Lesson 02 : Measuring angles

In topography, angles are always measured in a horizontal or vertical plane (never in an oblique plane), using instruments such as the level, the tachometer, the theodolite or the goniometer.

1. LE THEODOLITE :

A theodolite is a device for measuring horizontal angles (angles projected in a horizontal plane) and vertical angles (angles projected in a vertical plane), consisting essentially of three axes contributors and two chicks generally called circles (Figure 01)

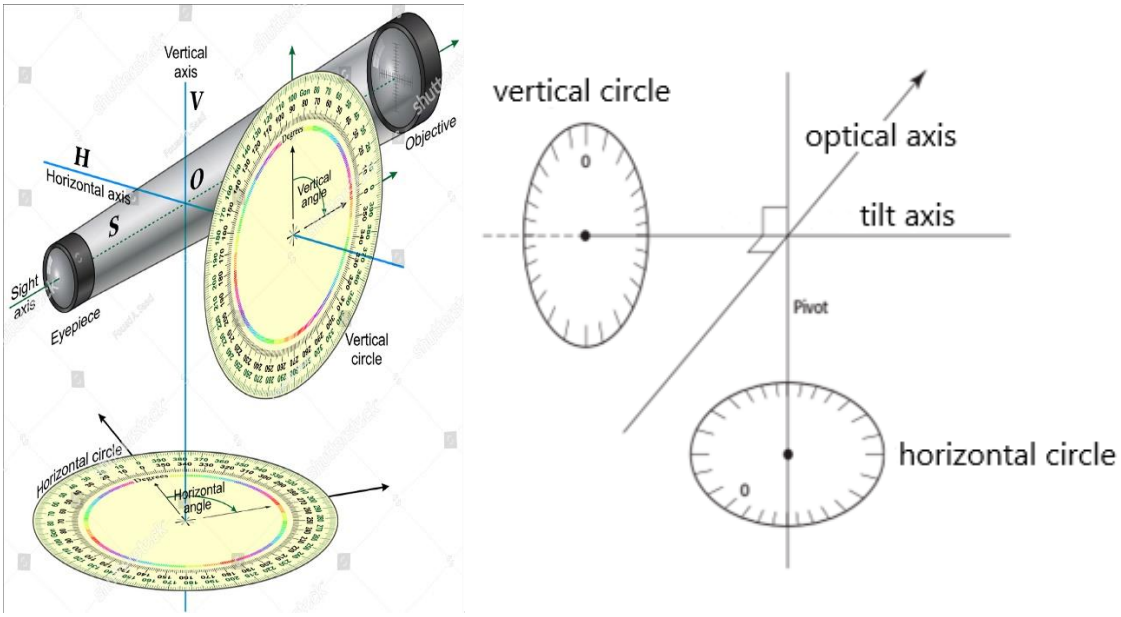


Figure 01: Theodolite principle diagrams

We distinguish:

– The pivot, or main axis, set vertically and centered, i.e., coincident with the vertical of the point on the ground or on the "roof" during underground work; the theodolite is then stationed, i.e., ready to measure horizontal and vertical angles;

– the tilt axis, also called the secondary axis or trunnion axis, perpendicular to the previous one, therefore horizontal at the time of observations;

– The optical axis of the telescope, perpendicular to the tilt axis, sweeps a vertical sighting plane.

– The Horizontal Circle, centered on the pivot, allows the measurement of horizontal angles;
– The Vertical Circle, or eclimeter, centered on the tilt axis, allows the measurement of vertical angles.
À l'heure actuelle, deux catégories d'instruments sont utilisés :



Figure 02: optical theodolite (left) and electronic theodolite (right)

- The optical theodolites (Figure 2 on the left), ancient instruments, with which the operator proceeds an optical reading by generally estimating milligrade for ordinary theodolites, the decimilligrade for precision theodolites;

- Electronic theodolites (Figure 2 on the right), with automatic reading, the integrated microprocessor managing the processing of the measurement and transmitting to the liquid crystal display the readings of the horizontal and vertical circles, with a resolution which can reach 0.1 mgr.

European Leica and Trimble companies have ceased the manufacture of optical theodolites, now supplanted by electronic theodolitis in all precision orders.

2. Precision of angle measures

A measure is marred by a certain error, of an uncertainty. It comes from various factors: the method used, the instrument used, the operator's experience, the size measured ... Different concepts are used to qualify the quality of the measurement, and various means exist to distribute the residues of a series of measurement.

