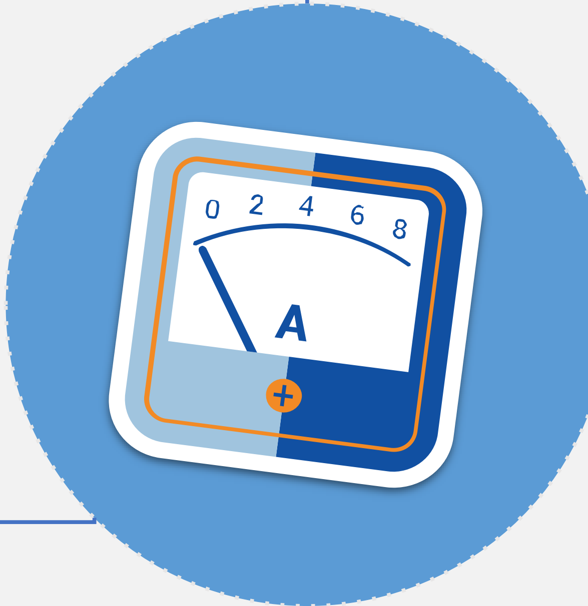
Chapter 3

Electrokinetic



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In this chapter, we discuss the physics of electric currents—that is, charges in motion. Examples of electric currents abound and involve many professions: Meteorologists, Biologists, physiologists, engineers working in medical technology and electrical engineers.

In this chapter we discuss the basic physics of electric currents and why they can be established in some materials but not in others. We begin with the meaning of electric current.

1- Electric Current

When there is a net flow of charge across any area, we say there is **an electric current** (or simply **current**) across that area.

To maintain a continuous current, we must maintain a net force on the mobile charge in some way. The net force may result, for example, from an electrostatic field

$\vec{F} = q\vec{E}$ → **the particle's driving force**

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1- Electric Current

To define the current, we consider positive charges moving perpendicularly onto a surface area A as shown in Fig 1.

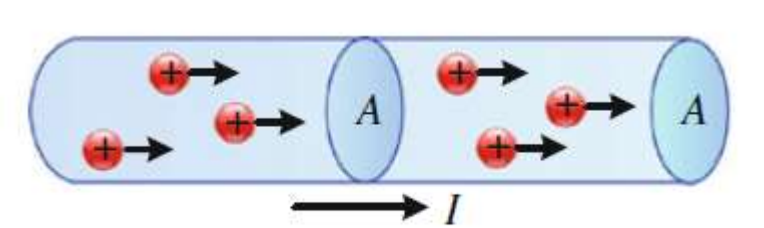

Fig 1:Charged particles in motion perpendicular onto an area A.

Thus, if a net charge ΔQ flows across an area A in a time Δt, the average current I_{av} across the area is:

$$I_{\text{av}} = \frac{\Delta Q}{\Delta t}$$

When the rate of flow varies with time, we define the instantaneous current (or the current) I as:

$$I = \frac{dQ}{dt}$$

The SI unit of the current is ampere (abbreviated by A)

$$1\,\text{A} = \frac{1\,\text{C}}{1\,\text{s}}$$

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1- Electric Current

Currents can be due to positive charges, or negative charges, or both. In conductors, the current is due to the motion of only negatively charged free electrons (called conduction electrons). By convention, the direction of the current is the direction of the flow of positive charges. Therefore, the direction of the current is opposite to the direction of the flow of electrons.

