Chapter 4

Magnéto-static

Magnéto-statique

The word **“magnetism”** derives from the name of the region called “magnesia”, located on the west coast of present-day Turkey, where the magnetic phenomenon has been observed for a very long time. This region contained deposits of the ore called “magnetite” which has specific properties. In 1819 the Swedish physicist Orsted demonstrated for the first time the effect of an electric current on a magnet and he proved that a wire passing through an electric current acquires magnetic properties quite similar to those possessed by a natural magnet.

In the previous chapters, we limited ourselves to the study of static electric charges and moving electric charges. In the final chapter, we will address magnetostatics. **Magnetostatics** is the study of magnetic fields in the steady state, that is, the state in which the magnetic field is independent of time; in other words, a state where its magnitude and direction at a given point remain constant and depend only on the position of that point.

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1- Properties of magnets.

In general, permanent magnets possess a set of properties that can be summarized as follows:

- Every magnet, no matter how much it is divided, always has two poles: a north pole (N), which points toward the Earth's north pole, and a south pole (S), which points toward the Earth's south pole.

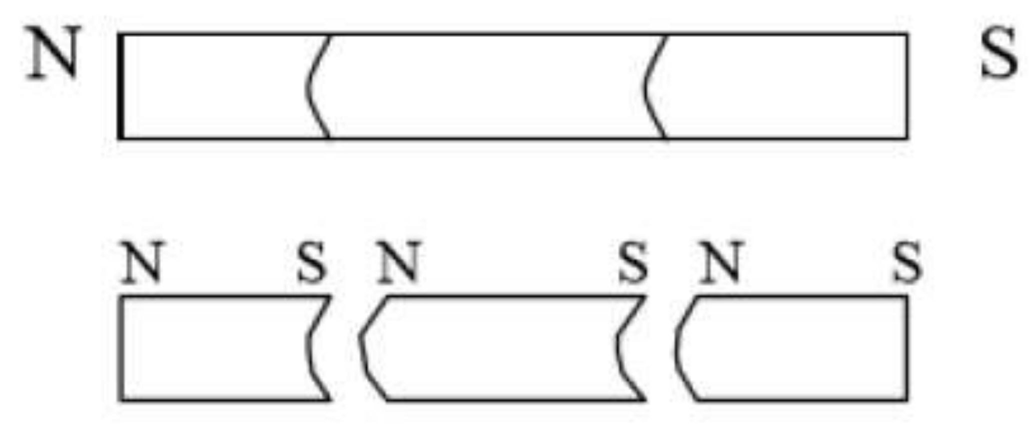

Fig 1: A magnet divided in three pieces.

Magnéto-statique

1- Properties of magnets.

- Like poles repel each other, while unlike poles attract each other.
- Magnetic field lines are closed loops; they are directed such that they emerge from the N pole and enter the S pole.

We can observe the fringes or field lines by sprinkling iron filings around a magnet; we notice patterns similar to the field lines of an electric dipole.

