\pi\mu^x r^y v^z$

where μ is a dimension coefficient : $\mu = ML^{-1}T^{-1}$

1- Find x, y and z

When the speed is a little high, the expression for the force becomes $F = kSv^2$,where k is a constant and S is the area of the great circle.

2- Find the dimension k.

3- Demonstrate that the kinetic energy ($Ec = \frac{1}{2}mv^2$) has the same dimension as a work $\omega = FL$.

Solution

1- $F = 6\pi\mu^x r^y v^z$, $\mu = ML^{-1}T^{-1}$

$$[F] = [\mu]^x[r]^y[v]^z = M^xL^{-x}T^{-x}L^yL^zT^{-z} = M^xL^{-x+y+z}T^{-(x+z)} \qquad (1)$$

On the other hand, we have

$$[F] = [ma] = MLT^{-2} \qquad (2)$$

$$(1) = (2)\begin{cases} x = 1 \\ -x + y + z = 1 \\ -(x + z) = -2 \end{cases}$$

$$x = 1, y = 1, z = 1$$

So $F = 6\pi\mu rv$

2- $E_c = \frac{1}{2}mv^2 \rightarrow [E_c] = \left[\frac{1}{2}\right][m][v]^2 = ML^2T^{-2} \qquad (1)$

The work $\boldsymbol{\omega = FL}$ $\rightarrow$ $[\omega] = [F][L] = MLT^{-2}L = ML^2T^{-2} \qquad (2)$

$$(1) = (2) \Rightarrow [E_c] = [\omega]$$

$$(Kg.m^2.s^{-1})$$

Exercise 1.3:

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. L is about 10 cm and is known to 1 mm accuracy. The period of oscillations is about 0.634 second. The time of 100 oscillations is measured with a wristwatch of 1s resolution. What is the accuracy in the determination of g?

Solution

The accuracy in determination of g is found in terms of minimum percentage error in calculation. The percentage error in g = $\frac{\Delta g}{g} x\ 100\%$, where $\frac{\Delta g}{g}$ the relative error in determination of g.

$T = 2\pi\sqrt{\frac{L}{g}}$ $or\ T^2 = 4\pi^2\frac{L}{g}$ $or\ g = \frac{4\pi^2\ L}{T^2} = \frac{4\pi^2\ 0.1}{(0.634)^2} = 9.81m/s^2;$

Now , $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2 \times \frac{\Delta T}{T}$

In terms of percentage, $100 \times \frac{\Delta L}{L} = 100 \times \frac{0.1}{10} = 1\%$

Percentage error in $T\ is\ 100 \times \frac{\Delta T}{T} = 100 \times \frac{1}{100 \times 0.634} = 1.57\%$

Thus percentage error in g $= \frac{\Delta g}{g} \times 100\% = 1\% + 2 \times 1.57\% = 4.14\%$

$g = (g \mp \Delta g)\ m/s^2$

$g = (9.81 \mp 4.14)\ m/s^2$

Exercise 1.4:

The error in measuring the radius of the sphere is 0.5%. What is the permissible percentage error in the measurement of its (a) surface area and (b) volume?

Solution

Percentage error in determination of any quantity = Relative error in determination of quantity ×100%. The relative error in area and volume of sphere are:
$\frac{\Delta A}{A} = \frac{2\Delta r}{r}\ and\ \frac{\Delta V}{V} = \frac{3\Delta r}{r}$ respectively.

Given $\frac{\Delta r}{r} = 0.5\%$

(a) The surface area of a sphere of radius r is $A = 4\pi r^2$

Percentage error in $A = \frac{\Delta A}{A} \times 100 = \frac{2\Delta r}{r} \times 100 = 2 \times 0.5\% = 1\%$

(b) The volume of a sphere with radius r is $V = \frac{4\pi}{3} r^3$

Percentage error in $V = \frac{\Delta V}{V} \times 100 = \frac{3\Delta r}{r} \times 100 = 3 \times 0.5\% = 1.5\%$

Tutorial N°2

2. Exercises on the propagation of light, plane diopters and the prism.

REMINDER

2.1 RETRACTION INDEX:

The term "refraction of light" refers to how light bends when it travels perpendicularly through transparent media. The refractive index is a measurement of how much a light beam bends as it passes through various materials. The speed of light in a medium reduces as it moves from a rarer to a denser one.

2.2 DIOPTRES:

A diopter is a surface separating two different medium indexes. Apart from those with mirrors or diffracting surfaces, usual optical systems (camera objective lenses, projection lenses, glasses, microscopes) are exclusively made of a number of diopters.
Optical systems generally have a revolution axis and diopters used are generally spherical or plane. The system axis is the line goring through the diopters centers of curvature; it is perpendicular to the diopters planes.

2.3 THE LAW OF REFRACTION:

When light travels from one medium to another, it generally bends, or *refracts*. The law of refraction gives us a way of predicting the amount of bend. This law is more complicated than that for reflection, but an understanding of refraction will be necessary for our future discussion of lenses and their applications. The law of refraction is also known as Snell's Law, named for Willobrord Snell, who discovered the law in 1621.

2.4 Snell's LAW

Like with reflection, refraction also involves the angles that the incident ray and the refracted ray make with the normal to the surface at the point of refraction. Unlike reflection, refraction also depends on the media through which the light rays are travelling. This dependence is made explicit in Snell's Law via *refractive indices*, numbers which are constant for given media.

Snell's Law is given in the following diagram.

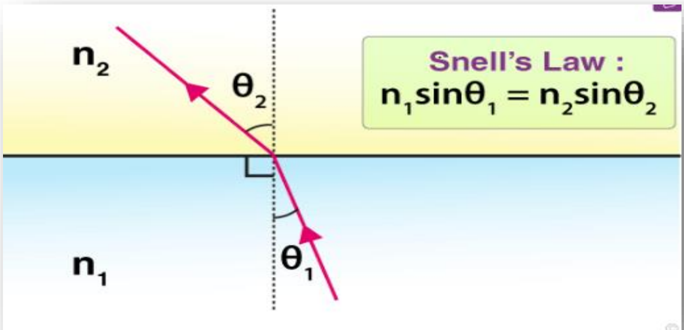


Figure 2.1: Snell's Law.

The normal on the surface is used to gauge the angles that the refracted ray creates at the contact point.

2.5 COMPLEX Snell's LAW DIAGRAM:

A complex diagram of Snell's Law displays something that is not directly obvious. A ray of light passes through the glass and standing behind it the viewer experiences refraction through three media. The situation is represented in the following diagram :

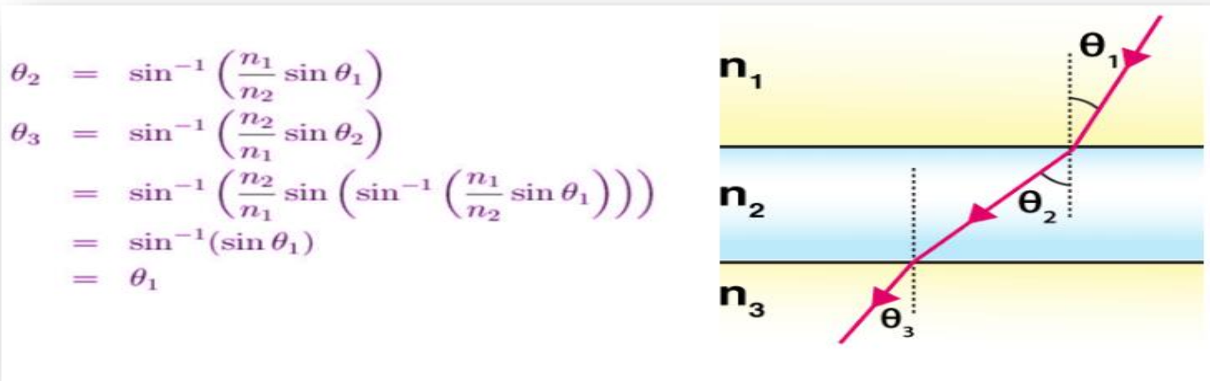


Figure 2.2: A complex diagram of Snell's Law.

2.6 CRITICAL ANGLE:

The critical angle is the angle of incidence where the angle of refraction is 90 degrees. Light must travel from an optically more denser medium to an optically less dense medium.

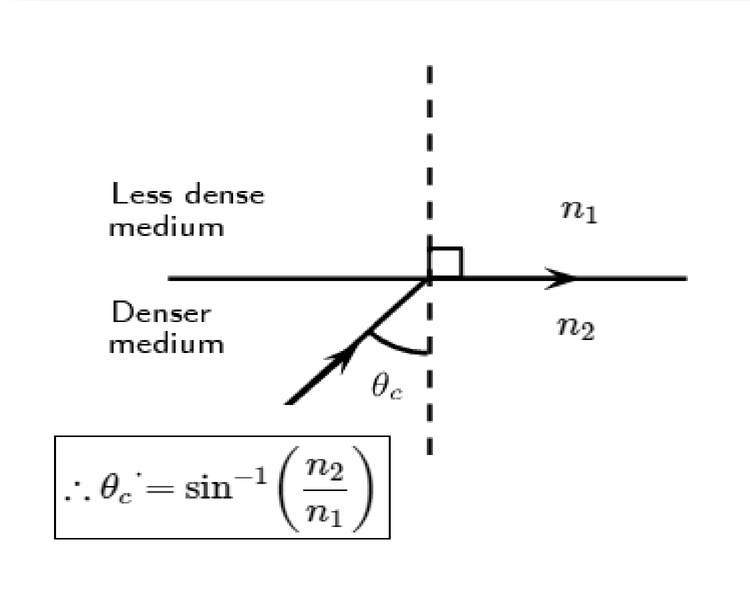


Figure 2.3: When the angle of incidence is equal to the critical angle, the angle of refraction is equal to $\mathbf{90°}$.

If the angle of incidence is bigger than this critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called **total internal reflection**.

The conditions for total internal reflection are:

1. Light is travelling from an optically denser medium (higher refractive index) to an optically less dense medium (lower refractive index).
2. The angle of incidence is greater than the critical angle.

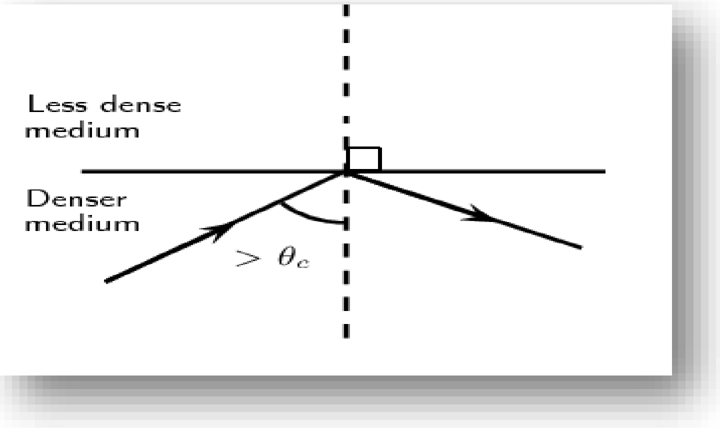


Figure 2.4: When the angle of incidence is greater than the critical angle, the light ray is reflected at the boundary of the two media and total internal reflection occurs.

2.7 PRISM

A prism of index n is composed of two dioptric planes forming an angle A. Following figure 2.5, a luminous ray enters from side 1 under incidence i and comes out of side 2 under incidence i', the corresponding refraction angles in the prism are r and r', D is the deviation from the ray prorogued by the prism. The angular sign convention is normal for side 1 and inversed for side 2. We have the following form:

$$\sin \mathrm{i} = n \sin r$$

$$\sin \mathrm{i}' = n \sin r'$$

$$A = r + r'$$

$$D = \mathrm{i} + \mathrm{i}' - A$$

At the minimum of deviation: i = i' et r = r', we obtain a relationship between n, A and D, allowing index measures of optical material:

$$n = \frac{\sin[(D + A)/2]}{\sin(A/2)}$$

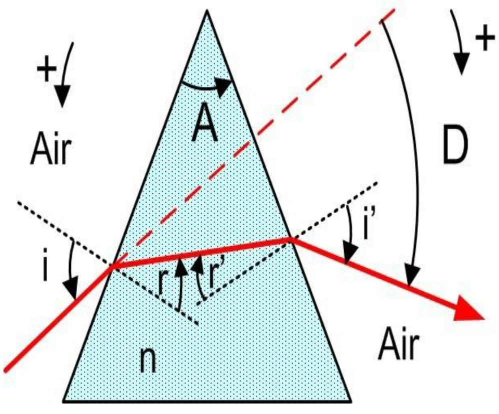
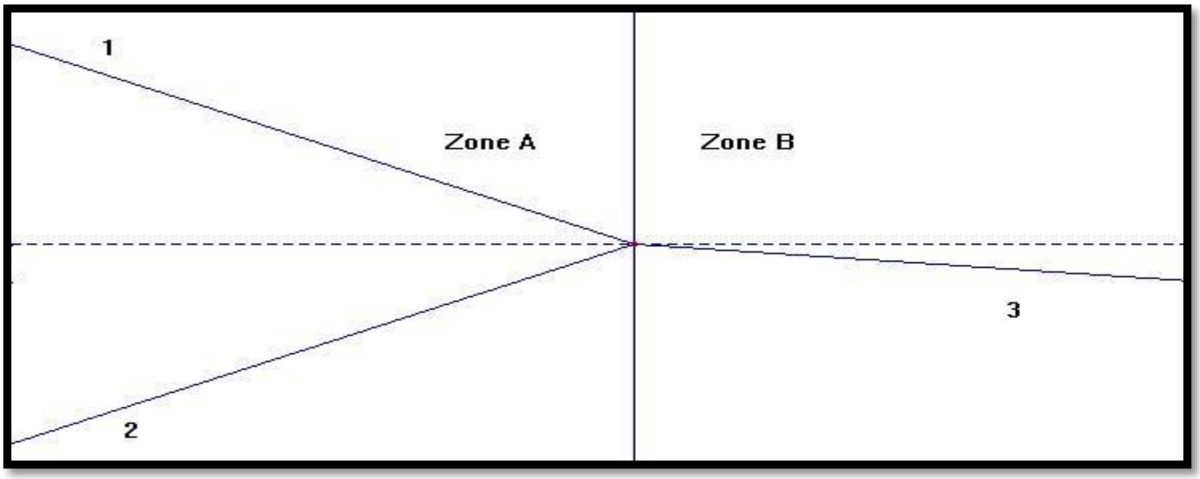


Figure 2.5: Path of a light ray passing through a prism.

Tutorial N°2: Exercises on the propagation of light, plane diopters and the prism.

Exercise 2.1:

A fine luminous brush arrives on a flat diopter separating water from air. We give $n_{water} = 1.33$. We represent the rays observed in the figure below:

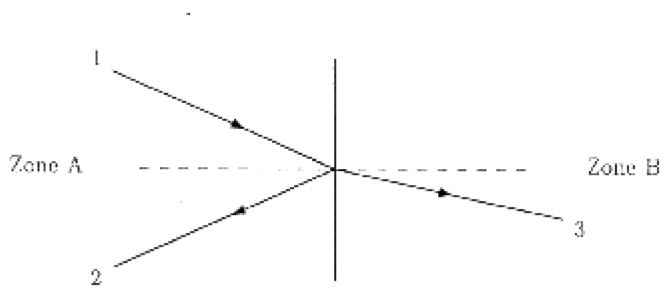


By justifying your answers:

1. Identify the different rays.
2. Indicate the direction of light propagation
3. In what zone is the water located.
4. Calculate the limiting angle of refraction
5. Generalize the result by specifying the zone where the limiting angle is located according to the difference in refraction of the media present and the consequences on the propagation of light from one medium to the other.