3. Measurement of a horizontal angle

The device is at the station at point S. The operator aims at point A (building summit) and regulates the zero of graduations on this point. By targeting point B, he reads in the theodolite the horizontal angle a's 'b' (of which a ', s', b 'are the projections of A, B and S on the horizontal plane

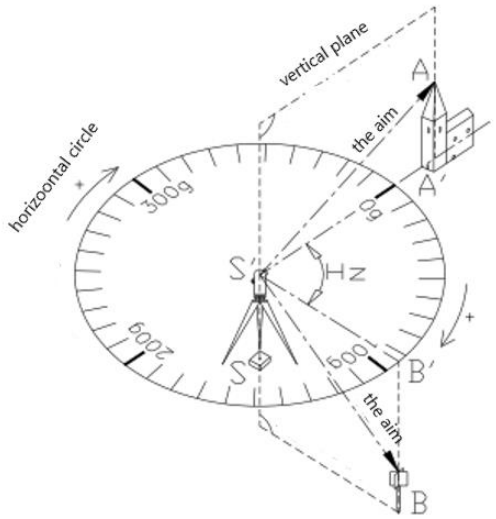


Figure 03: Measuring horizontal angles

4. Measuring a Zenith Angle

In a theodolite, the vertical circle is attached to the telescope. Its zero is placed on the telescope's axis (sight).

The reading index is fixed and positioned vertically (zenith) to the station point S.

When the line of sight passes through a point M, the index then gives the reading of the angle z (or V) intercepted on the vertical circle

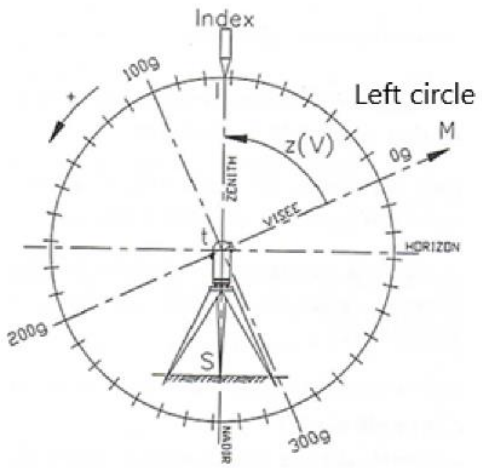


Figure 04 : Measuring vertical angles

5. Orientation

The direction SM, defined by the station point S and the signal M (Figure 5), is oriented in the plane relative to the "north" of point S:

- grid north, i.e., the positive ordinate axis of the projection system
- geographic north
- magnetic north.

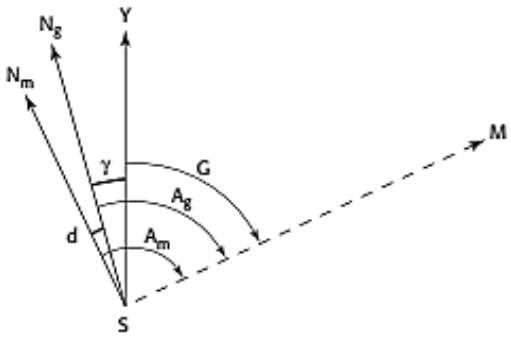


Figure 05 : Orientation

- Geographic north is defined as the direction of the North Pole.

- Magnetic north is indicated at a given moment by the direction taken by the magnetic needle. It is not stable and varies over centuries, years, and hours. It is also influenced by all the magnetic bodies surrounding a location of operation.

- the gisment (Bearing) is defined as the angle, in the horizontal plane, between a vector, defined by two points known in coordinates, and the direction of cartographic north. It is counted in a clockwise direction.

5.1 Bearing Calculation

The Gisement (Bearing) is a horizontal angle widely used by surveyors because it is very useful in calculations.

The bearing of an AB direction is the horizontal angle measured positively in a clockwise direction between the y-axis of the projection system used and this AB direction (Figure 6).