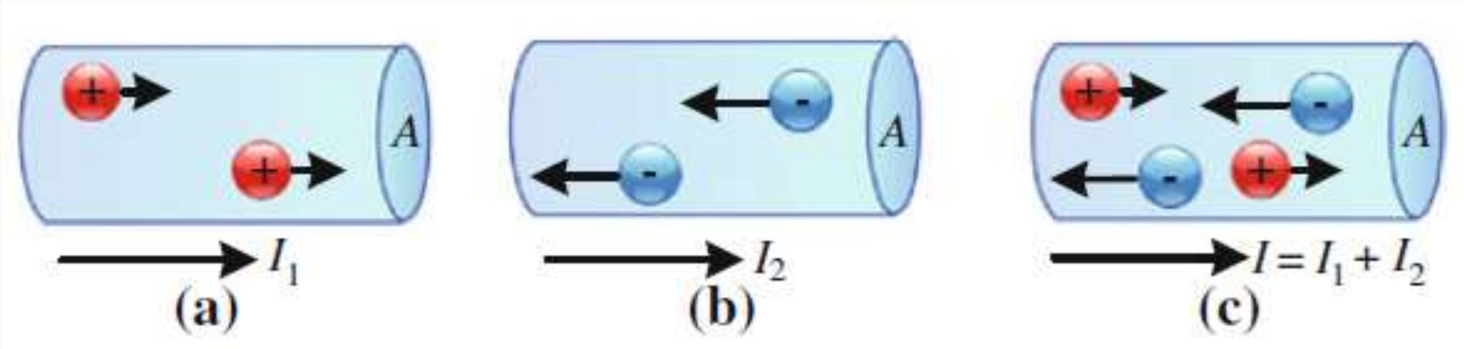

Fig 2:Direction of current due to (a) positive charges, (b) negative charges, and (c) both positive and negative charges.

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2- Electric Current Density

The current across an area can be expressed in terms of the motion of the charge carriers. To achieve this we consider a portion of a cylindrical rod that has a cross-sectional area A, length Δ x, and carries a constant current I.

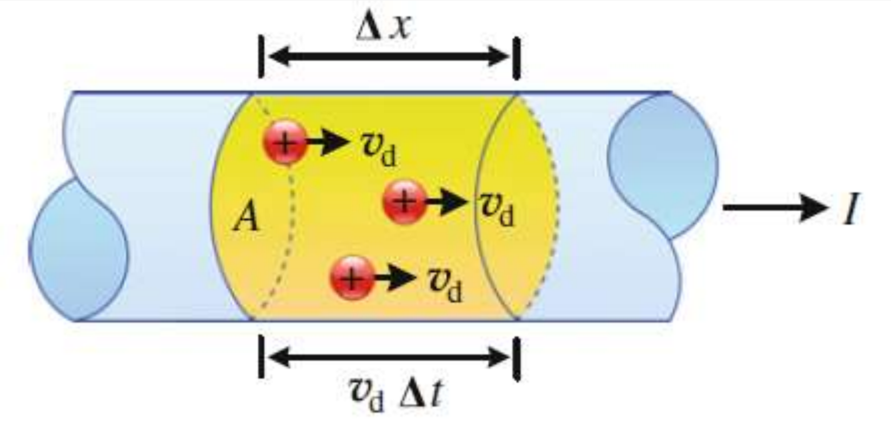

Fig 3: A portion of a straight rod of uniform cross-sectional area A.

For convenience we consider positive charge carriers each having a charge q, and the number of carriers per unit volume in the rod is n. Therefore, in this portion, the number of carriers is n A Δx and the total charge ΔQ is:

$$\Delta Q = (n\,A\,\Delta x)\,q$$

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2- Electric Current Density

Suppose that all the carriers move with an average speed v_d (called the **drift speed**). Therefore, during a time interval Δt, all carriers must achieve a displacementΔx = v_d Δt in the x direction.

$$\Delta Q = (n\, A\, v_{\mathrm{d}}\, \Delta t)\, q$$

Therefore, the current I = ΔQ/Δt in the rod will be given by:

$$I = n\, q\, v_{\mathrm{d}}\, A$$

The current density J is defined as the current per unit area:

$$J = \frac{I}{A}$$

Using the relation I = n q v_d A, we get:

$$J = n\, q\, v_{\mathrm{d}}$$

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3- Ohm’s Law and Electric Resistance

As a result of maintaining a potential difference V across a conductor, an electric field $\vec{E}$ and a current density $\vec{J}$ are established in the conductor. For materials with electrical properties that are the same in all directions (isotropic materials), the electric field is found to be proportional to the current density. That is:

$$\vec{E} = \rho \vec{J}$$ → **Ohm’s law**

where the constant ρ is called **the resistivity** of the conductor. Materials that obey this relation are said to obey Ohm’s law.

Since it is difficult to measure $\vec{E}$ and $\vec{J}$ directly, we need to put Ohm’s law into a more practical form. This can be obtained by considering a portion of a straight conductor that has a uniform cross-sectional area A and length L, as shown

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3- Ohm's Law and Electric Resistance

In addition, a potential difference ΔV = $V_b - V_a$ between the ends of the conductor (denoted by a and b) will create a straight electric field and current. Since charge carriers in conductors are electrons, they will drift from face a to face b, against the field $\vec{E}$.