Solution

A - To every incident ray, there corresponds a reflected ray on the same side of the diopter, and in the other medium, a refracted ray. The reflected ray and the refracted ray are on the same side of the normal to the diopter.



1. It follows that ray (1) is the incident ray, (2) is the reflected ray and (3) is the refracted ray.
2. From the above, the direction of light is as shown in the figure.
3. The water index $n_{water} = 1.33$ is greater than that of air that is equal to one. The ray (3) approaches to the normal, it therefore propagates in the most refractive medium: water is therefore in zone B.
4. Sin (i_{Bl}) = 1/1.33 so i_{Bl} = 48,75°
5. The limiting angle of refraction is always found in the most refractive medium (with the greatest index n).

Exercise 2.2:

One of the rays of a beam of light propagating in air enters a diamond with a refractive index of 2.43.

a. Schematize the situation.

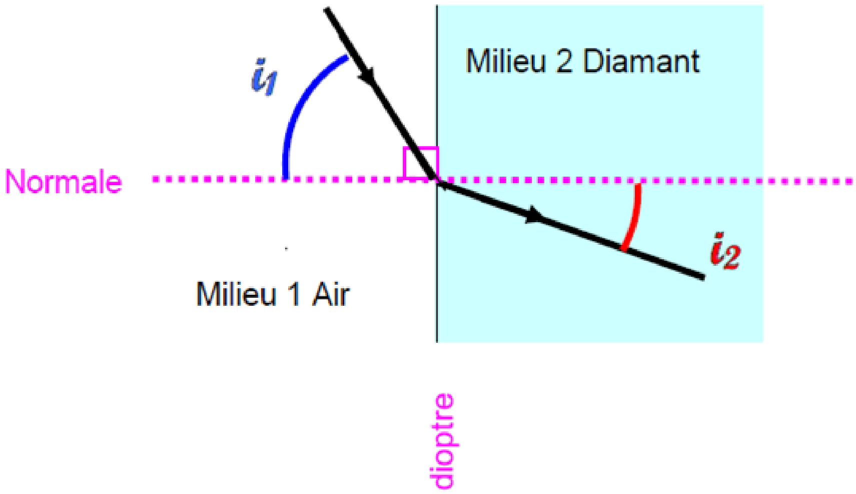
b. Write Descartes' second law.

c- Calculate the angle of incidence to obtain an angle of refraction of 20°.

Solution

a) Schematization

See diagram below



b- Second law of refraction Descartes $n_1 \sin \mathrm{i}_1 = n_2 \sin \mathrm{i}_2$

c- Calculation of the angle of incidence making it possible to obtain an angle of refraction $i_2 = 20°$ according to the previous law of Descartes

$$\sin \mathrm{i}_1 = \frac{n_2 \sin \mathrm{i}_2}{n_1} = \frac{2.43 \sin 20^0}{1} = 0.83$$

$$\mathrm{i}_1 = \sin^{-1} 0.83 = 56^0$$

We verify that the second medium being more refractive than the first, the deviation of the ray is such that the refracted ray approaches to the normal: $\mathrm{i}_2 = 20^0 < \mathrm{i}_1 = 56^0$.

Exercise 2.3:

Rayon light consists of the superposition of three colors: violet, yellow and red. This ray propagates in a glass whose refractive indices for violet, yellow and red radiation are respectively $n_v = 1.530, n_y = 1.517 and\ n_r = 1.513$. The rayon arrives on the diopter plan separating the glass from the air.

1. Calculate the limiting angles of incidence for the colors violet, yellow and red on the diopter separating the glass from the air. The air index being equal to 1.

2. What colors are observed in the air if the ray arrive the diopter at an angle of incidence $\mathrm{i} = 38^0$?

3. Same question if the angle of incidence $\mathrm{i} = 41.38^0$?

4. What can this assembly be used for?

Solution

1. La loi de diffraction dans le dioptre plan : $n \sin \mathrm{i} = \sin r$

The refractive index of glass is greater than that of air; the angle of refraction is greater than the angle of incidence: the limiting angle of incidence i_{lim} corresponds to an angle of refraction $r = 90^0$.

The diffraction law is then: $n \sin \mathrm{i}_{lim} = \sin 90^0$

From where: $\sin \mathrm{i}_{lim} = \frac{1}{n}$

$$\mathrm{i}_{lim} = \sin^{-1}\frac{1}{n}$$

For violet radiation: $\mathrm{i}_{lim} = \sin^{-1}\frac{1}{1.530} = 40.81^0$

For yellow radiation: $\mathrm{i}_{lim} = \sin^{-1}\frac{1}{1.517} = 41.24^0$

For red radiation: $\mathrm{i}_{lim} = \sin^{-1}\frac{1}{1.513} = 41.37^0$

2- The angle of incidence $\mathrm{i} = 38^0$ is less than the limit angles of the three colors, the three colors will emerge in the air. The refraction angles are different for the three radiations; hence, they will be separated after refraction by diopters in air.

3- The angle of incidence $\mathrm{i} = 41.30^0$ is greater than the limiting angles of incidence of the colors violet and yellow; these two colors will not emerge in the air and will be reflected in the glass on the surface of the diopter. On the other hand, $\mathrm{i} = 41.30^0$is less than the limiting angle of red radiation, only this radiation will be refracted.

4- This assembly can be used as a disperser in spectroscopy to separate colors from a light beam or as a chromatic filter to eliminate certain radiation by reflection on the surface of the diopter.

Exercise 2.4:

We want to determine the refractive index n of a glass. To do this, we use this glass to make a prism whose base is an equilateral triangle. We place it at the minimum deviation. The minimum deviation angle D measured is42^0.

Calculate its refractive index.

Solution

The deviation is minimal for $i = i' = \sin^{-1}\left(n\sin\frac{A}{2}\right)$

$$D = 2\times\sin^{-1}\left(n\sin\frac{A}{2}\right) - A$$

$$2\times\sin^{-1}\left(n\sin\frac{A}{2}\right) = D + A$$

$$\sin^{-1}\left(n\sin\frac{A}{2}\right) = \frac{D+A}{2}$$

$$n\sin\frac{A}{2} = \sin\left(\frac{D+A}{2}\right)$$

$$n = \sin\left(\frac{D+A}{2}\right)\Big/\sin\frac{A}{2}$$

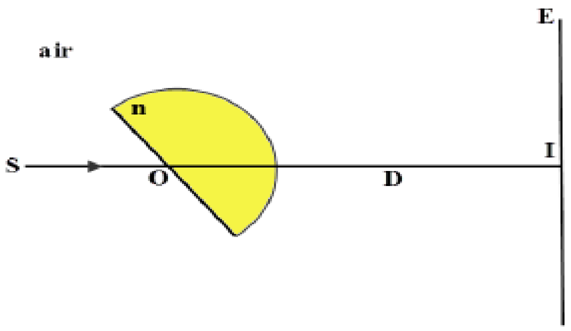
$$n = \sin(51^0)/\sin(30^0)$$

$$n \approx 1.55$$

Exercise 2.5:

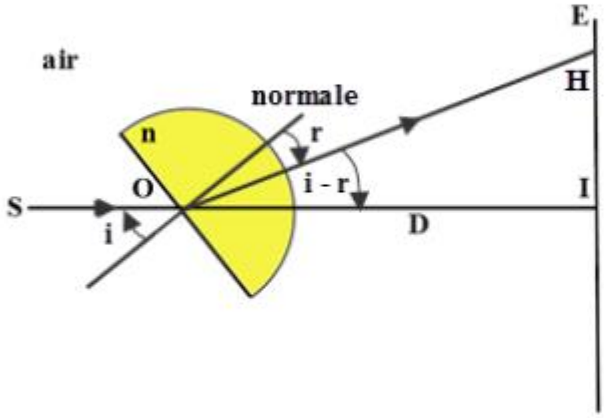
A point source S emits a monochromatic light ray, which arrives on the flat face of a half-cylindrical block, of index n, at its center O with an angle of incidence $i = 40^0$(figure below). We place a screen E at a distance D=1m from the center O. Consider H the point of impact of the ray emerging from the block on the screen E. The deviation observed is IH=0.24m.

1- Trace the path of the light ray from S until H .
2- Calculate the angle of refraction r at the point O.
3- Calculate the index n of this block



Solution

1. Walk of the ray coming from S.



2. In the triangle OIH: $\tan(i - r) = \frac{IH}{D} = \frac{0.24}{1} = 0.24$ $(i - r) = 13.5^0$ $r = i - 13.5^0 = 40^0 - 13.5^0 = 26.5^0$
3. The law of refraction at the point A is written:

$\sin i = n \sin r$ $n = \frac{\sin i}{\sin r} = \frac{\sin 40^0}{\sin 26.5^0} = 1.44$

Tutorial N°3

3. Exercises on spherical diopters and thin lenses.

REMINDER

3.1 DIOPER:

Diopter is defined as the surface that separates tow transparent media with different refractive index $n_1 and\ n_2$. This surface can be flat or curved. If it is a spherical surface, it is a spherical diopter (Figure 3.1). C is the center and S the top of curvature.

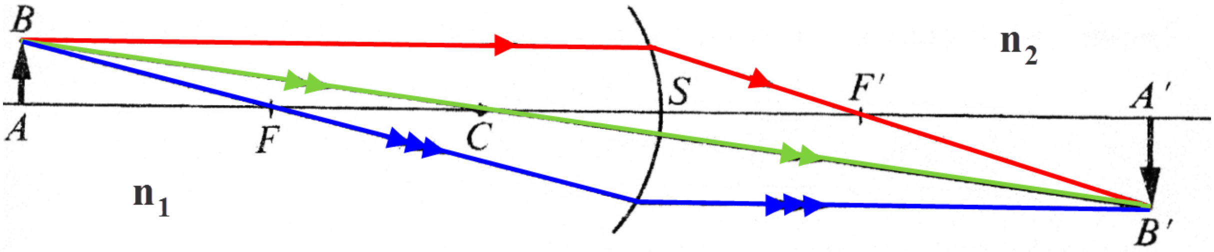


Figure 3.1: Construction of an image through a spherical diopter for $n_1 > n_2$

- The conjugation relationship for the spherical diopter is donated by:

$$\frac{n_2}{\overline{SA'}} - \frac{n_1}{\overline{SA}} = \frac{(n_2 - n_1)}{\overline{SC}} \qquad \overline{SC} = R$$

- The magnification γ of a spherical diopter

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_1}{n_2}\frac{\overline{SA'}}{\overline{SA}}$$

- The focal lengths SF and SF' have the expressions:

$$\overline{SF} = \frac{n_1}{(n_1 - n_2)}\overline{SC}$$

$$\overline{SF'} = \frac{-n_2}{(n_1 - n_2)}\overline{SC}$$

3.2 CASE OF THE DIOTRE PLAN:

For a plane diopter separating two homogeneous media of refractive indices n_1 and n_2 $\overline{SC} = \infty$

The conjugation relation is then written:

$$\frac{n_2}{\overline{SA'}} = \frac{n_1}{\overline{SA}}$$

The magnification γ in this case is:

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_1}{n_2}\frac{\overline{SA'}}{\overline{SA}} = 1$$

The image $A'B'$is the same size as the object AB.

3.3 THIN LENSES:

A lens is a transparent medium of refractive n index and limited by two diopters which can be spherical or one is spherical and the other plane (Figure 3.2). It is said to be thin if its diameter is very large compared to its thickness. The point O is its optical center. There are two types of lenses:

The distance from the lens to the focal point is called the focal length. For converging lenses, the focal length is always positive $\overline{OF'} > 0$, while diverging lenses always have negative focal lengths$\overline{OF'} < 0$.

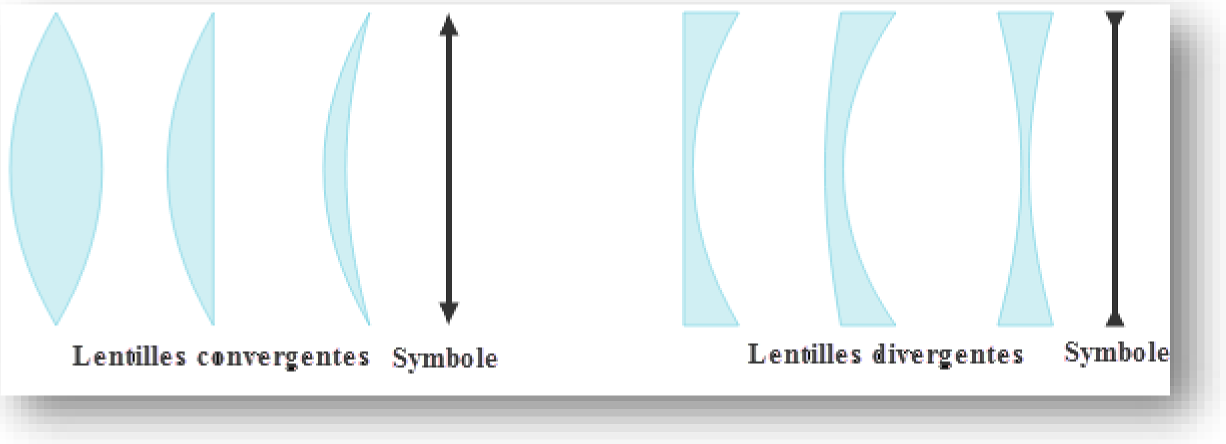


Figure 3.2: The six categories of chopped lentils.

Figure 3.3 shows the geometric construction of a real and virtual object through a thin lens.

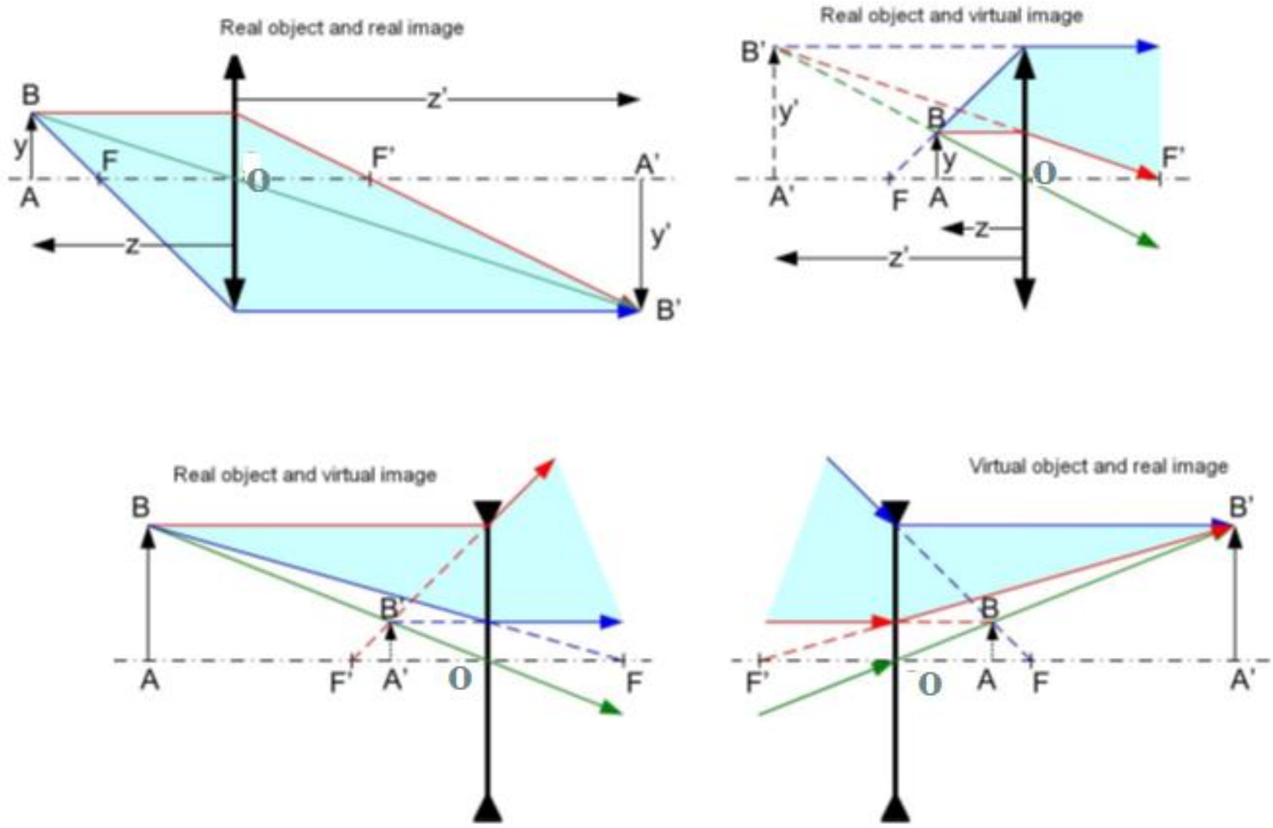


Figure 3.3: geometric construction of a real and virtual object through:
(a) a converging lens (b) a diverging lens.