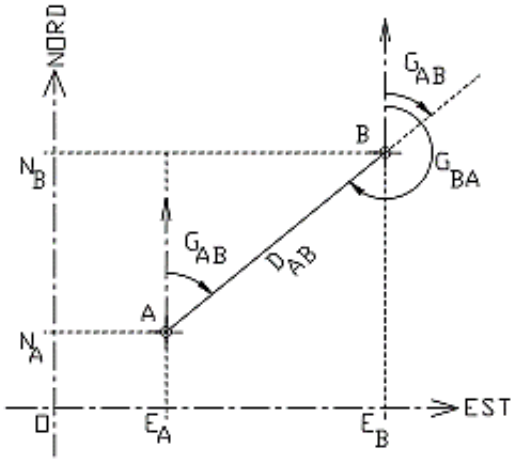


Figure 06 : Bearing of th AB direction

We denote G AB (or also VAB).

Mathematically, it is the positive clockwise angle between the coordinate system's ordinate axis and the vector. G is between 0 and 400 gr.

For example (Figure 6): GAB is the angle between North (ordinates) and the AB direction.

G BA is the angle between North and the BA direction.

The relationship between GAB and GBA is: GBA = GAB + 200

5.2 Calculating a Bearing from Cartesian Coordinates

Consider the coordinates of two points A (EA, NA) and B (EB, NB).

The following relationship is used to calculate **GAB: tan GAB = (EB – EA)/(NB – NA)**

Note

To obtain the value of G, use the tan–1 ( ) or inverse tangent function.

5.3 Using the Bearing for Coordinate Calculations

In topography, it is very common to know a point S (ES, NS) and to search for the coordinates of a point P visible from S. We say that P is radiated from S if we can measure the horizontal distance D SP and the bearing G SP (Figure 7).

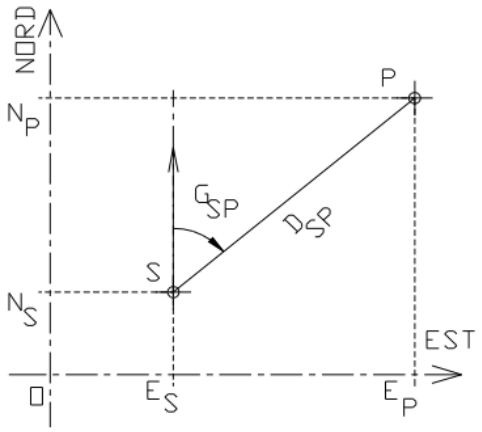


Figure 07 : Calcul de cordonnées

Regardless of the quadrant, the coordinates of point P can then be calculated using the following formulas:

EP = ES + DSP. Sin GSP

Np = NS + DSP · cos GSP

If GSP cannot be measured directly, an angle α can be measured with a direction whose bearing is known, or a mean station G0 can be calculated.

To obtain GAB, the position of point B relative to point A must be taken into account; these are called quadrants (Figure 8):

- **Quadrant 1: B is east and north of A (ΔE > 0 and ΔN > 0).**
  GAB = g
  • Quadrant 2: B is east and south of A (ΔE > 0 and ΔN < 0).
  GAB = 200 + g (with g < 0)
  • Quadrant 3: B is west and south of A (ΔE < 0 and ΔN < 0).
  GAB = 200 + g (with g > 0)

• Quadrant 4: B is west and north of A (ΔE < 0 and ΔN > 0).

GAB = 400 + g (with g < 0)

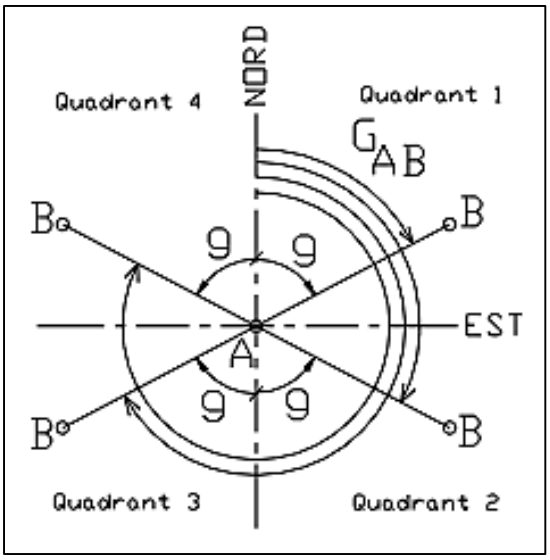


Figure 08 : Différents quadrants

5.4 Determining the G0 Mean Station

When positioning a theodolite at a point S known in Lambert coordinates, the position of the limb zero is initially arbitrary.