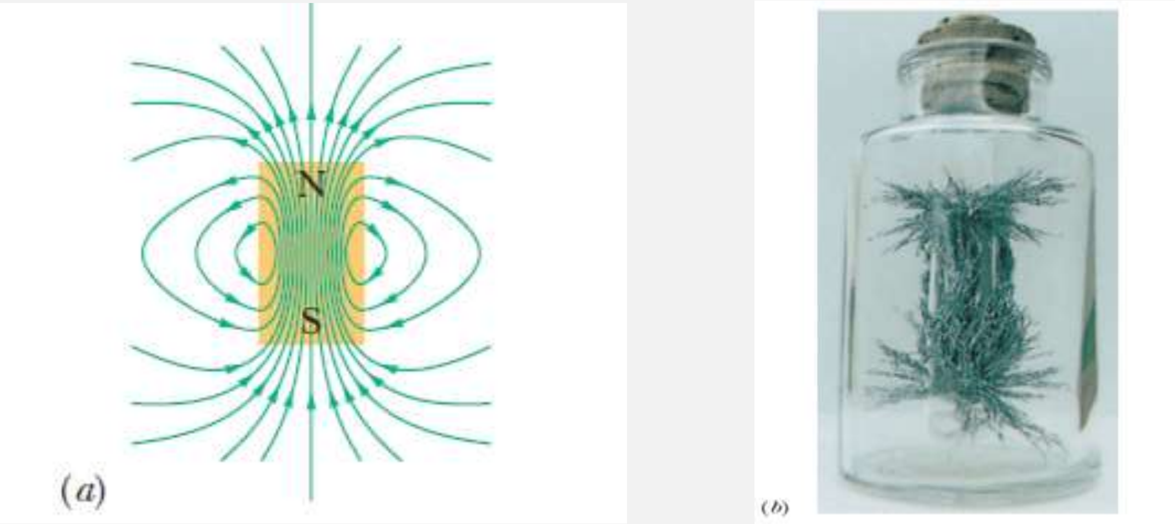

Fig 2: (a) The magnetic field lines for a bar magnet. (b) A “cow magnet”.

Magnéto-statique

2- What Produces a Magnetic Field?

Because an electric field $\vec{E}$ is produced by an electric charge, we might reasonably expect that a magnetic field $\vec{B}$ is produced by a magnetic charge. Although individual magnetic charges (called magnetic monopoles) are predicted by certain theories, their existence has not been confirmed. How then are magnetic fields produced?

There are two ways

→ **One way** is to use moving electrically charged particles, such as a current in a wire, to make an electromagnet. The current produces a magnetic field that can be used, for example, to control a computer hard drive or to sort scrap metal (Fig. 28-1). In Chapter 29, we discuss the magnetic field due to a current.

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2- What Produces a Magnetic Field?

- **The other way** to produce a magnetic field is by means of elementary particles such as electrons because these particles have an intrinsic magnetic field around them. That is, the magnetic field is a basic characteristic of each particle just as mass and electric charge are basic characteristics. The magnetic fields of the electrons in certain materials add together to give a net magnetic field around the material. Such addition is the reason why a **permanent magnet**, the type used to hang refrigerator notes, has a **permanent magnetic field**. In other materials, the magnetic fields of the electrons cancel out, giving no net magnetic field surrounding the material. Such cancellation is the reason you do not have a permanent field around your body, which is good because otherwise you might be slammed up against a refrigerator door every time you passed one.

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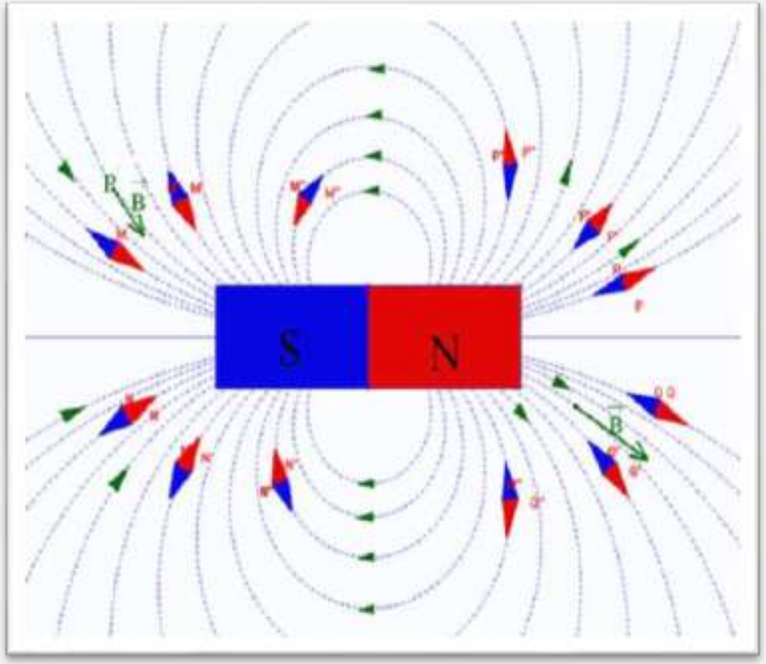
2- What Produces a Magnetic Field?