- The conjugation relation for a thin lens:

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}}$$

- The magnification γ of a thin lens:

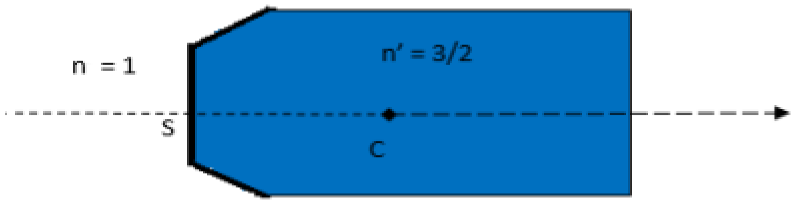
$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$

- The vergence V of a lens is the inverse of the algebraic value of its image focal length $\overline{OF'}$. Its unit is the diopter (δ): V $= \frac{1}{\overline{OF'}}$ (δ)

Tutorial N°3: Exercises on spherical diopters and thin lenses.

Exercise 3.1:

A spherical diopter with a radius of curvature of 10 cm separates two media with indices n =1 and n'=3/2.



Determine the position of the focal lengths, Calculate and draw the position of the image of an object AB.

Place a:

a) 60 cm from the top and real;

b) 10 cm from the top and real;

c) 5 cm behind the diopter (virtual object).

Same questions if we reverse the indices

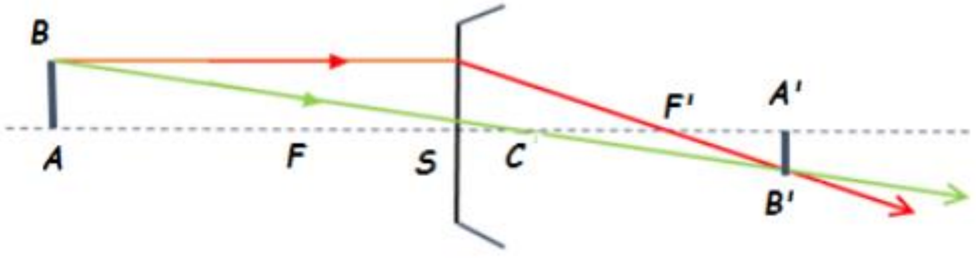
Solution

We suppose (p' =$\overline{SA'}$) and (p =$\overline{SA}$)

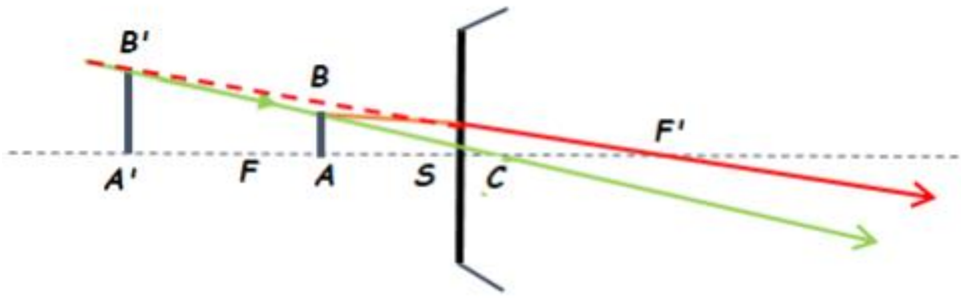
a) r is positive, the diopter is convergent. We then have $\overline{SF} = f = -2r = -20\ cm\ and$

$$\overline{SF'} = f' = 3r = 30\ cm.$$

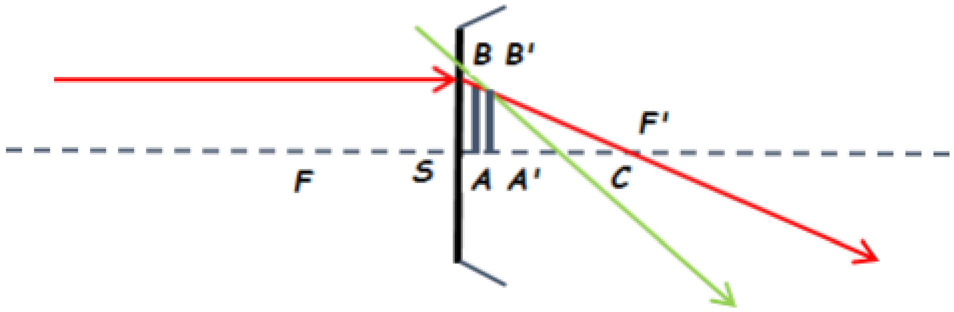
If $\overline{SA}$= −60 cm, $\overline{SA'}$ = 45 cm. The image is real and reversed



b) If $\overline{SA} = -10\ cm, \overline{SA'}\ = -30\ cm$. The image is virtual in the same side as the object.

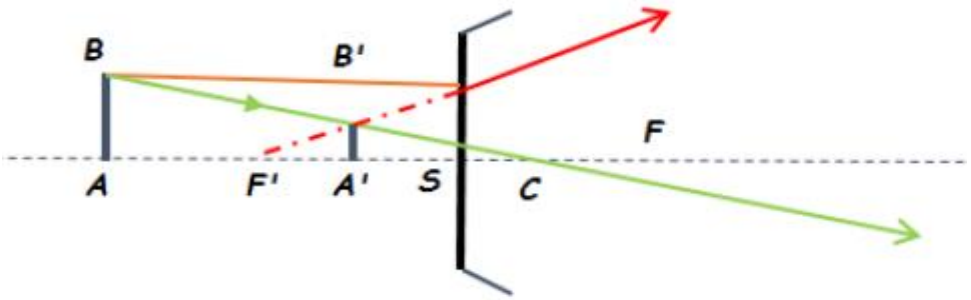


c) If p=5 cm, p' = 6 cm. The object is virtual and the image is real

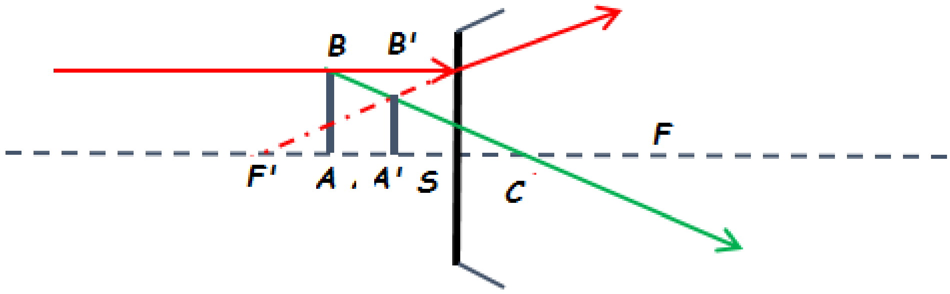


If we reverse the indices, f' =−20 cm and f =30 cm. The diopter is divergent.

a) If p= − 60 cm, p' =−13.33 cm. The object is real and the image is virtual in the same side as the object.



b) If p=−10 cm, p=−5 cm. The object is real and the image is virtual in the same side as the object.



c) If p=5 cm, p' =4 cm. The object is virtual and the image is real.

Exercise 3.2:

A spherical diopter with top S and center C separating 2 media with indices n =1 and n'=4/3 has a radius of curvature |r|=4 cm.

1) Write the formulas of the spherical diopter without demonstration: relation of conjugation, transverse magnification and focal lengths.

2) This diopter gives an image A'B' (p' = SA') of a real object AB (p = SA') such that the magnification γ is equal to +2.

a- Calculate the distances p and p' and on a scale figure, place the points S, C, A and A'.

b- Calculate the focal lengths f and f'.

c- Is the diopter convergent or divergent; convex or concave? Place the points on the diagram.

Solution

1- Conjugation relationship: $\frac{n'}{p'} - \frac{n}{p} = \frac{(n'-n)}{r}$

Transverse magnification: $\gamma = \frac{n}{n'}\frac{p'}{p}$

Image focal length: $f' = \frac{n'r}{n'-n}$

Object focal length: $f = \frac{-nr}{n'-n}$

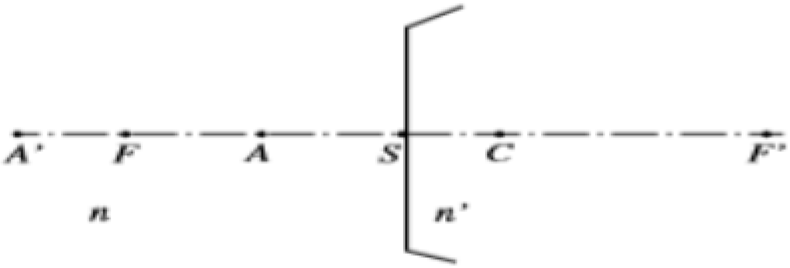
a) We obtain: $\gamma = 2 = \frac{3p'}{4p} \Longrightarrow p' = \frac{8}{3}p$

We replace: $p' = \frac{8}{3}p$ in the conjugation relation, we find:

$$\frac{4}{3p'}-\frac{1}{p}=\frac{1}{3r}=\frac{1}{2p}-\frac{1}{p}=-\frac{1}{2p}\Rightarrow p=-\frac{3r}{2}$$

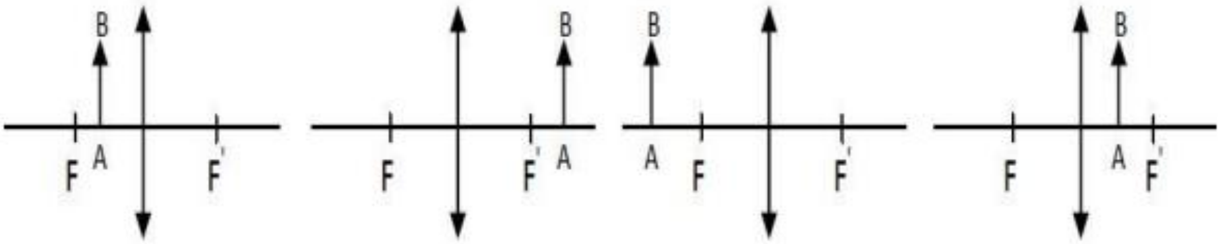
p is negative, The object is real and the image is virtual. $So\ r>0, r=4\ cm, p=-6\ cm\ and\ p'=-16cm.$

c) $f'=16cm\ and\ f=-12cm$. The diopter is convergent and convex.

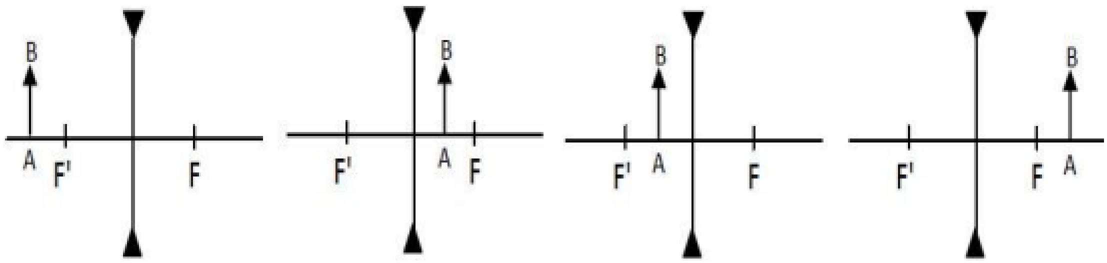


Exercise 3.3:

1. When the lens is convergent, complete the following constructions:



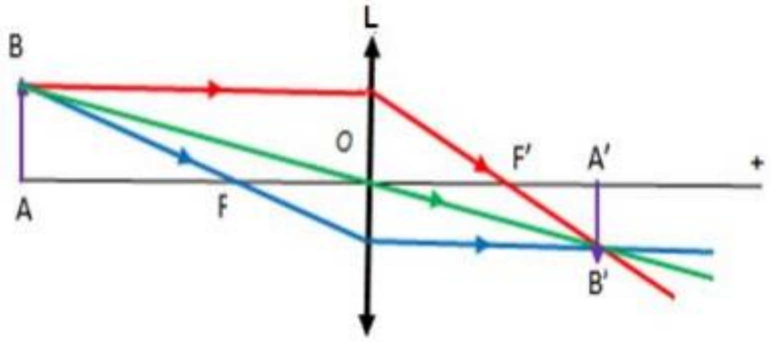
2. When the lens is divergent, complete the following constructions:



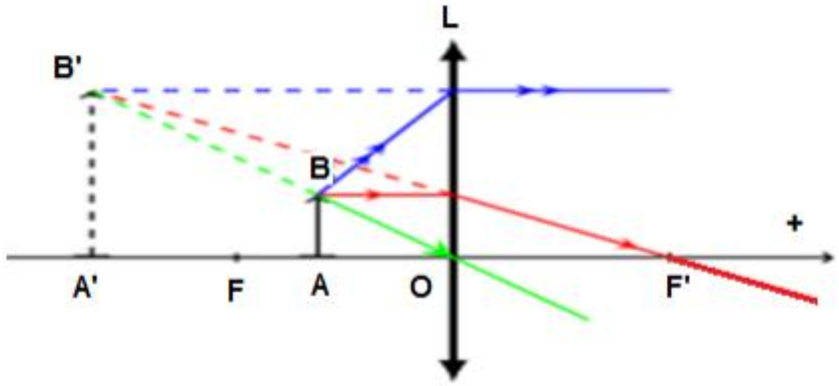
Solution

1. Construction of the image by a converging lens of an object

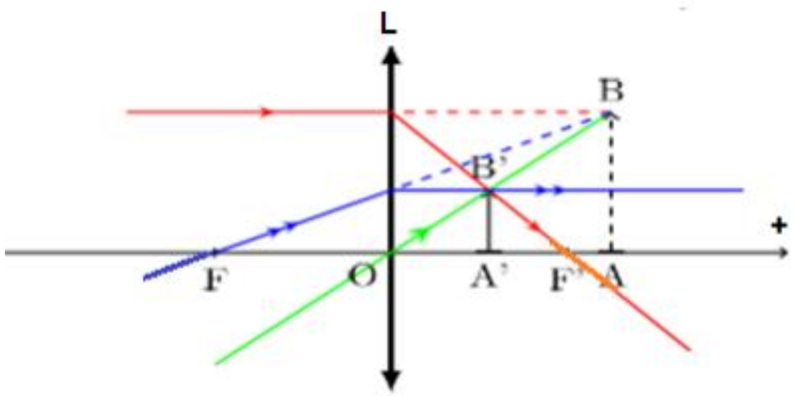
a- A real object, right reversed image



b- Real object, virtual image

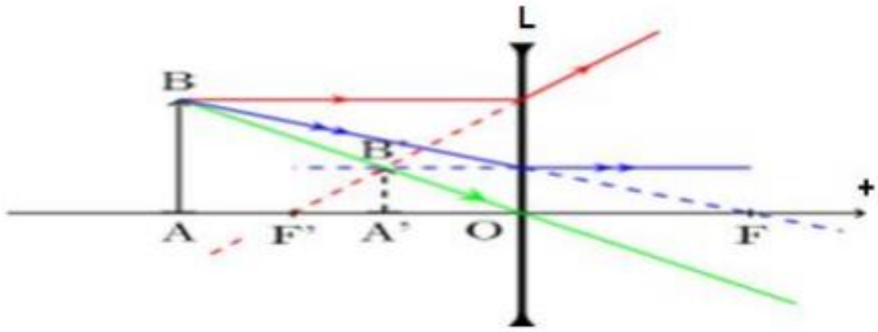


c- Virtual object, real image

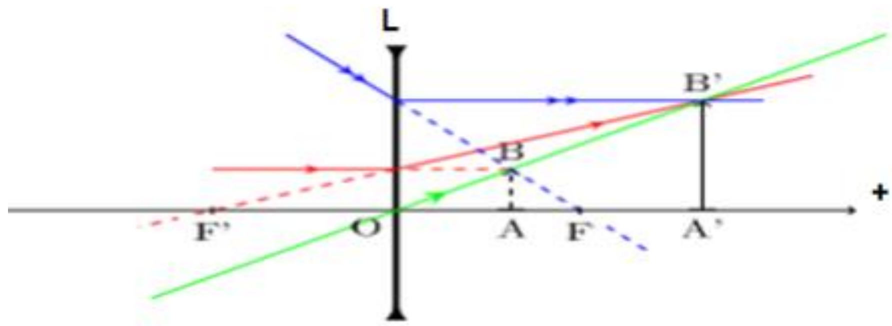


2. Construction of the image by a lens diverging from an object

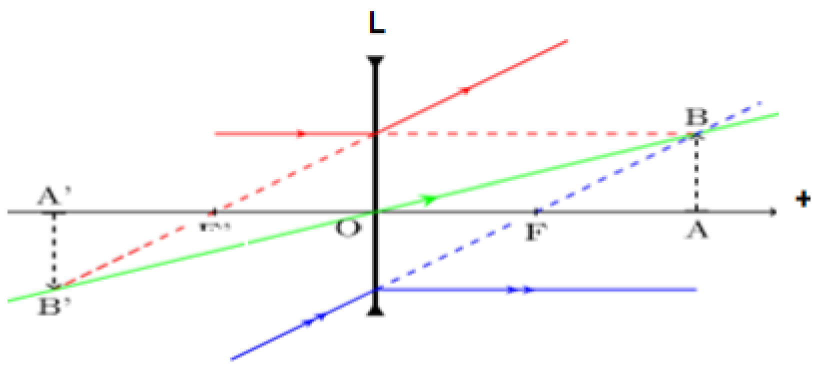
a- A real object, right virtual image



b- A virtual object, real right image



c- A virtual object, reversed virtual image



Exercise 3.4:

A lens forms an image of an object 20 cm away from it. The image is at 6 cm from the lens and on the same side as the object.

a) What is the focal length of the lens?

b) Determine the nature of the lens.

c) If the object is 0.4 cm in size, what is the size of the image?

d) Determine the nature of the image.

e) Make the diagram

Solution

a) $\overline{\mathrm{OA}} = -20\mathrm{cm}$ and $\overline{\mathrm{OA'}} = -6\mathrm{cm}$

Conjugaison relationship: $\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}} = \frac{1}{f'}$ $given\ f' = -8.57cm$

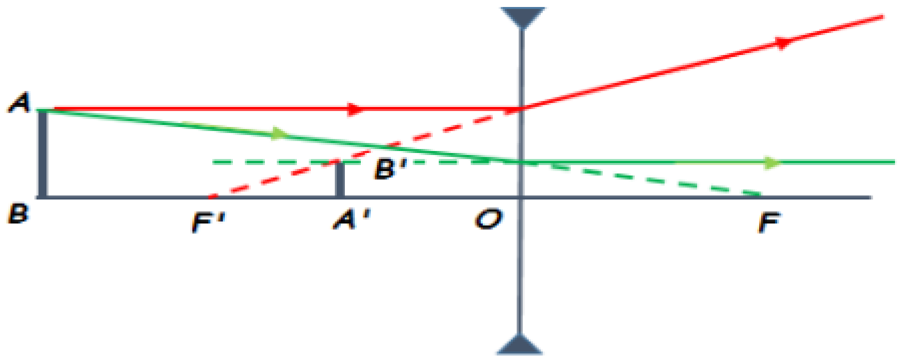
b) It is a divergent lens, $\overline{OF'} < 0$

c) The magnification is given by:

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}} = \frac{p'}{p} = 0.3, \overline{A'B'} = 0.12cm$$

d) It is a virtual image, straight and reduced 0.3 times.

e) Diagram



Tutorial N°4

4. Exercises on plane and spherical mirrors and the reduced eye.

REMINDER

4.1 THE SPHERICAL MIRROR AND MIRROR PLAN

DESCRIPTION

A spherical mirror is a spherical cap with center C and vertex S made reflective. The axis of symmetry is the optical axis of the mirror. This axis is usually oriented from left to right because light arrives from the left (by convention). There are two types of spherical mirrors:

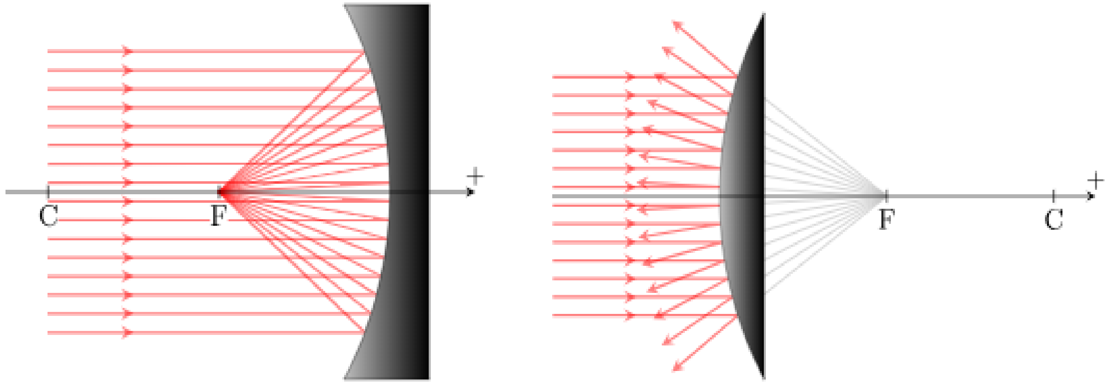


Figure 4.1: Concave and convex spherical mirrors.

- the concave mirror is a spherical mirror such that $\overline{SC} < 0$
- the convex mirror is a spherical mirror such that $\overline{SC} > 0$

In the case of spherical mirrors, the principle of inverse return of light implies: $f = f'$

- The conjugation relation for a spherical mirror:

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{1}{\overline{SF}} = \frac{1}{f'} \qquad \overline{SF} = \overline{SF'} = f'$$

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}}$$

Case of the concave mirror $\overline{SC} < 0$, $\overline{SC} = -R$,

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = -\frac{2}{R}$$

- The magnification

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$$

- The focal lengths $\overline{SF}$ and $\overline{SF'}$ have the expressions:

$$\overline{SF} = \overline{SF'} = \frac{\overline{SC}}{2}$$

Case of the mirror plan:

For a plane mirror $\overline{SC} = \infty$

The conjugation relation is then written: $\overline{SA'} = -\overline{SA}$

The object and the image are equidistant from the mirror.

The magnification γ in this case is: $\gamma = 1$

The image $A'B'$is the same size as the object AB

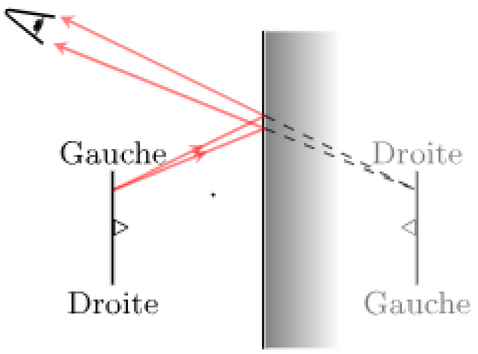
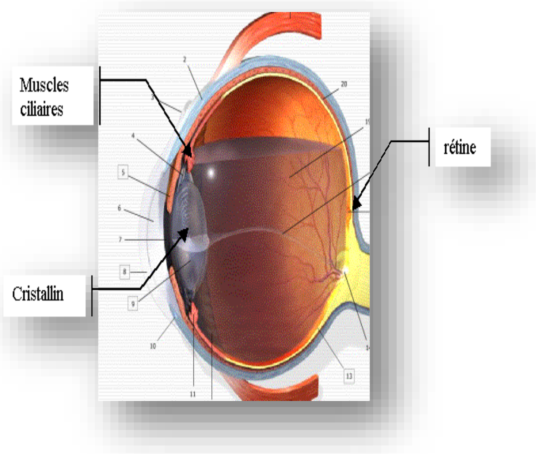


Figure 4.2: Formation of an image with a plane mirror. The image is reversed (left/right)

4.2 THE EYE

REMINDERS ON THE STRUCTURE OF THE EYE

The eye is an organ that allows humans to analyze light, which allows us to analyze the environment in which it is located.



- **The retina** is comparable to a background screen on which the rays form images. When a group of rays coming from an object forms a beam converge on a point of the retina, a clear image is then interpreted by the brain.

- **The cornea** corresponds to a rigid surface allowing to converge the rays of light.

- **The crystalline lens** corresponds to a flexible lens which is deformable thanks to the ciliary muscles.

Figure 4.3: The eye.

4.3 EYE MODELING:

The eye can be modeled as a converging lens (cornea-crystalline assembly) with variable focal length OF' to accommodate the position of objects and form their images on the retina. The distance between this lens and the retina is called eye depth d. For a normal eye, the focal length is equal to the depth of the eye. The furthest point of distinct vision (the Punctum Remotum PR) is at infinity. The closest point of distinct vision (the Punctum Proximum PP) is at the minimum distance $d_m = 25cm$ from the lens.

We define the amplitude of accommodation by:

$$A = \mathrm{V}_{\max} - \mathrm{V}_{\min}$$

$$\mathrm{V}_{\max} = \frac{n'}{d} - \frac{1}{\overline{PP}}$$

$$\mathrm{V}_{\min} = \frac{n'}{d} - \frac{1}{\overline{PR}}$$

$$A = \frac{1}{\overline{PR}} - \frac{1}{\overline{PP}}$$

$n'$is the refractive index inside the eye after the lens. d is the distance between the cornea-lens assembly and the retina: $\frac{n'}{d} \approx 59\delta$. $\overline{PP}$ is the distance between the lens and the Punctum Proximum. $\overline{PR}$ is the distance between the lens and the Punctum Remotum PR.

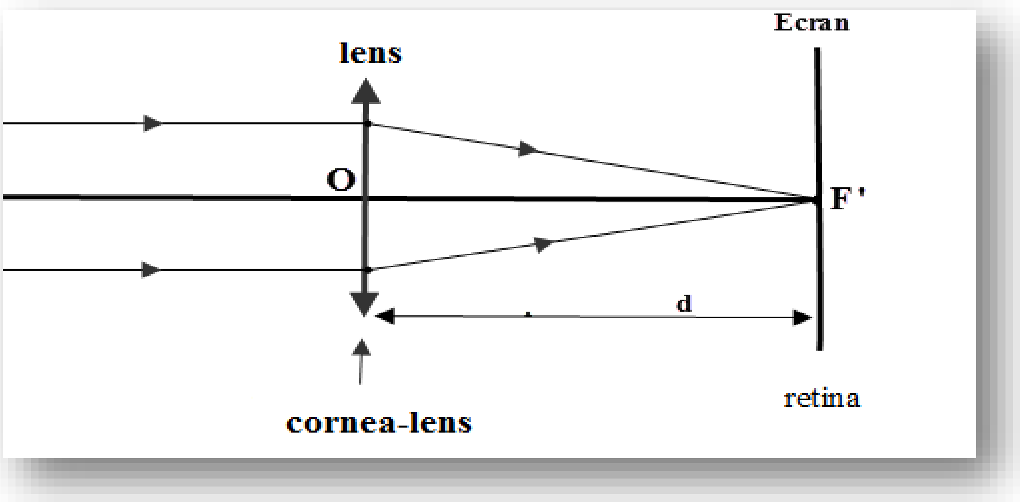


Figure 4.4: Representative diagram of a normal eye.

4.4 THE DIFFRENT AREAS OF VISION

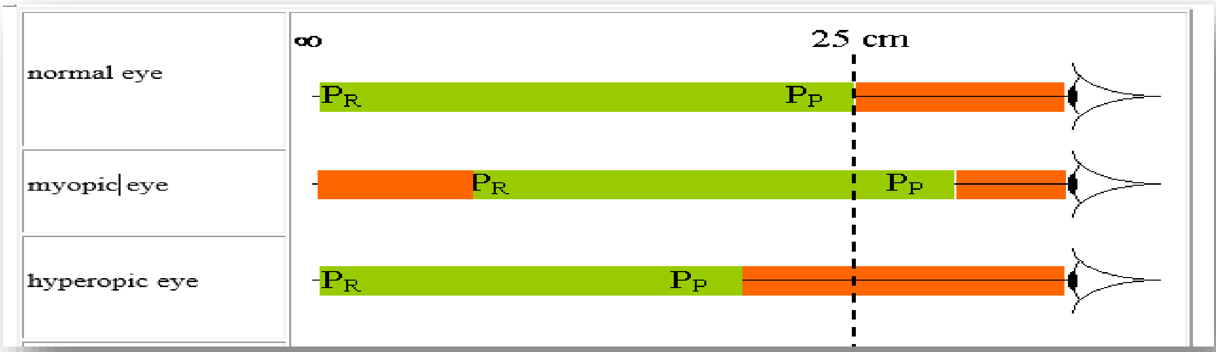


Figure 4.5: Representative the different areas of vision:

4.5 DEFECTS OF VISION AND THEIR CORRECTIONS:

4.5.1 Myopia (Short sightedness):

It is a kind of defect in human eye due to which a person can see near objects clearly but he cannot see the distant objects clearly. Myopia is due to :(i) Excessive curvature of cornea, (ii) Elongation of eye ball.

- In this case $OF' < d$ ($F'$is in front of the retina**).**

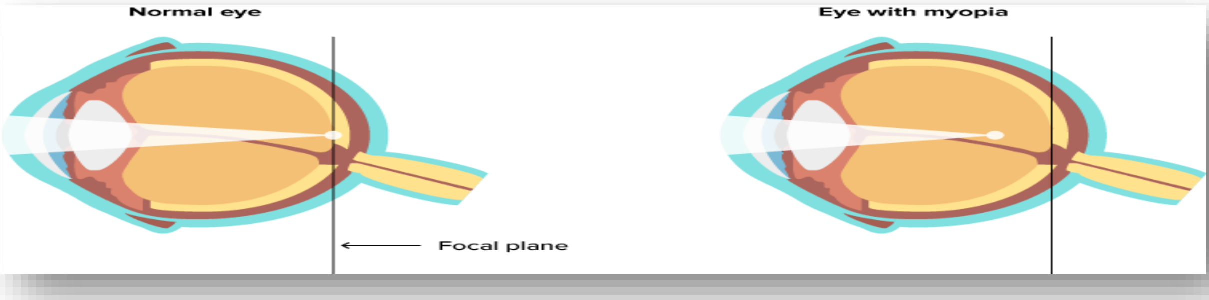


Figure 4.6: Eye normal and myopia.