If the operator wishes to fix the reference frame in which to work, they orient the limb (the horizontal circle of the instrument) to another known point P. Two cases then arise:

• The point is known in local coordinates:

The work is performed in local coordinates. The operator generally places the limb zero at point P and thus defines the reference frame Sxy (Figure 9).

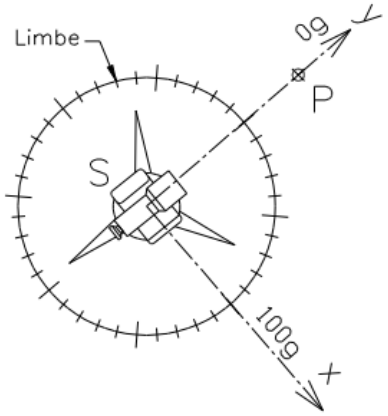


Figure 09 : Origin of limb on P

• The point is known in Lambert coordinates:

It is then possible to calculate the bearing of the SP direction and display it on point P. This defines the SEN reference frame in Figure 09. The axes of which are parallel to the general Lambert reference frame. The advantage of this second manipulation is to work directly in the Lambert reference frame and therefore to limit intermediate calculations. In practice, we will not move the limb of the device. We will simply aim at point P and deduce the bearing of the limb zero, called G0 (Figure 10). This orientation can be done on a single pair of SP points, but, for greater precision, we can calculate an average over several other known points well distributed around the station, hence the term G0 station average.

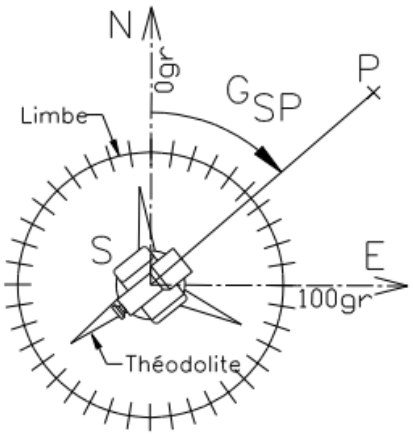


Figure 10: Origin of the limb on the north Lambert

5.5 Calculating the Station G0

The station G0 (also denoted V0) is a station orientation constant S which, added to a horizontal angle reading at a target point P, gives the bearing of the SP direction.

It is also the bearing of the limb zero, i.e., the angle between the direction of the Lambert coordinate system's y-axis and the limb zero of the stationed aircraft (Figure 10).

The bearing of the SP direction is defined by:

GSP = G0 + LS→ P

G0 = GSP – LS→ P

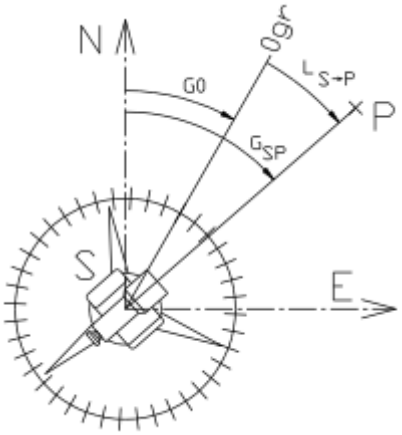


Figure 09 : G_0 station

5.6 Defining the Average Station G0

To improve the accuracy of the station's orientation, several readings are taken at known coordinate points: these points are called "old points."

To obtain a correct orientation, at least two sightings are required (three or four are preferable) distributed over the four quadrants around the station point S.

Given that four old points M, N, W, and P are sighted from station S, we obtain four G0 values:

Aiming at M: G01 = GSM – LS →M Aiming at N: G02 = GSN – LS →N	Aiming at O: G03 = GSO – LS →O Aiming at P: G04 = GSP – LS →P

The G0average is then the weighted average of the G0i. It is only weighted if the sights are of unequal length (the weighting is proportional to the length of each sight because the longer a sight, the more precise its theoretical angular orientation).

So we have:

$$G0_{\text{moyen}} = \frac{\sum_{i=1}^{i=n} pi \cdot G0i}{\sum_{i=1}^{i=n} pi}$$

n is the number of orientation sights (number of calculated G0i).

pi represents the weight of each sight, i.e., its length in kilometers: pi = Dikm.

Weights ***pi*** are rounded to the nearest meter in manual calculations.