The space surrounding a magnet is characterized by a field called the magnetic field, denoted by $\vec{B}$, whose direction is indicated by a compass. At any point, it is tangent to the magnetic field lines.

The magnetic field has a magnitude, a line of action, and a direction. Its characteristics at a point M in space are:



- **Point of application**: the considered point M.
- **Line of action**: the axis of the magnetic needle placed at that point.
- **Direction**: from the south pole of the needle toward its north pole.
- **Magnitude:** measured in tesla (Tesla).

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2- What Produces a Magnetic Field?

If several magnetic fields act on a moving electric charge or on a magnetized needle, the resultant magnetic field $\vec{B}$ is equal to the vector sum of all the acting fields (the principle of superposition applies).

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \cdots \dots \dots \dots + \vec{B}_n$$

3- The magnetic force acting on a moving electric charge (Lorentz law).

At the end of the nineteenth century, the Dutch physicist Hendrik Lorentz formulated the expression for the force $\vec{F}$ acting on a point electric charge q moving with velocity $\vec{v}$ in the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$.

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3- The magnetic force acting on a moving electric charge (Lorentz law).

$$\vec{F} = \vec{F}_C + \vec{F}_m = q\vec{E} + q.\vec{v}\wedge\vec{B} = q\left(\vec{E} + \vec{v}\wedge\vec{B}\right)$$

If $\vec{E} = \vec{0}$ a Lorentz force becomes:

$$\vec{F} = \vec{F}_m = q\left(\vec{v}\wedge\vec{B}\right)$$

In vector form, these results can be written as follows:

$$\vec{F}_B = q\,\vec{v}\times\vec{B} = q\begin{vmatrix} \vec{\mathrm{i}} & \vec{\mathrm{j}} & \vec{\mathrm{k}} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

Therefore, the magnitude of the magnetic force on q is:

$$F_B = |q|\,vB\sin\theta$$

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3- The magnetic force acting on a moving electric charge (Lorentz law).

To find the direction of $\vec{v} \times \vec{B}$ and the direction of $\overrightarrow{F_B}$ for both positive and negative q, we use the right-hand rule, as shown in Fig.