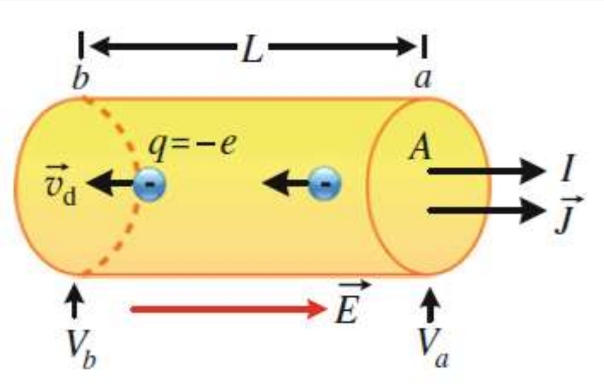

Fig 4: A potential difference ΔV cross a conductor of cross-sectional area A and length L sets up a field $\vec{E}$ and current I

Recall that for uniform electric fields we have:

$$\Delta V = E L$$

$$\frac{\Delta V}{L} = \rho J$$

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3- Ohm’s Law and Electric Resistance

Also, using J =I/A, the potential difference ΔV can be written as:

$$\Delta V = \left(\rho \frac{L}{A}\right) I$$

The quantity in brackets is called the electrical resistance (or simply resistance) of the conductor and is denoted by the symbol R; that is:

$$R = \rho \frac{L}{A}$$

We can define the resistance R as a proportionality constant to the relation $V \propto I$ and write the equivalent Ohm’s law as:

$$\Delta V = I R$$

Equivalent form of Ohm’s law

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3- Ohm's Law and Electric Resistance

Also, using J =I/A, the potential difference V can be written as:

$$\Delta V = \left(\rho \frac{L}{A}\right) I$$

The quantity in brackets is called the electrical resistance (or simply resistance) of the conductor and is denoted by the symbol R; that is:

$$R = \rho \frac{L}{A}$$

We can define the resistance R as a proportionality constant to the relation V ∝ I and write the equivalent Ohm's law as:

$$\Delta V = I R$$

Equivalent form of Ohm's law

The SI unit of resistance is ohm (abbreviated by Ω).

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3- Ohm’s Law and Electric Resistance

The inverse of resistivity is called the conductivity σ, thus:

$$\sigma = \frac{1}{\rho}$$

→ The resistivity of a material increases with temperature. It generally follows an empirical formula given by:

$$\rho = \rho_0[1+\alpha(T-T_0)]$$

ρ and ρ_0 are the resistivities of the material at temperatures T and T_0 and α is the temperature coefficient of resistivity.

4- Electric Power

In an electric circuit, as charge moves through the wires of an electric circuit, electrical energy is continuously converted into other forms of energy (they lose electric potential energy).

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4- Electric Power

The rate of energy loss is the power P delivered to the circuit elements.

$$P = \frac{\Delta q . V}{\Delta t} = I.V$$

The units of electric power is joules per second. $\frac{J}{s}$=W

Using Ohm's law:

$$\begin{cases} V = I.R \\ I = \frac{V}{R} \end{cases} \Rightarrow P = I.V = I^2.R = \frac{V^2}{R}$$

The energy used by a device with a power P over a time t is: $E = \frac{P}{t}$

5- Resistors in Series and Parallel

Resistors in a circuit may be used in different combinations, and we can sometimes replace a combination of resistors with one equivalent resistor.

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5- Resistors in Series and Parallel

Resistors in a Series Combination

Figure 5 shows two resistors R1 and R2 that are connected in series with a battery.

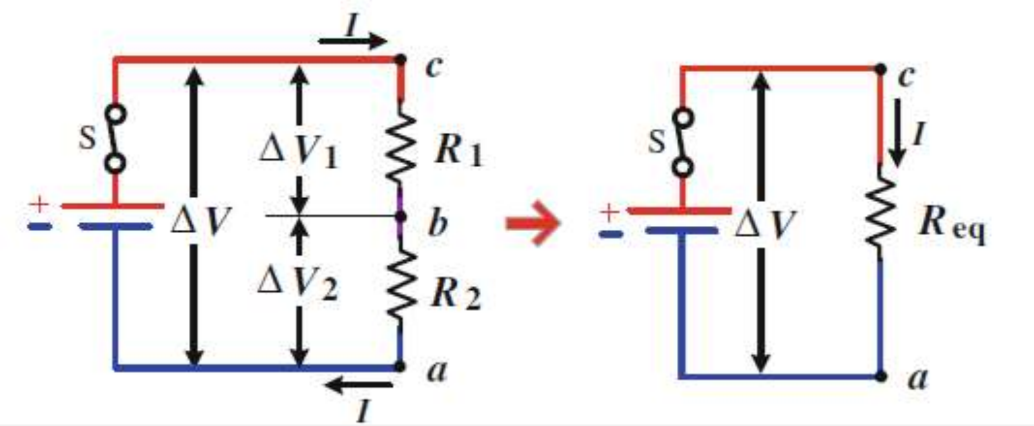

Fig 5: Two resistors are connected in series to a battery B that has a potential difference ΔV

When the circuit is connected, the amount of charge that passes through R_1 must Also pass through R_2 in the same time interval. Otherwise, charge will accumulate on the wire between resistors. Thus, for series combination of resistors, the current I is the same in both resistors.

