Université Mohamed - KHIDER - BISKRA
Département de Génie Mécanique – MMC / Master – 2026/2027
Responsable du Module : Pr. HECINI M.

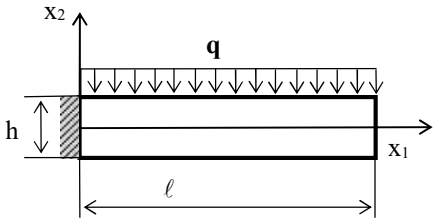
Travaux Dirigés (Série N° 6)

Exercise N° 1 :

Consider a beam of length ℓ, height h, and thickness e, clamped at its end $x_1=0$ and subjected to a load q per unit length, uniformly distributed along its upper surface (y=h/2) (body forces are neglected).

1°) Show that the problem can be solved using a 5th-degree stress function of the form:

$$\phi(x_1,x_2) = A\,x_1^2x_2^3 + B\,x_2^5 + C\,x_1^2x_2 + D\,x_2^3 + E\,x_1^2$$



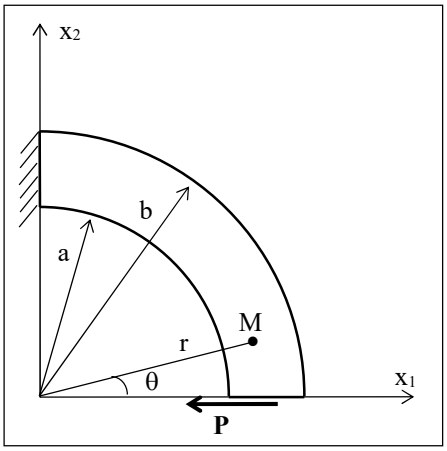
2°) Determine σ_{11} , σ_{22} , σ_{12} at a point M(x_1 , x_2).

Considering the following boundary conditions:

* pour x_2= +h/2 ⇒ σ_{22}=-q et σ_{12}=0
* pour x_2= -h/2 ⇒ σ_{22}= 0 et σ_{12}=0
* pour x_1= ℓ/2 ⇒ $\int_{-h/2}^{h/2} \sigma_{11}\, x_2\, dx_2 = 0$

Exercise N° 2 :

Consider a beam with a rectangular cross-section, clamped at one end ($\theta = \pi/2$) and subjected to a concentrated load P at the other end ($\theta = 0$) in the radial direction.
(Body forces are neglected.)



Given that the problem can be solved using an Airy stress function of the form:

$$\phi(r,\theta) = \left[Ar^3 + \frac{B}{r} + C\,r + D\,r\,Ln(r) \right] \sin(\theta)$$

determine the stresses σ_{rr} , $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$, in the beam,
considering the following boundary conditions:

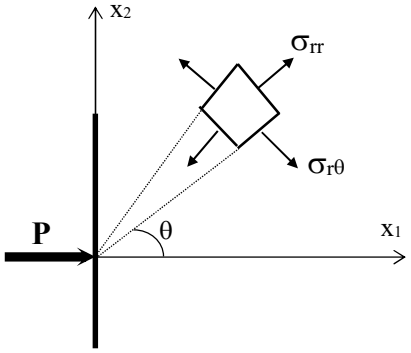
* free surface : σ_{rr}= 0 et $\sigma_{r\theta}$= 0 si r = a and r = b ∀θ.
* loaded surface : $\int_a^b \sigma_{r\theta}\, dr = P$ si θ = 0

Exercise N° 3 :

Using a stress function of the form :

$$\phi(r,\theta) = K\,r\,\theta\,\sin(\theta),$$

determine the stress distribution resulting from a concentrated normal force P acting on the straight boundary of a semi-infinite plate.

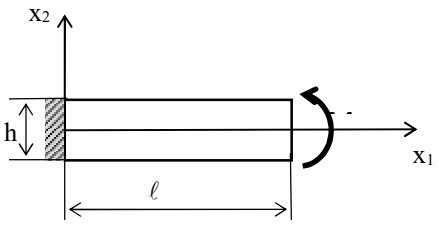


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Exemples

Example 01:

Consider a beam of length ℓ height **h**, and thickness **e**, clamped at its end $\mathbf{x_1 = 0}$ and subjected to a bending moment M at its end $\mathbf{x_1 = \ell}$ (body forces are neglected).



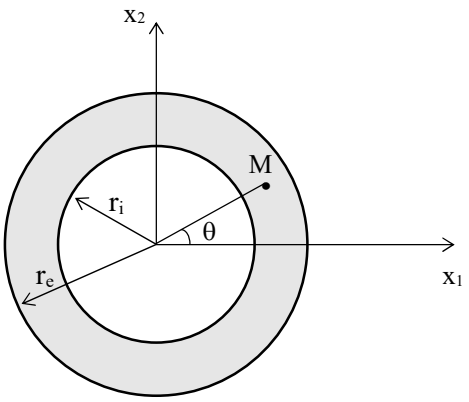
1°) Show that the problem can be solved using a third-degree stress function; then determine σ_{11} , σ_{22} , σ_{12} at a point $M(x_1 , x_2)$.

2°) Considering the case of plane stress, determine the displacements at a point $M(x_1, x_2)$. From this, deduce the deformed shape of the beam.

Example 02:

A circular cylinder with inner radius $\mathbf{r_i}$ and outer radius $\mathbf{r_e}$ is subjected to both an internal pressure $\mathbf{p_i}$ and an external pressure $\mathbf{p_e}$. The problem is solved using a stress function of the form:

$$\phi(r,\theta) = A\ Ln(r) + C\ r^2 \ .$$



1°) Determine the stresses σ_{rr} , $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the cylinder

2°) Considering the case of plane stress, determine the displacements at a point. M(r, θ).

3°) Application: A thick-walled steel cylinder with an internal diameter of 8 cm is subjected to an internal pressure of 400 kg/cm². There is no external pressure. The allowable stress for the material is 1260 kg/cm². Calculate the external diameter of the cylinder.