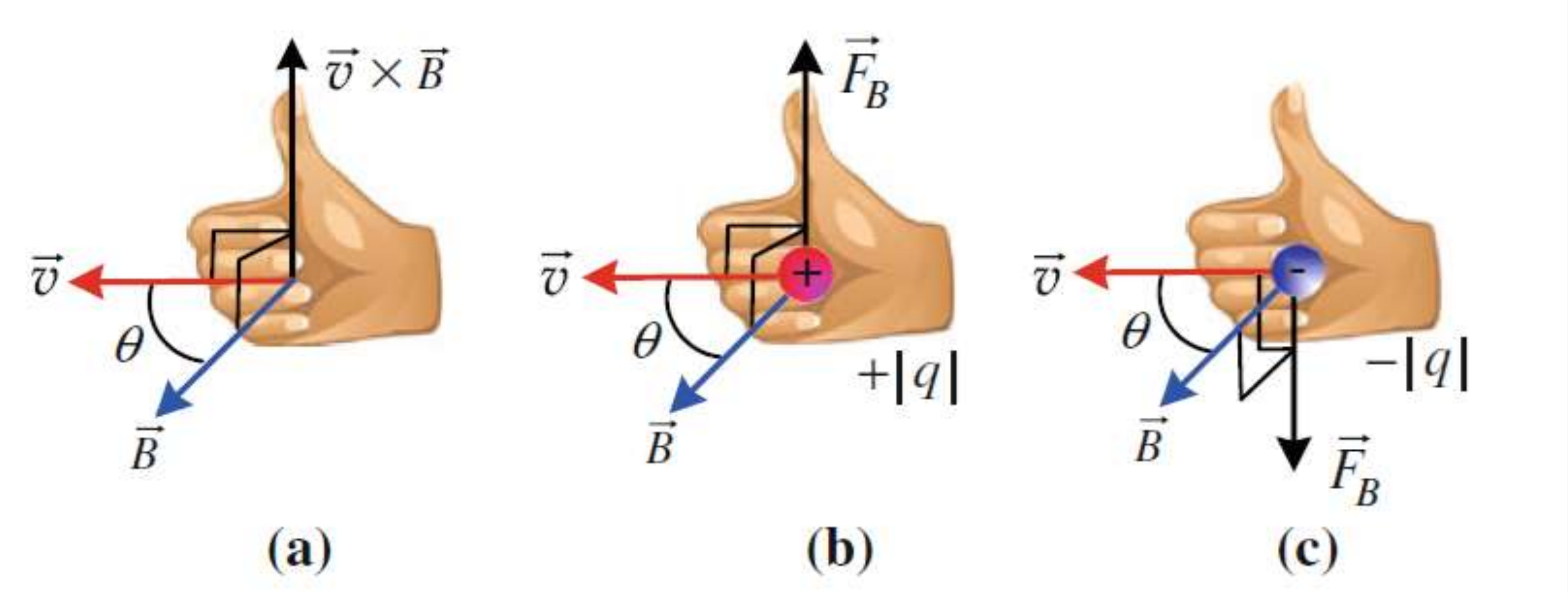

Fig 2: the right-hand rule for positive charge (a,b) and negative charge.

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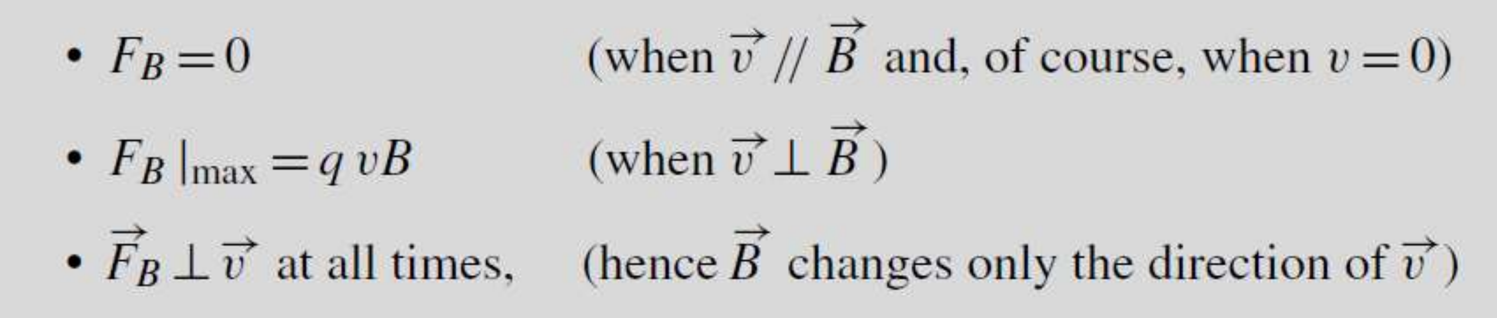
3- The magnetic force acting on a moving electric charge (Lorentz law).



- $F_B = 0$ (when $\vec{v} \,//\, \vec{B}$ and, of course, when $v = 0$)
- $F_B|_{\max} = q\,vB$ (when $\vec{v} \perp \vec{B}$)
- $\vec{F}_B \perp \vec{v}$ at all times, (hence $\vec{B}$ changes only the direction of $\vec{v}$)

For convenience, we label the magnetic field coming out of the page by black dots (or blue dots), as shown in Fig. a and the magnetic field going into the page by black crosses (or blue crosses), as shown in Fig. b.