Correction: Myopia or short-sightedness can be corrected by wearing spectacles containing concave lens.

4.5.2 Hypermetropia (Long sightedness):

I t is a kind of defect i n human eye due to which a person can see distant objects properly but cannot see the nearby objects clearly. I t happens due to: (i) Decrease in power of eye lens i .e., increase in focal length of eye l ens. (ii) Shortening of eye ball.

- In this case $OF' > d$ ($F'$is in behind of the retina**).**

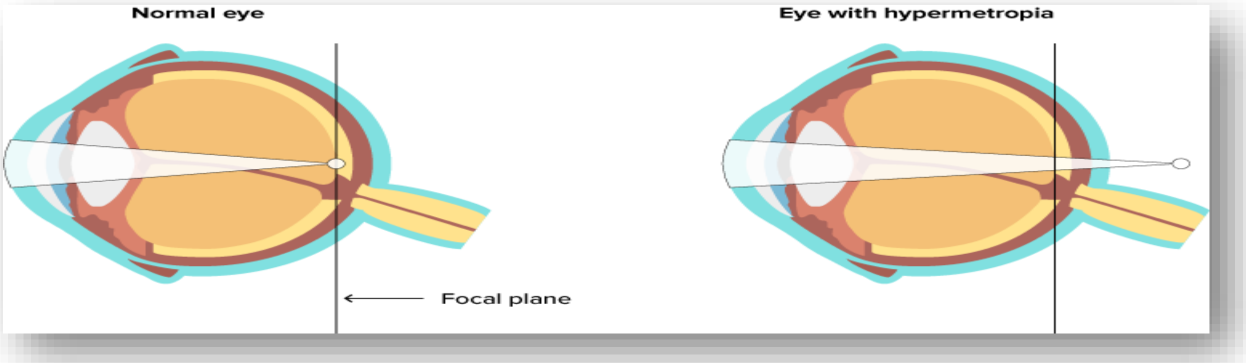


Figure 4.7: Eye normal and hypermetropia.

Correction: The near-point of an eye having hypermetropia is more than 25 cm. The condition of hypermetropia can be corrected by putting a convex lens i n front of the eye.

Presbyopia: is the loss of the power of accommodation with age.

✓ Astigmatism: the focal length is not the same in all viewing directions.

4.6 MAGNIFIYING GLASS

A magnifying glass is a convex lens that is used to produce a magnified image of an object. The lens is usually mounted in a frame with a handle. A magnifying glass can be used to focus light, such as to concentrate the sun's radiation to create a hot spot at the focus for fire starting.

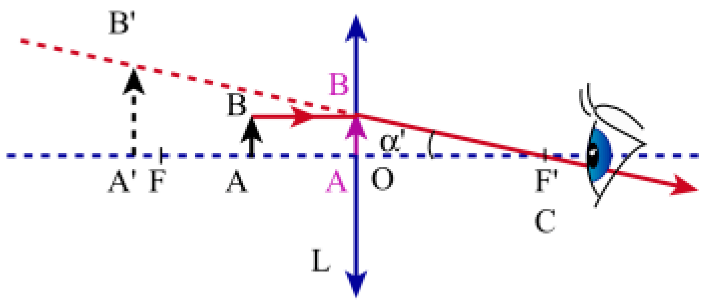


Figure 4.8: The magnifying glass.

- The algebraic distance $\overline{OA'}$ of the image $A'B'$ of an object AB is calculated from the conjugation relation:

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{OF'}}$$

- The magnification γ of the magnifying glass is calculated by the relation:

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$

- The power P of the magnifying glass is defined by the relationship:

$$P = \frac{\tan\alpha'}{AB}$$

α' is the angle with which the observer sees the image (Figure 2). Its unit is diopter (δ).

- The magnification of the magnifying glass is: $G = \frac{\tan\alpha'}{\tan\alpha}$

α' is the angle from which we observe the image $A'B'$

α the angle at which we observe the object AB from a distance d_m where d_m represents the minimum distinct vision distance: $d_m = 25cm = 0.25m$.

- Relationship between G and P: $G = P.d_m$.

4.7 THE OPTICAL MICROSCOPE:

The optical microscope consists of two thin converging lenses (Figure 4.9):

- The lens with a focal length $\overline{OF'}$ of a few millimeters gives a real enlarged and reversed image A_1B_1 of the object AB. The magnification of the objective is defined by:

$$\gamma = \frac{\overline{A_1B_1}}{\overline{AB}} = \frac{\overline{O_1A_1}}{\overline{O_1A}}$$

- The eyepiece with a focal length O_2F_2' of a few centimeters works like a magnifying glass. It allows you to observe a virtual image $A'B'$ enlarged and rejected to infinity A_1B_1. To do this, the image A_1B_1 of the object AB must be in the focal plane of the eyepiece. The point A_1 must therefore merge with the object focal point F_2 of the eyepiece (Figure 4.9).

The magnification G of the eyepiece is defined by the ratio:

$$G = \frac{\tan\alpha'}{\tan\alpha} = \frac{\frac{A_1B_1}{O_2F_2}}{\frac{A_1B_1}{d_m}} = \frac{d_m}{O_2F_2}$$

- The power of an optical microscope is defined by the relationship:

$$P = \frac{\tan\alpha'}{AB} = \frac{\frac{A_1B_1}{O_2F_2}}{AB} = \frac{A_1B_1}{AB}.\frac{1}{O_2F_2} = |\gamma|.\frac{1}{O_2F_2}$$

We also demonstrate that: $P = \frac{\Delta}{O_1F_1'.O_2F_2}$

Where .$\Delta = F_1'F_2$ represents the optical interval of the microscope.

- The commercial magnification of the optical microscope is defined by the relationship:

$$G_C = \frac{\tan\alpha'}{\tan\alpha} = \frac{\frac{A_1B_1}{O_2F_2}}{\frac{AB}{d_m}} = \frac{d_m}{O_2F_2}.\frac{A_1B_1}{AB} = G.|\gamma|$$

Noticed:

$$G_C = \frac{d_m}{O_2F_2}.\frac{A_1B_1}{AB} \; and \; P = \frac{A_1B_1}{AB}.\frac{1}{O_2F_2} \;\; from\, where\; G_C = P.d_m$$

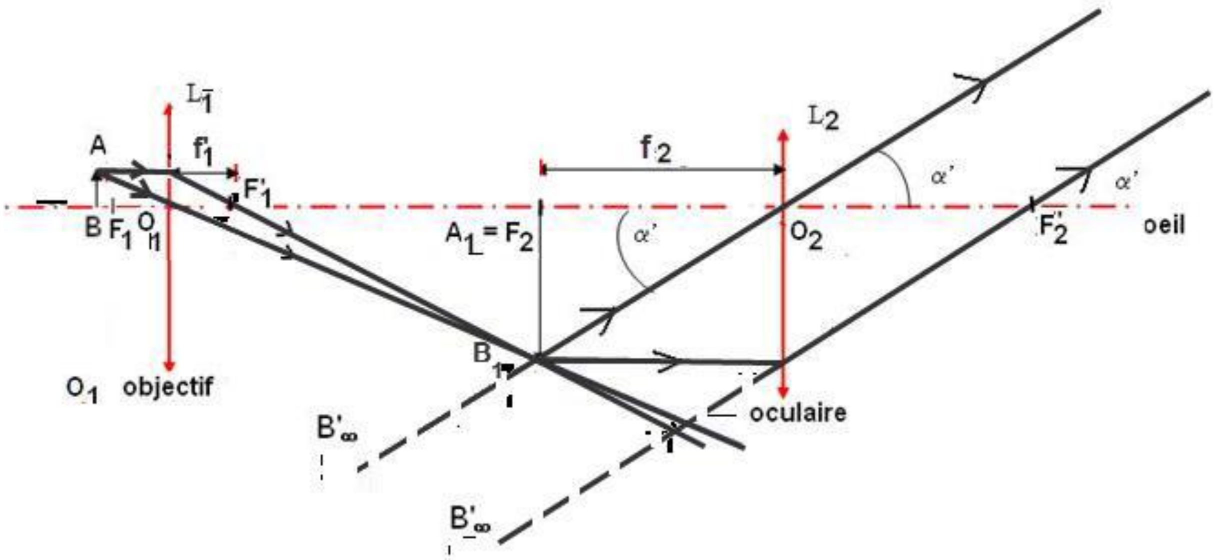


Figure 4.9: Building the image of an object through an optical microscope.

- The numerical aperture (maximum angle under which rays from the object can penetrate the optical system)
- Separating power is the ability to distinguish two adjacent points as distinct. The eye has the ability to distinguish particles with a diameter of up to $0.1\mu m$. However, they must be separated from each other by a distance of at least $5\mu m$. The resolving power of the eye is $5\mu m$.

Tutorial N°4: Exercises on plane and spherical mirrors and the reduced eye.

Exercise 4.1:

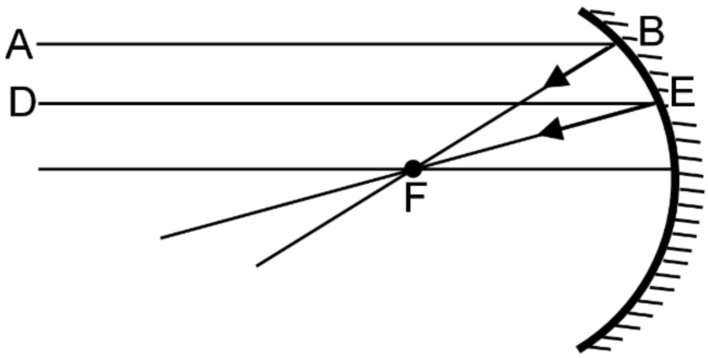
Formation of different types of images by concave mirror.

1. At the infinity
2. Beyond the center of curvature
3. At the center of curvature
4. Between center of curvature and principal focus
5. At the principal focus
6. Between the principal focus and pole

Solution

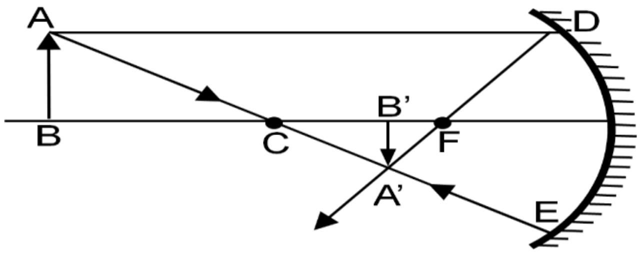
1. **Image formed by a concave mirror when the object is placed at infinity**

When the object is placed at infinity, the two rays AB and DE running parallel to the principal axis are reflected at point B and E respectively and intersect each other at the principal focus F on the principal axis. Therefore, in this case the image is formed at the principal focus, which is highly diminished, real and inverted.



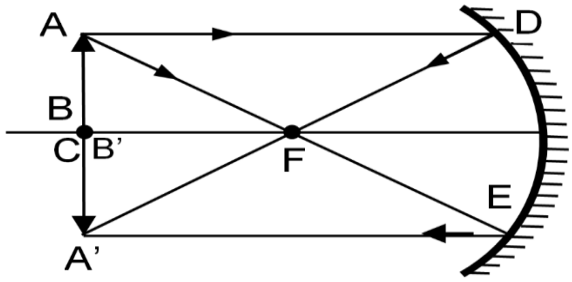
2. **When the object is placed beyond the centre of curvature**

When the object AB is placed beyond the center of curvature then a ray of light AD which is parallel to the principal axis and another ray AE which pass through the center of curvature intersect each other after reflection at point A’ between the focus and center of curvature. Thus, the image formed is between the principal focus F and center of curvature C, diminished, real and inverted.



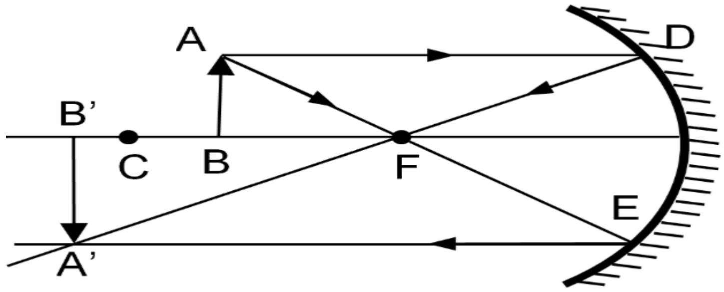
3. **When the object is placed at the center of curvature**

When the object AB is placed at the center of curvature C, then a ray of light AD which is parallel to the principal axis and another ray AE which pass through the principal focus intersect each other after reflection at point A' just below the position of the object. Thus the image formed in this case is at the center of curvature, of same size as the object, real and inverted.



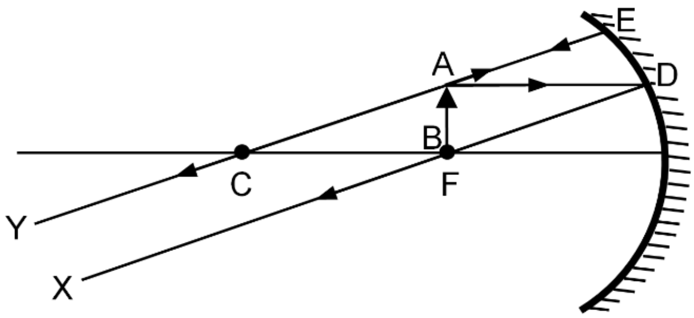
4. **When the object is placed between the center of curvature and principal focus**

When the object AB is placed between the center of curvature and principal focus, then the ray AD running parallel to the principal axis and another ray AE passing through the principal focus F intersect each other at point A' beyond the center of curvature. Thus, the image formed in this case is beyond C, enlarged, real and inverted.



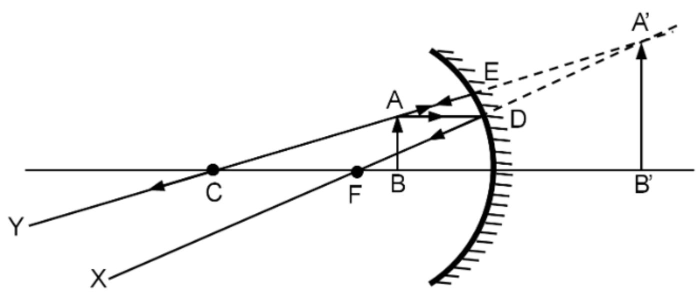
5. **When the object is placed at principal focus**

When the object AB is placed at the principal focus, then the parallel ray of light AD passes through the principal focus F giving us the reflected ray DX. And the second ray of light AE passing through the center of curvature C is reflected along the same path forming the reflected ray EY. In this case, both the reflected rays i.e. DX and EY become parallel to each other so these rays cannot intersect each other and the image will be formed at infinity. The image formed in this case will be highly enlarged, real and inverted.