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5- Resistors in Series and Parallel

Resistors in a Series Combination

Figure 5 shows a single resistor R_{eq} that is equivalent to this combination and has the same effect on the circuit. This means that when the potential difference ΔV is applied across the equivalent resistor, it must produce the same current I as in the series combination.

→ The potential difference ΔV is divided to ΔV1 and ΔV2 across the resistors R1 and R2, respectively. Thus:

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V_1 = V_c - V_b = IR_1 \quad \text{and} \quad \Delta V_2 = V_b - V_a = IR_2$$

$$\Delta V = IR_1 + IR_2$$

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5- Resistors in Series and Parallel

Resistors in a Series Combination

The equivalent resistor R_{eq} has the same applied potential difference ΔV and the same circuit current I flowing through it; thus:

$$\Delta V = IR_{eq} = IR_1 + IR_2$$

Canceling I, we arrive at the following relationship:

$$R_{eq} = R_1 + R_2$$

We can extend this treatment to n resistors connected in series as:

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

→ The equivalent resistor of a series combination of resistors is simply the algebraic sum of the individual resistances and will always be greater than any one of them.

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5- Resistors in Series and Parallel

Resistors in a Parallel Combination

Figure 5 shows two resistors R_1 and R_2 that are connected in parallel with a battery.

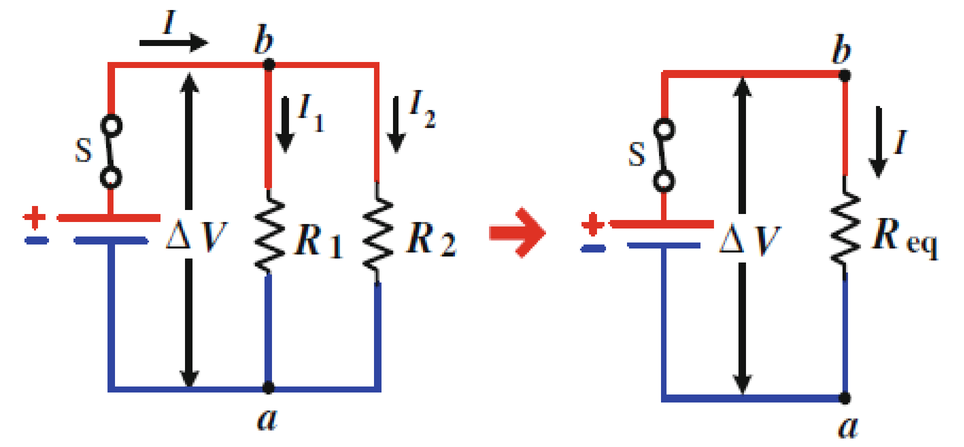

Fig 6: Two resistors are connected in parallel to a battery B that has a potential difference ΔV

When the current I reaches junction b, it will split into two parts, I_1 in R_1 and I_2 in R_2. Because electric charge is conserved, the current I that enters junction b must equal the total current leaving that junction; that is:

$$I = I_1 + I_2$$

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5- Resistors in Series and Parallel

Resistors in a Parallel Combination

Because the potential difference ΔV across the resistors is the same, then from Fig 6, we have:

$$\Delta V = I_1 R_1 \quad \text{and} \quad \Delta V = I_2 R_2$$

$$I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Delta V$$

An equivalent resistor with the same applied potential difference ΔV and total current I has a resistance R_{eq} given by:

$$I = \frac{\Delta V}{R_{\text{eq}}}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

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5- Resistors in Series and Parallel

Resistors in a Parallel Combination

We can extend this treatment to n resistors connected in parallel as:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

→ The equivalent resistance of a parallel combination of resistors is simply the algebraic sum of the reciprocal of the individual resistances and is less than any one of them.

6- Kirchhoff's Rules

Electrical circuits

An electrical circuit is a closed loop or path that allows electrical current to flow from the source and back to it. It consists of electrical components such as a power source (battery), conductors (wires), and loads (resistors, lamps, or devices) connected together.