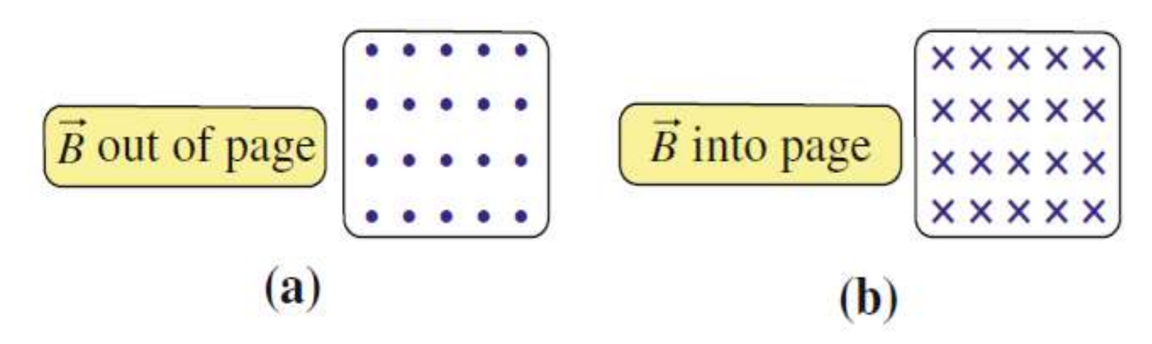


Fig 2: Magnetic field lines

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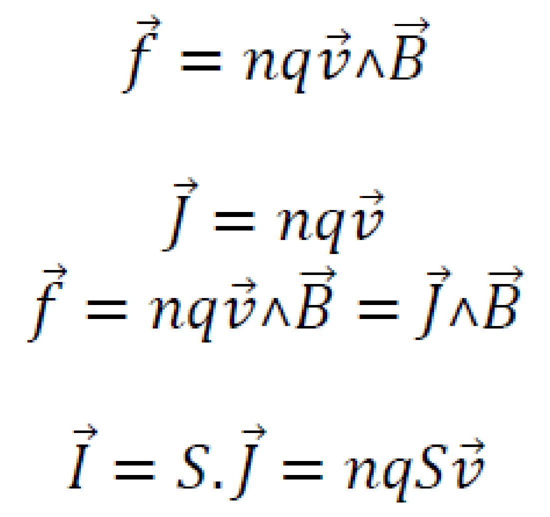
3- The magnetic force acting on an electric current (Laplace’s law).

Consider a conducting wire carrying an electric current I in a magnetic field $\vec{B}$. Each infinitesimal volume element dV of the conductor, of length $\overrightarrow{dl}$ and cross-sectional area S, contains charges that are subject to a magnetic force given by: q($\vec{v} \times \vec{B}$). Thus, n charges are subjected to a magnetic force.

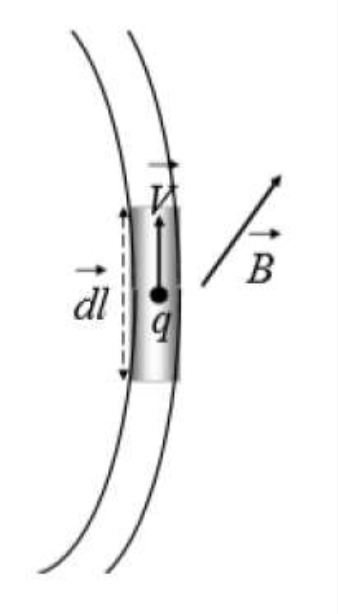


$$\vec{f} = nq\vec{v}\wedge\vec{B}$$

$$\vec{J} = nq\vec{v}$$

$$\vec{f} = nq\vec{v}\wedge\vec{B} = \vec{J}\wedge\vec{B}$$

$$\vec{I} = S.\vec{J} = nqS\vec{v}$$



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3- The magnetic force acting on an electric current (Laplace’s law).