6. **When the object is placed between the principal focus and the pole**

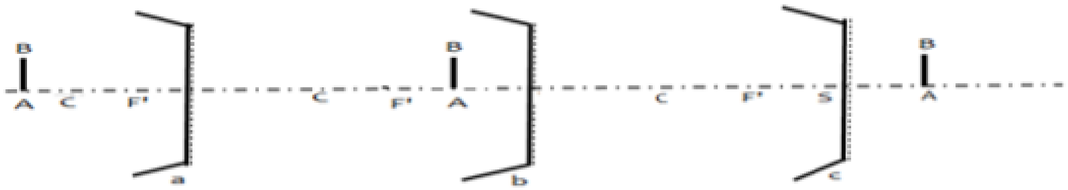
When the object AB is placed between the principal focus and the pole, then the parallel ray of light AD passes through the focus F giving us the reflected ray DX. And the second ray AE passing through the center of curvature C is reflected along the same path forming the reflected ray EY.



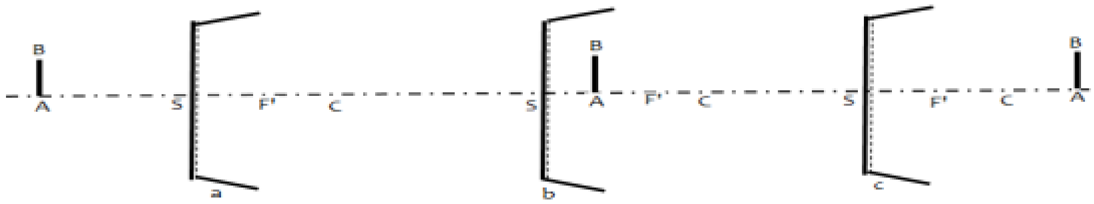
Exercise 4.2:

1. Construct the image from the object:

a- Concave mirror

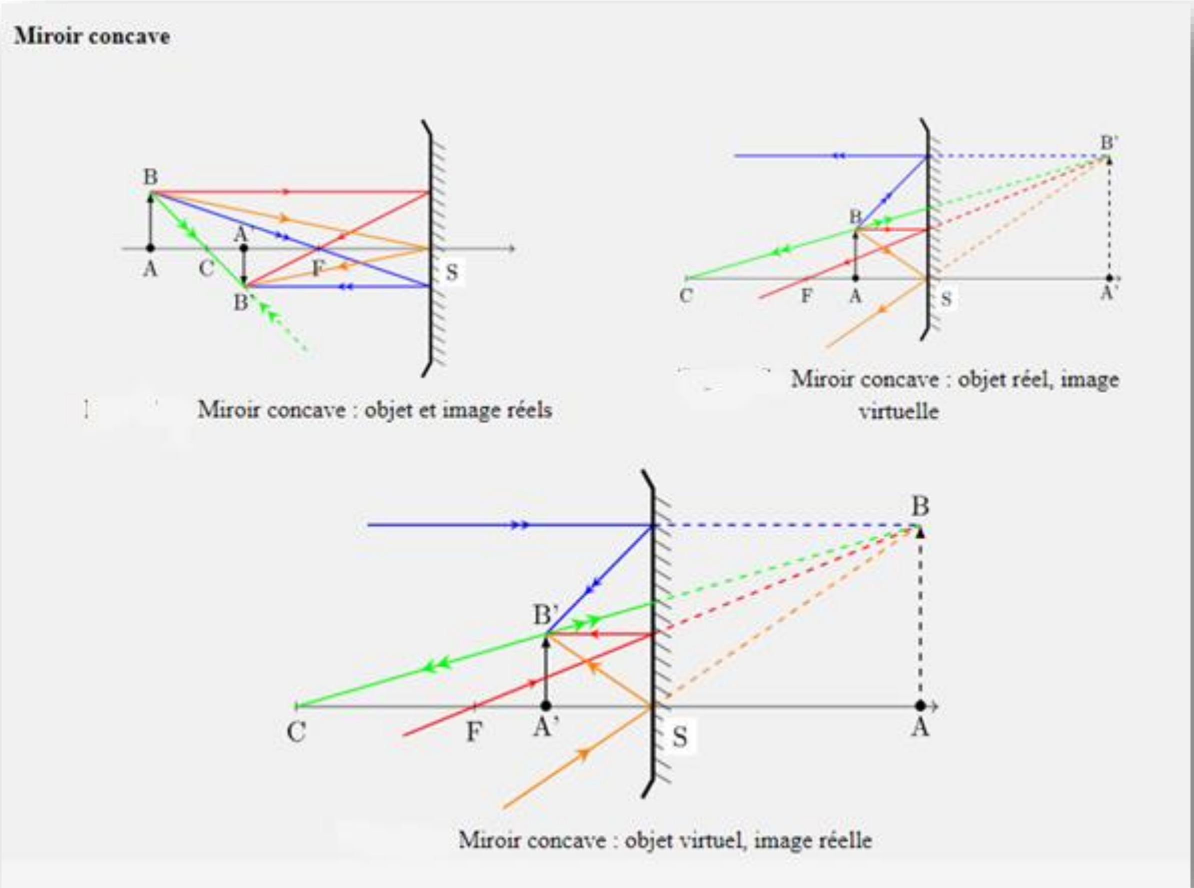


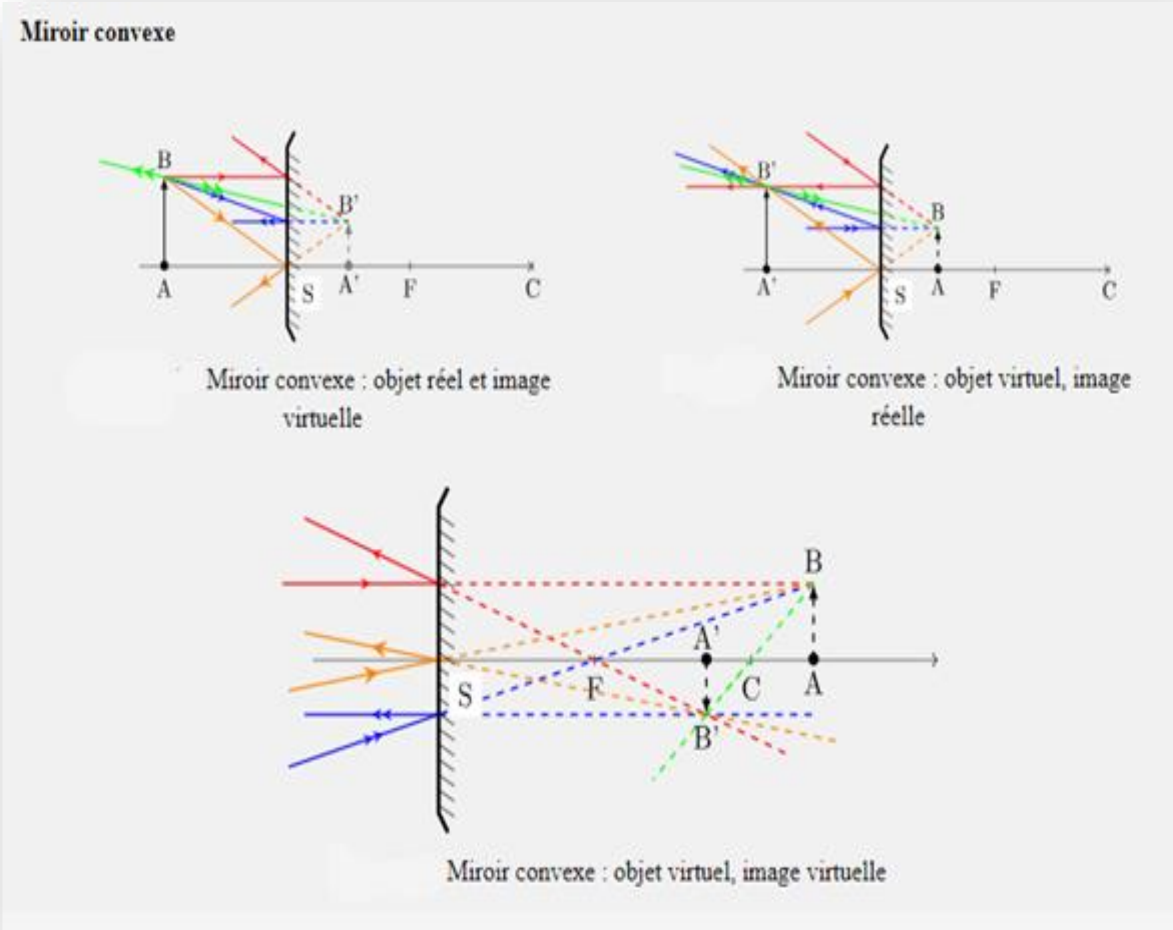
b- Convex mirror



Solution

https://www.physagreg.fr/optique-12-generalites-systemes-miroirs.php





Exercise 4.3:

We consider a convex spherical mirror, with center C, vertex S, radius of curvature $R = \overline{SC} = +30\ cm$ and an object of height1 cm.

1) Give the position of the focus$F$.

2) Determine the image $\overline{A'B'}$of the object $\overline{AB}$ by specifying its position, its magnification, its size and its nature in the case where$\overline{SA} = -30\ cm$.

3) Construct the image (Diagram).

Solution

1- The position of focus F.

The focus F of the convex spherical mirror is in the middle of the segment $[SC]$ and $\overline{SF'} = \overline{SF} = 15\ cm.$

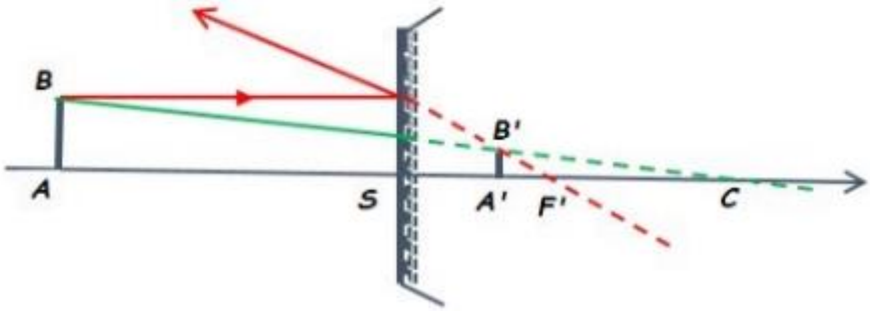
2- The position of A' :

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{1}{\overline{SF}} \Rightarrow \overline{SA'} = \frac{\overline{SA}.\overline{SF}}{\overline{SA} - \overline{SF}} = \frac{(-30).(15)}{-30-15} = +10cm$$

The magnification γ :$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}} = -\frac{10}{-30} = 0.33 \Rightarrow \overline{A'B'} = 0.33.1cm = 0.33cm$

Nature of mage: The image is virtual, vertical and smaller than the object

3-



Exercise 4.4:

A- A myopic eye is comparable, when it does not accommodate, to a 15mm lens focal distance. The retina is then located 1 mm beyond the image focus F_O'.

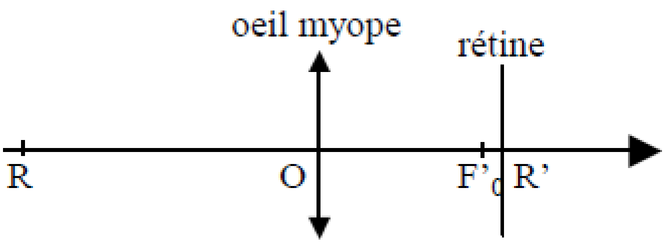
Determine:

1. The distance from the eye to the Remotum point.
2. The number of the corrective lens to use

B- Same questions as part A for a hyperopic eye whose focal length is 15 mm when it does not accommodate and the retina is then located 1 mm below the focus image F'_H.

Solution

1- the distanceOR:$\frac{1}{\overline{OR'}} - \frac{1}{\overline{OR}} = \frac{1}{\overline{OF_O'}}$



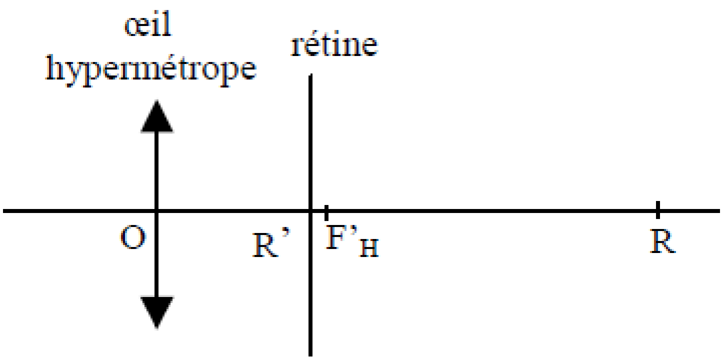
with $\overline{OF'_o} = 15mm$ and $\overline{OR'} = \overline{OF'_o} + \overline{F'_oR'} = 15 + 1 = 16mm$

$$\overline{OR} = \frac{\overline{OF'_o}.\overline{OR'}}{\overline{OF'_o} - \overline{OR'}} = \frac{15.16}{15 - 16} = -240mm = -0.24m$$

2-Corrective lens: The corrective lens is a divergent lens whose image focus is in R.

Its focal length is therefore $F'_C = -0.24m$ and its vergence is:$C = -\frac{1}{0.24} = -4.16\delta$ this is the requested number.

B- the distance OR:$\frac{1}{\overline{OR'}} - \frac{1}{\overline{OR}} = \frac{1}{\overline{OF'_H}}$



with $\overline{OF'_H} = 15mm$ and $\overline{OR'} = \overline{OF'_H} - \overline{F'_HR'} = 15 - 1 = 14mm$

$$\overline{OR} = \frac{\overline{OF'_H}.\overline{OR'}}{\overline{OF'_H} - \overline{OR'}} = \frac{15.14}{15 - 14} = +210mm = +0.21m$$

2-Corrective lens: The corrective lens is a convergent lens whose image focus is in R.

Its focal length is therefore $F'_C = +0.21m$ and its vergence is:$C = +\frac{1}{0.21} = +4.76\delta$ this is the requested number.

Exercise 4.5:

A myopic person cannot see objects clearly further than $2m$ and their minimum distinct vision distance is $10\ cm$.

1. What is its amplitude of accommodation?

2. What must be the vergence V_1 of the contact lens L_1 of the corrective lenses so that he can see objects at infinity? Deduce the type of lens of these corrective lenses.

3. What happens to the Punctum Proximum of these eyes when wearing these corrective lenses?
4. Instead of the contact lens, a lens L_2 is placed $2\ cm$ on the cornea. What should be the vergence $V_2$of this lens to maximize the visual field of these eyes at infinity.

Solution

1. The amplitude of accommodation A is defined by:

$$A = \mathrm{V_{max}} - \mathrm{V_{min}} = \frac{1}{\overline{PR}} - \frac{1}{\overline{PP}}$$

$\overline{PR} = -2cm$ Punctum Remotum .

$\overline{PP}$=$-10cm = 0.1m$ Punctum Proximum

$$A = \frac{1}{(-2)} - \frac{1}{(-0.1)} = 9.5\delta$$

2- The contact lens L_1 of optical center O_1(confused with the optical center of the lens of the crystalline lens) and of vergence V_1 shape of the object located at infinity ($\overline{O_1A} = -\infty$) form an image A' that must be confused with PR:$\overline{O_1A'} = \overline{PR} = -2m$

The conjugation relation for the contact lens L_1:

$$\frac{1}{\overline{O_1A'}} - \frac{1}{\overline{O_1A}} = V_1$$

The vergence $V_1$of corrective lenses:

$$V_1 = \frac{1}{(-2)} - \frac{1}{(-\infty)} = -0.5\delta$$

$V_1 < 0$ The lens is divergent.

