Tutorial Series N°05: Matrix Reduction

Exercise 1. Let A be the following 2×2 matrix:

$$A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

1) Calculate the characteristic polynomial $P_A(\lambda) = \det(A - \lambda I)$.
2) Deduce the eigenvalues of the matrix A.
3) Determine the eigenvectors associated with each eigenvalue.

Exercise 2. Consider the matrix B given by:

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

1) Without calculating the determinant, what are the eigenvalues of B? Justify your answer.
2) Are all the eigenvalues distinct? What can you conclude about the diagonalizability of matrix B?
3) Find the eigenvector associated with the smallest eigenvalue.

Exercise 3. Let C be the following matrix:

$$C = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

1) Find the characteristic polynomial of C.
2) State the **Cayley-Hamilton theorem** and apply it to matrix C to find an equation relating C^2, C, and the identity matrix I.
3) Deduce the inverse matrix C^{-1} using the result from the previous question.