The total force acting on a length element:

$$\overrightarrow{dF} = \vec{f}.dV = \vec{f}.S.dl = S.dl.\vec{J}\wedge\vec{B} = I\,\overrightarrow{dl}\wedge\vec{B}$$

Thus, the total force is:

$$\vec{F} = I.\int \overrightarrow{dl}\wedge\vec{B}$$

This force is perpendicular to the plane defined by the magnetic field $\vec{B}$ and the element $\overrightarrow{dl}$ of the wire.

If α is the angle between the straight conductor and the magnetic field vector, then:

$$F = B.I.l.\sin\alpha$$

This expression is known as **Laplace’s law**.

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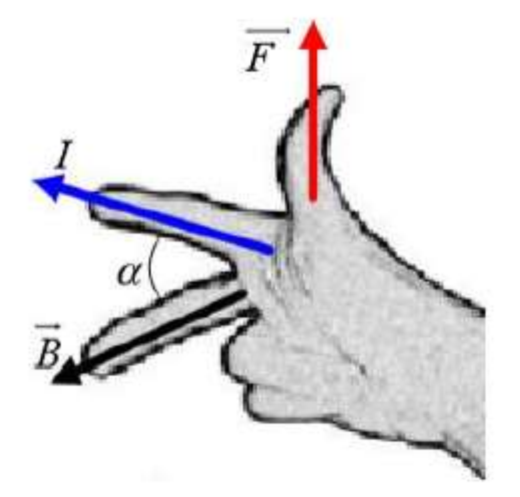
3- The magnetic force acting on an electric current (Laplace’s law).

To determine the direction and sense, we use the right-hand rule.



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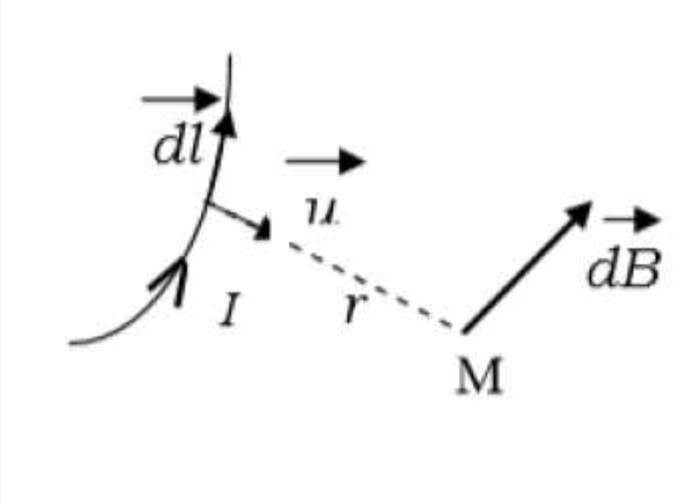
3- The magnetic field produced by an electric current (Biot–Savart law).

In 1820, Jean-Baptiste Biot and Félix Savart established a law to determine the magnitude of the magnetic field at a point in the vicinity of a conductor carrying a steady current.

Consider an element of length dl, represented by the vector $\overrightarrow{dl}$. This element produces, at a point M, an elementary magnetic field $\overrightarrow{dB}$ given by the Biot–Savart law:

$$d\vec{B}(M) = \frac{\mu_0}{4\pi}\frac{I\vec{dl} \times \vec{u}}{r^2}$$



where μ_0 is a constant called the permeability of free space:

$$\mu_0 = 4\pi.10^{-7}\, T.m / A$$

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3- The magnetic field produced by an electric current (Biot–Savart law).

The total magnetic field $\vec{\mathbf{B}}$ at point M, produced by the entire circuit, is given by:

$$\vec{B}(M) = \int d\vec{B}\,(M) = \frac{\mu_0}{4\pi}\int \frac{Id\vec{l} \times \vec{u}}{r^2}$$