3- The new punctum proximum PP_N is the object through which the lens L_1 gives an image to the “natural” punctum proximum PP of the uncorrected eye.

From the conjugation relation of L_1, we have:

$$\frac{1}{\overline{PP}} - \frac{1}{\overline{PP_N}} = V_1 \Rightarrow \frac{1}{\overline{PP_N}} = \frac{1}{\overline{PP}} - V_1 \Rightarrow \overline{PP_N} = \frac{\overline{PP}}{1 - V_1.\overline{PP}} = \frac{(-0.1)}{1-(-0.5).(-0.1)}$$
$$= -0.105m$$

4- Eye-lens distance L_1:$\overline{OO_2} = -0.02m$

$\overline{O_2A'} = \overline{O_2O} + \overline{OA'} = +0.02 - 2 = -1.98m$

the distance $O_2A' = -1.98m$ for an object A placed at infinity $\overline{O_2A} = -\infty$

$$\frac{1}{\overline{O_2A'}} - \frac{1}{\overline{O_2A}} = V_2 \Rightarrow V_2 = \frac{1}{(-1.98)} = -0.505\delta$$

Tutorial N°5

5. Exercises on Pascal's law and Archimedes thrust. (Hydrostatic)

REMINDER

5.1 HYDROSTATIC

Hydrostatic: v = 0

1-Pressure force $\vec{F}$ (normal to the surfaces), due to the impacts of the fluid particles on the surface. The molecules of the fluid have disordered movements, which causes shocks on the surfaces of objects immersed in the fluid. With each shock, there is a change in the momentum of the molecule, and therefore a force is exerted on the surface.If we change the orientation of the surface on which this force is exerted, we always obtain a force of the same standard, which leads to using another modeling, and we instead define a new scalar quantity (therefore without orientation):

the pressure: **P = F / S,**

where F is the norm of the force which would be exerted on the surface S of an object if we placed this object at this location, but it is not necessary that there be an object in the fluid to calculate a pressure.

unité :

-SI : Pascal (Pa) -l'atmosphère, 1 atm = 101 300 Pa , -1bar=10^5 Pa

Forces of cohesion and adhesion= force between the molecules of the liquid (cohesion) and between the molecules of the liquid and those of its container (adhesion).

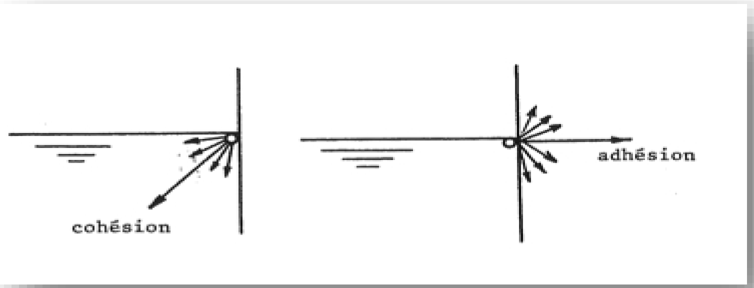
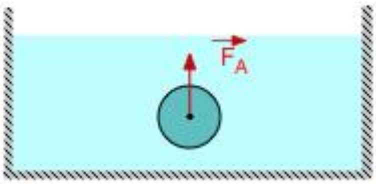


Figure 5.1 Forces of cohesion and adhesion

5.2 CONSEQUENCES OF PRESSURE FORCES -Archimedes' force-

1-Archimedes' force: resulting from pressure forces on the walls of an object. It is vertical upwards, and $\pi_A = \rho . g . V$



5.3 FUNDAMENTAL RELATIONSHIP OF HYDROSTATIC:

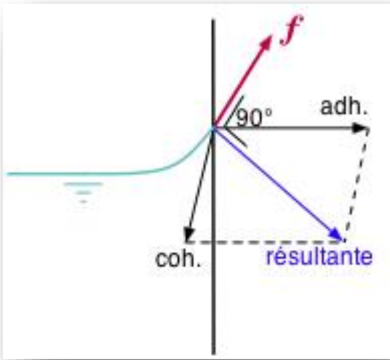
- At two points located at the same height relative to the bottom, if we have the same liquid, we will have the same hydrostatic pressure.
- If we descend from a depth x in a liquid of density ρ, we have an increase in the hydrostatic pressure: **$\Delta p = \rho . g . x$**

5.4 CONSEQUANCES OF COHESION AND ADHESION FORCES- SURFACE TENSION:-

1-SURFACE TENSION:

The liquid rises (or falls) along the wall of the container AS IF there was a force γ pulling on the surface. In this model, this force is the capillary force acting on the surface of the liquid, tangentially to it.
This force tends to minimize the surface area of the liquid.
In practice, we use surface tension instead: $f = \gamma / l$

Another definition is often preferred: $\gamma = dW / dS$ work per unit of surface that must be provided to enlarge the surface. SI unit: **N/m** or **J/m²**



Jurin's law : Capillaries: difference in liquid level with the outside of the capillary of $h = 2\gamma \cos\theta / r\rho g$ (Jurin's law)

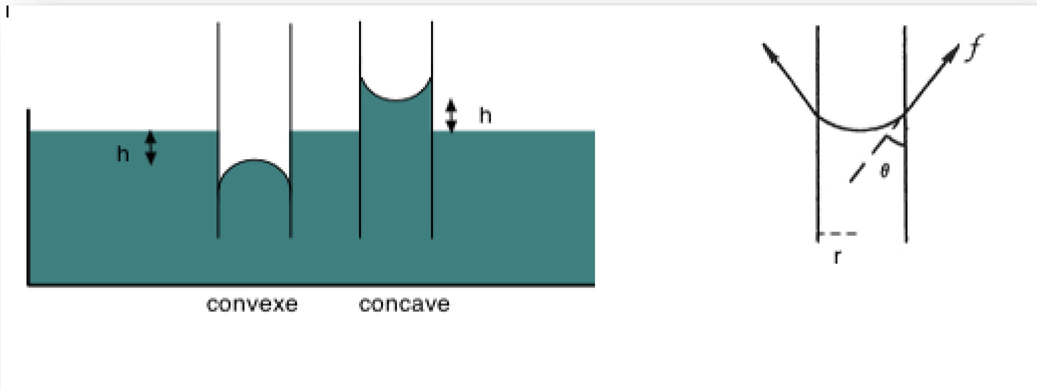


Figure 5.2 : Difference in liquid level with the outside and inside of the capillary.

Pressure variation at the surface of a liquid:

• if flat surface, $P(in) = P(out)$
• if convex surface,

$$\boldsymbol{P(in) = P(out) + 2\gamma/R}$$

• if concave surface,

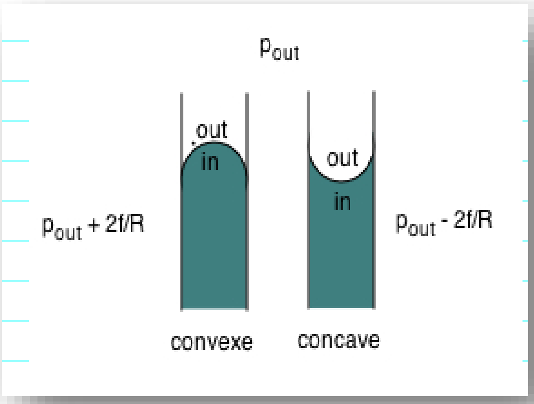
$$\boldsymbol{P(in) = P(out) - 2\gamma/R}$$

In the last 2 cases, we have:

$$\boldsymbol{P(concave\ side) - P(convex\ side) = 2\gamma/R}$$

$$\boldsymbol{(Laplace's\ law).}$$

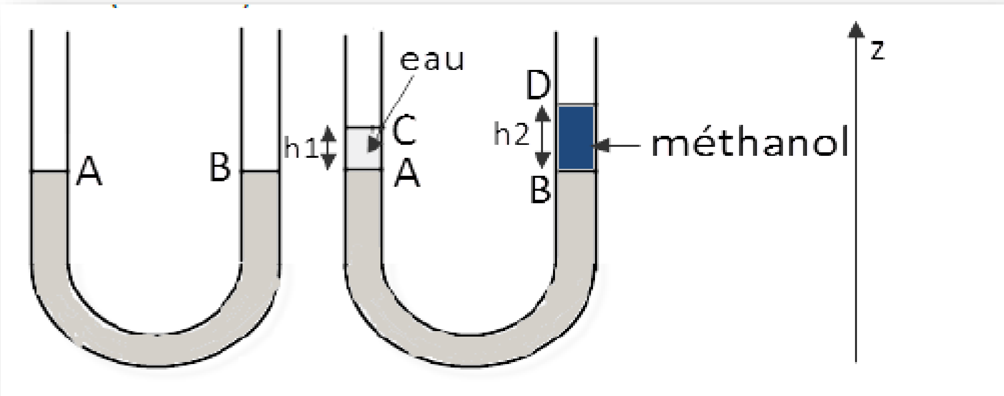
where R is the radius of curvature of the surface.



Tutorial N°5: Exercises on Pascal's law and Archimedes thrust. (Hydrostatic)

Exercise 5.1:

To determine the volume mass of ethanol ρ_{ethanol}, glycerin is introduced into a U tube. In the left branch, water of density $\rho_{\text{water}} = 1000\ kg.m^{-3}$ is poured over a height $h_1 = 10cm$, which causes a difference in level between the points A and B . To bring the points A and B back to the same height, methanol is poured over a height $h_1 = 12.5\ cm$ (diagram).



1. Write the fundamental hydrostatic relationship for the three fluids.

2. Deduce the volumic mass (density) of ethanol ρ_{ethanol}

Solution

1- The fundamental hydrostatic relationship for the three fluids:

Glycerin: $P_A - P_B = 0$ (1)

Water: $P_A - P_C = \rho_{water}.h_1 . g$ (2)

Methanol $P_B - P_D = \rho_{\text{Methanol}}.h_2 . g$ (3)

2- So we have:

from (1) $P_A = P_B$

from (2) $P_A = P_C + \rho_{water}.h_1 . g$

from (3) $P_B = P_D + \rho_{\text{Methanol}}.h_2 . g$

We also have: $P_C = P_D = P_{atm}$

from where: $\rho_{\text{Methanol}}.h_2 .g = \rho_{water}.h_1 .g$

$$\rho_{\text{Methanol}} = \frac{\rho_{water}.h_1}{h_2} = \frac{1000 \times 10}{12.5} = 800Kg.m^{-3}$$

Exercise 5.2:

A hollow steel sphere of density $\rho_{steel} = 7600\ Kg.m^{-3}$ and radius $r = 20cm$ and thickness$e = 5mm$.

1- Determine the weight of this sphere.

2- Determine the Archimedes' thrust that would be exerted on this sphere if it were totally immersed in water.

3- Determine the force that Archimedes would exert on this ball if it were completely submerged in water.

4- Could this sphere float on the surface of water? If yes, then what is the fraction of its submerged volume?

Solution

1-The volume of the hollow sphere V_{HC} is

Volume of the hollow sphere $V_{HC} =$ Volume of the sphere$V_S -$ vacuum volume V_V

$$V_{HC} = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi(r-e)^3 = \frac{4}{3}\pi[r^3 - (r-e)^3] = \frac{4}{3}\pi[(0.2)^3 - (0.2 - 0.008)^3]$$

$$V_{HC} = 3.86 \times 10^{-3} m^3$$

The weight P_{HC} of the hollow sphere is then:

$$P_{HC} = mg = \rho_{steel}.V_{HC}.g = 7600 \times 3.86 \times 10^{-3} \times 9.81 = 287.79N$$

2. The Archimedes thrust for the totally submerged ball is the weight of the displaced volume of the water therefore:

$$\boldsymbol{\pi = \rho_{water}.V_S.g = \rho_{water}.}\frac{4}{3}\pi r^3\boldsymbol{.g = 1000.}\frac{4}{3}\pi.(0.2)^3.9.81 = 328.74N$$

The ball will float because the Archimedes π thrust is greater than its weightP_{HC}:

$$\pi > P_{HC}$$

3- The volume of the submerged part of the ball is equal to the volume of the water V_{water} displaced. At equilibrium, the Archimedes thrust π is equal to the weight of the volume of water displaced: $\pi = P_{HC}$

$$\rho_{water} V_{water} \cdot g = P_{HC}$$

$$V_{water} = \frac{P_{HC}}{\rho_{water} \cdot g} = \frac{287.79}{1000 \times 9.81} = 2.93 \times 10^{-2} m^3$$

4- Knowing that the volume of the sphere is:

$$V_S = \frac{4}{3} \times \pi \times (0.2)^3 = 3.35 \times 10^{-2} m^3$$

The fraction of the submerged volume of the ball compared to its volume:

$$\frac{V_{water}}{V_S} = \frac{2.93 \times 10^{-2} m^3}{3.35 \times 10^{-2} m^3} = 0.87 = 87\%$$

Exercise 5.3:

A very fine capillary tube with a radius r is introduced into a tank filled with water.

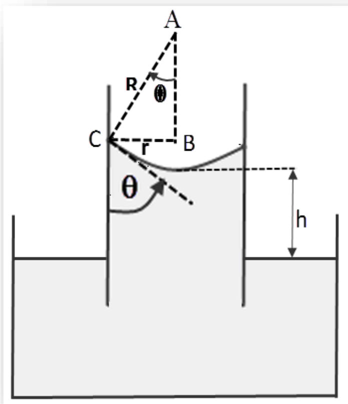
1. What phenomenon do we observe? Explain the phenomenon.

2. Demonstrate Jurin's law; the height h as a function of surface tension γ, contact angle θ, radius r of the capillary tube, density of water ρ and acceleration of gravity g.

3. Assuming that the raw sap is perfectly wetting and has the same properties as water: and, calculate the height of ascent: $\rho = 1000\ Kg.m^{-3}\ and\ \gamma = 73.10^{-3} N/m$. calculate the height of sap rise in rayon xylene channels $r = 25 \mu m$

Solution

1. We observe the rise of water in the capillary tube of an height h due to the Laplace pressure difference. The reverse pressure due to the weight of the riser in the capillary will limit the rise of the water to a height h



2- The pressure difference ΔP Laplace due to surface tension γ is expressed by:

$$\Delta P = \frac{2\gamma}{R}$$

R : is the radius of curvature of the meniscus (interface between water and air).

In the triangle ABC the cosine of the contact angle θ verifies $\cos\theta = \frac{r}{R}$ hence $R = \frac{r}{\cos\theta}$

r is the radius of the capillary tube. ΔP is then written:

$$\Delta P = \frac{2\gamma\ \cos\theta}{r}$$

The water rises to a height h until the hydrostatic pressure π of the water riser in the balance tube ΔP, knowing that $\pi = \rho.g.h$, at pressure equilibrium we have: $\pi = \Delta P \Rightarrow \rho.g.h = \frac{2\gamma\ \cos\theta}{r}$

hence Jurin's law: $h = \frac{2\gamma\ \cos\theta}{r.\rho.g}$

3. The sap is perfectly wet: $\theta^0 = 0$ $r = 25\mu m = 25.10^{-6}m$

$$h = \frac{2 \times\ 73.10^{-3} \times \cos 0}{25.10^{-6} \times 1000 \times 9.81} = 0.595m$$

Tutorial N°6

6. Exercises on Bernoulli's law (hydrodynamics)

REMINDER

6.1 DYNAMICS OF INCOMPREHENSIBLE FLUIDS

Definitions:

The principle of continuity expresses the conservation of mass, which means that no fluid cannot be created nor disappear in a given volume.

