Statistical Inference

TD Sheet 1: Sampling and Sampling Distributions

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Exercise 1: Population, Sample, Parameter and Statistic

A company manufactures light bulbs. The lifetime of each bulb is measured in hours. A random sample of 50 bulbs is selected from the daily production.

For the sample, the average lifetime is found to be 1240 hours and the sample standard deviation is 85 hours.

1. Identify the population.
2. Identify the sample.
3. Give two possible population parameters of interest.
4. Identify two statistics computed from the sample.
5. Explain the difference between a parameter and a statistic.

Exercise 2: Sampling Methods and Experimental Design

For each of the following situations, identify the sampling method used.

1. A university randomly selects 200 students from the complete list of registered students.
2. A population is divided into four age groups, and a random sample is selected from each group.
3. A researcher selects every 20th name from an ordered list after randomly selecting the first name.
4. Ten schools are randomly selected from a region, and all students in these schools are surveyed.
5. A researcher interviews the first 100 people entering a shopping center.

A medical experiment compares a new treatment with a standard treatment. Two hundred patients are randomly assigned to the two treatments.

6. Identify the experimental units.
7. Identify the factor and its levels.
8. Identify the response variable if the objective is to compare recovery time.
9. Explain why randomization is important.

Exercise 3: Sampling Distribution of the Sample Mean

The weight of a certain manufactured product is normally distributed with

$$\mu = 80 \text{ grams}, \qquad \sigma = 12 \text{ grams}.$$

A random sample of size

$$n = 36$$

is selected.

1. Determine the distribution of the sample mean $\overline{X}$.
2. Compute $E(\overline{X})$.
3. Compute $\text{Var}(\overline{X})$.
4. Compute the standard error of $\overline{X}$.
5. Find
$$P(78 \leq \overline{X} \leq 84).$$

Exercise 4: Sampling Distribution of a Proportion

In a large population, the proportion of individuals having a certain characteristic is

$$p = 0.40.$$

A random sample of size

$$n = 100$$

is selected, and $\widehat{p}$ denotes the sample proportion.

1. Compute $E(\widehat{p})$.
2. Compute $\text{Var}(\widehat{p})$.
3. Compute the standard error of $\widehat{p}$.
4. Check whether the normal approximation is appropriate.
5. Using the normal approximation, compute approximately
$$P(0.35 \leq \widehat{p} \leq 0.45).$$

Exercise 5: Difference of Two Sample Means

Two independent populations have the following characteristics:

$$\mu_1 = 50, \qquad \sigma_1 = 8,$$

and

$$\mu_2 = 46, \qquad \sigma_2 = 6.$$

Independent random samples of sizes

$$n_1 = 64 \qquad \text{and} \qquad n_2 = 36$$

are selected.
Let $\overline{X}$ and $\overline{Y}$ denote the corresponding sample means.

1. Compute
$$E(\overline{X} - \overline{Y}).$$
2. Compute
$$\text{Var}(\overline{X} - \overline{Y}).$$
3. Compute the standard error of
$$\overline{X} - \overline{Y}.$$
4. Assuming that the two populations are normal, determine the distribution of
$$\overline{X} - \overline{Y}.$$
5. Compute
$$P(\overline{X} - \overline{Y} > 6).$$

Exercise 6: Sample Variance and Central Limit Theorem

Part A: Sample Variance

Let

$$X_1, \ldots, X_{20} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu, 25).$$

Let

$$S^2 = \frac{1}{19} \sum_{i=1}^{20} (X_i - \overline{X})^2.$$

1. Determine the distribution of
$$\frac{19S^2}{25}.$$
2. How many degrees of freedom are involved?
3. Let $\chi^2_{19,0.95}$ denote the 95th percentile of the chi-square distribution with 19 degrees of freedom. Find a value c such that
$$P(S^2 \leq c) = 0.95.$$

Part B: Central Limit Theorem

Suppose that the lifetime X of a component has a non-normal distribution with

$$E(X) = 10 \qquad \text{and} \qquad \text{Var}(X) = 100.$$

A random sample of size

$$n = 100$$

is selected.

4. Use the Central Limit Theorem to approximate the distribution of $\overline{X}$.

5. Compute approximately

$$P(8 \leq \overline{X} \leq 12).$$