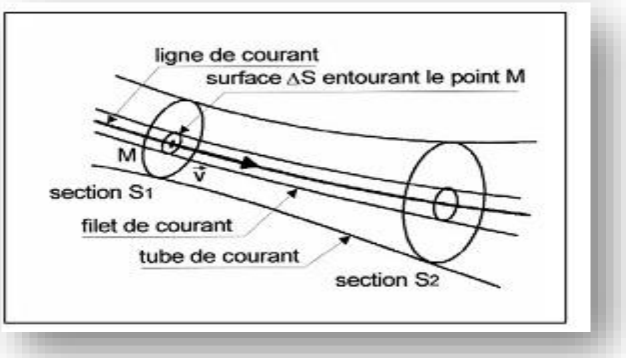


Figure 6.1: The principle of continuity expresses the conservation of mass.

- Flow: is the quantity of material that passes through a straight section of pipe during the unit of time.
- Mass flow: If dm is the elementary mass of fluid having traveled a straight section of the carried out during the time interval dt, the mass flow is written:

$$q_m = \frac{dm}{dt}\,[Kg.s^{-1}]$$

- Volume flow: If dV is the elementary volume of fluid having traveled a straight section of rolling during the time interval dt, the volume flow is written:

$$q_V = \frac{dV}{dt}\,[m^3.s^{-1}]$$

- The relation between $q_m and\ q_V$: The density ρ is given by the relation:$\rho = \frac{dm}{dV}$. from where : $q_m = q_V$

Since the flow rate always remains constant in a steady state), **the continuity equation** is written as: $Q = S_1V_1 = S_2V_2$

6.2 GENERAL Flow EQUATION OR Bernoulli EQUATION

A flow regime is said to be permanent or stationary if the parameters, which characterize (pressure, temperature, speed, density, etc.), have a constant value over time:

a- Case of Perfect Fluids (non-viscous)

Bernoulli's equation expresses that, all along a fluid stream

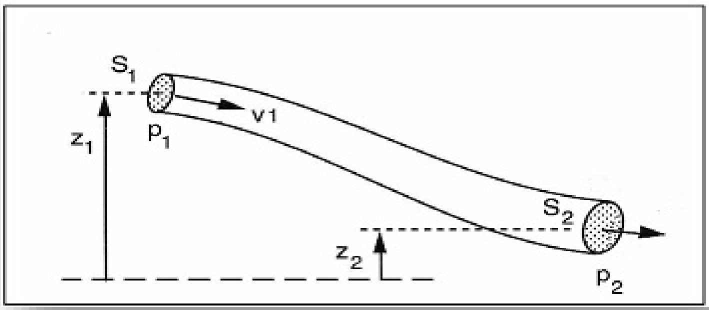


Figure 6.2: A flow regime permanent or stationary

Bernoulli's equation expresses that, throughout a fluid stream in permanent (stationary) motion, the total energy per unit weight of the fluid remains constant and the equation is written:

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} = H = constant$$

b- Case of real fluids (viscous)

In the case of real fluids, the energy decreases in the direction of flow. this is due to the viscous nature of the fluid which dissipates part of the energy: this loss of energy is called pressure loss and the equation is written:

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h$$

h: which is the consequence of the viscosity of the fluid and the roughness of the walls of the section flow.

Tutorial N°6: Exercises on Bernoulli's law (hydrodynamics)

Exercise 6.1:

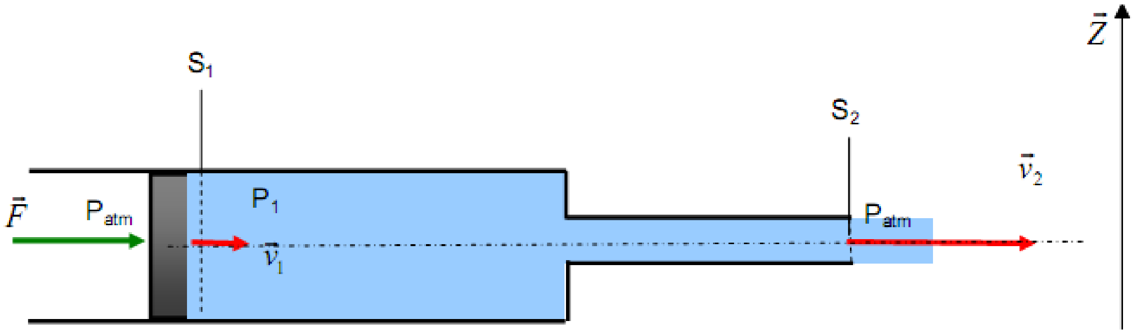
The figure below shows a piston that moves without friction in a cylinder of section S_1 and diameter $d_1 = 4\ cm$ filled with a perfect fluid of density$\rho = 1000\ kg/m^3$. A force F with an intensity of 62.84 N acts on the piston, at a constant speed V_1 .The fluid can escape to the outside through a cylinder of section S_2 and diameter $d_2 = 1\ cm$ at a speed V_2 and a pressure $P_2 = P_{atm} = 1\ bar$.

1- By applying the Fundamental Principle of Dynamics to the piston, determine the pressure P_1 of the fluid at section S_1 as a function of F, P_{atm} and d?

2- Write the continuity equation and determine the expression of the speed V_1 as a function of V_2?

3- By applying the Bernoulli equation, determine the flow speed V_2 as a function of P_1, P_{atm} and ρ?

We assume that the cylinders are in a horizontal position ($Z_1 = Z_2$)



Solution

1- Applying the Fundamental Principle of Dynamics, we obtain:

$$P_1 = \frac{4F}{\pi d_1^2} + P_{atm} = 1.5\text{bar}$$

2- The continuity equation:

$$\pi {d_1}^2 V_1 = \pi {d_2}^2 V_2 \text{ and } d_1 = 4d_2$$

$$\pi 16 {d_2}^2 V_1 = \pi {d_2}^2 V_2 \Rightarrow V_1 = \frac{1}{16} V_2$$

3- By applying the Bernoulli equation

$$\frac{P_1}{\rho}+\frac{v_1^2}{2}=\frac{P_{atm}}{\rho}+\frac{v_2^2}{2}\Longrightarrow\frac{P_1}{\rho}+\frac{v_2^2}{2\times 256}=\frac{P_{atm}}{\rho}+\frac{v_2^2}{2}$$

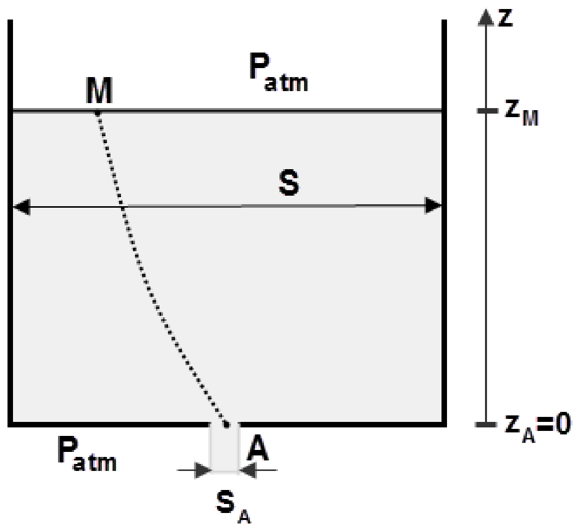
$$\frac{P_1}{\rho}-\frac{P_{atm}}{\rho}=\frac{v_2^2}{2}-\frac{v_2^2}{2\times 256}\Longrightarrow\frac{(P_1-P_{atm})}{\rho}=\frac{(256-1)\times v_2^2}{512}$$

$$\frac{(P_1-P_{atm})}{\rho}=\frac{(255)\times v_2^2}{512}$$

$v_2^2=\frac{512\times(P_1-P_{atm})}{255\times\rho}\Rightarrow v_2=\sqrt{\frac{512\times(P_1-P_{atm})}{255\times\rho}}=10m/s.$

Exercise 6.2:

A reservoir, cubic in shape and section $S=4m^2\ and\ a=2m$ .The reservoir is filled with liquid that can be emptied through an opening A pierced at its horizontal bottom and opening into the open air. A is section $S_A=8\ cm^2$. We will assume that when it is drained, the liquid is perfect, incompressible and its flow speed is constant.



1- When emptying this reservoir, consider the streamline joining points M and A. By applying the Bernoulli relation between these two points, give the expression for the flow speed v_A liquid to the point A depending on the acceleration of gravity g and the altitude Z_M of the point M.
2- Give the relation of the volume flowQ_v at the orifice A as a function of g, Z_M and M.
3- Establish the relationship of the flow speed v_M at point M according to $g, Z_M, S_A\ and\ S$
4- Calculate the time necessary for the total emptying of this reservoir.

Solution

1- The Bernoulli relation between the two points $M\ and\ A$ is written:

$$z_A + \frac{P_A}{\rho g} + \frac{v_A^2}{2g} = z_M + \frac{P_M}{\rho g} + \frac{v_M^2}{2g}$$

$$z_A = 0$$

The points $M\ and\ A$ and being in direct contact with the air, their pressures are equal to atmospheric pressure: $P_A = P_M = P_{atm}$

$$\frac{P_{atm}}{\rho g} + \frac{v_A^2}{2g} = z_M + \frac{P_{atm}}{\rho g} + \frac{v_M^2}{2g} \Rightarrow\ v_A^2 = v_M^2 + 2.g.Z_M$$

For a perfect liquid the volume flow Q_v is constant:$S_A.v_A = S.v_M$

$$v_M = \frac{S_A.v_A}{S}$$

$S_A = 8 \times 10^{-4}\ m^2$
$S = 4\ m^2$

$$v_M = \frac{8 \times 10^{-4}\ .v_A}{4} \Rightarrow v_M = 2 \times 10^{-4}\ .v_A$$

$v_M^2 = 4 \times 10^{-8}\ .v_A^2 \ll v_A^2, we\ then\ neglect\ v_M^2\ compared\ to\ v_A^2$

$v_A^2 = 2.g.Z_M \Rightarrow\ v_A = \sqrt{2.g.Z_M}$

2- $Q_v = S_A.v_A = S_A.\sqrt{2.g.Z_M}$

3- $v_M = \frac{S_A.\sqrt{2.g.Z_M}}{S}$

4- Speed v_M is expressed by:

$$v_M = -\frac{dZ_M}{dt}$$

$$-\frac{dZ_M}{dt} = \frac{S_A.\sqrt{2.g.Z_M}}{S}$$

$$\frac{dZ_M}{\sqrt{Z_M}} = -\frac{S_A.\sqrt{2.g}}{S}.dt$$

We integrate from $t = 0$ until the moment T when the reservoir has been completely emptied: $Z_M = Z_A = 0\ m$

$$\int_{Z_M}^{0} \frac{dZ_M}{\sqrt{Z_M}} = -\frac{S_A.\sqrt{2.g}}{S}.\int_0^{\tau} dt$$

$$-2\sqrt{Z_M} = \frac{S_A.\sqrt{2.g}}{S}.T$$

$$-2\sqrt{Z_M} = \frac{S_A.\sqrt{2.g}}{S}.T$$

$$T = \frac{2\sqrt{Z_M}.S}{S_A.\sqrt{2.g}} = \frac{2\sqrt{2}\times 4}{8\times 10^{-4}\times\sqrt{2\times 9.81}} \approx 3193 \text{ s}$$

Exercise 6.3:

The aorta is the largest artery in the body. It receives the blood that leaves the heart and distributes it to the arteries throughout the body. The heart rate for an adult is 80 beats per minute. With each beat, the heart injects a volume $v_b = 0.075\ L$ into the aorta.

1- Calculate total volume V_t of blood flowing through the aorta in one minute. Deduce the volume flow Q_v.

2- Calculate average speed v_{moy} of blood flow knowing that the diameter of the aorta is $d = 2cm$.

3- Calculate the Reynolds number R_e for flow in the aorta knowing that the dynamic viscosity of the blood is$\eta = 5 \times 10^{-3} Pa.s$ and its density is $\rho = 1060\ Kg.m^{-3}$. Deduce the flow regime.

4- Determine the critical speed v_{critical} at which the regime becomes turbulent.

5- The blood distributed by the aorta ultimately reaches the capillaries. A blood capillary is an extremely thin blood vessel of medium radius $r_c = 5\mu m$. The blood circulates there at an average speed $v_{cap} = 0.06\ cm\ s^{-1}$ Calculate the volume flow rate Q_{cap} of blood in this capillary.

6- Determine the average number N_{cap} of capillaries present in the body in a human being.

Solution

1- The total volume of blood flowing through the aorta in one minute is:

$$V_t = 80 \times v_b = 80 \times 0.075 = 6\ L$$

$$Q_v = 6L.minute^{-1} = \frac{6 \times\ 10^{-3} m^3}{60\ s} = 10^{-4} m^3.s^{-1}$$

2- $Q_v = v_{moy}.S$

S is the section of the aorta:

$$S = \pi.\left(\frac{d}{2}\right)^2 = \pi.\left(\frac{2 \times\ 10^{-2}}{2}\right)^2 = 3.14 \times\ 10^{-2} m^2$$

$$v_{moy} = \frac{Q_v}{S} = \frac{10^{-4}}{3.14 \times 10^{-2}} = 0.32\ m.s^{-1}$$

3- The Reynolds number is defined by:

$$R_e = \frac{\rho.v_m.d}{\eta} = \frac{1060 \times 0.32 \times 2 \times 10^{-2}}{5 \times 10^{-3}} = 1356.8$$

$$R_e < 2000\ The\ flow\ regime\ is\ laminar$$

4- The regime becomes turbulent for $R_e > 3000$. The critical speed from which the flow becomes turbulent is for $R_e = 3000$.

$$v_{\text{critical}} = \frac{\eta.R_e}{\rho.d} = \frac{5 \times 10^{-3} \times 3000}{1060 \times 2 \times 10^{-2}} = 0.71 m/s$$

5- $Q_{cap} = v_{cap}.S_{cap} = v_{cap}.\pi.r^2 = 6 \times 10^{-4} \times \pi \times (5 \times 10^{-6})^2 = 4.7 \times 10^{-14} m^3/s.$

6- $Q_v = N_{cap}.Q_{cap}$

$$N_{cap} = \frac{Q_v}{Q_{cap}} = \frac{10^{-4}}{4.7 \times 10^{-14}} = 2.13 \times 10^{10}$$

REFERENCES

1) Mokhtar FALEK, "Polycopié Biophysique Cours, Exercices et Travaux Pratiques'', 2ième année Licence Biologie', Université Biskra, janvier 2016.

2) BOUDAIRA Boukhemis, "Polycopié Biophysique Cours, Exercices et Travaux Pratiques ", 1ère année tronc commun Biologie, Université Biskra, Novembre 2020.

3) BAKHTATOU Ali, " Polycopié de Physique Travaux Dirigés ", 1ère année tronc commun Biologie, UNIVERSITÉ 8 MAI 1945 GUELMA, Décembre 2019.

4) Exercices de thermodynamique et d'optique, M. Bertin et J. Renault, éditions Dunod, Paris 1980.

5) Optique ; Optique géométrique et notions d'optique physique ; M. Bertin, J. P. Faroux et J. Renault, éditions Dunod, Paris 1981.

6) Hydrodynamique physique, Etienne Guyon, Jean-Pierre Hulin, Luc Petit et John Hinch, éditions CNRS, Paris 2001.

7) Optique géométrique, RAPPELS DE COURS ET EXERCICES, Agnès MAUREL et Jean-Marie MALBEC, éditions BELIN, Paris 2002.

8) Optique géométrique, COURS ET 134 EXERCICES CORRIGÉS, T. Bécherrawy, éditions De Boeck, Bruxelles 2006.

9) https://www.physagreg.fr/optique-12-generalites-systemes-miroirs.php