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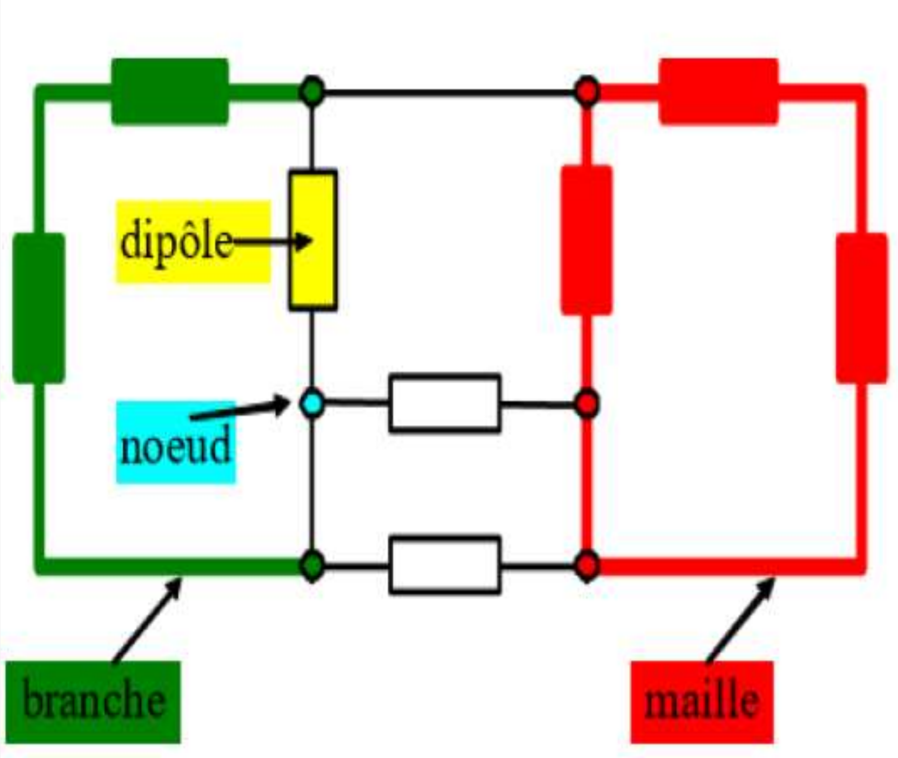
6- Kirchhoff's Rules

Electrical circuits



- **Node (Noeud)**: A point in a circuit where three or more wires meet.
- **Branch (branche)**: A part of a circuit between two nodes.
- **Loop (maille)**: A set of branches forming a closed loop.
- **Dipole (dipÔle)** (two-terminal element): A component in an electrical circuit with two terminals; current enters through one terminal and leaves through the other.
- **junction wires (Fils de jonction):** Their resistance is neglected compared to that of other circuit elements.
- **Network (Réseau):** A set of electrical circuits.

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6- Kirchhoff's Rules

Kirchhoff's Rules

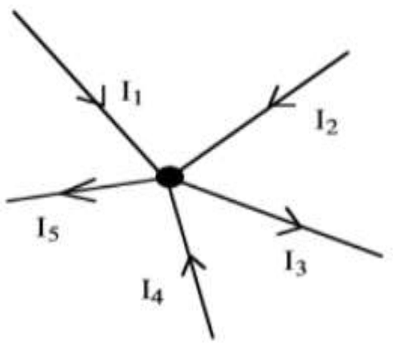
Not all circuits can be reduced to simple series and parallel combinations.A technique that is applied to loops in complicated circuits consists of two principles called Kirchhoff's Rules.

1. Junction rule

At any junction in a circuit, the sum of the ingoing currents must equal the sum of the outgoing currents. That is:

$$\sum I_{in} = \sum I_{out}$$

$$I_1 + I_2 + I_4 = I_3 + I_5$$



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6- Kirchhoff's Rules

2. Loop rule

For any closed loop in a circuit, the sum of the potential differences across all elements must be zero. That is:

$$\sum_{\text{closed loop}} \Delta V = 0 \quad \Rightarrow \quad \sum_{k=1}^{n} e_k = \sum_{k=1}^{n} R_k I_k$$

- We apply Kirchhoff's first law, if we have **n** nodes, we will obtain **(n-1)** equations.
- We apply Kirchhoff's second law, if we have **b** branches, then the number of loop equations is **m=b-(n-1).**
- Separate the network into its components of independent loops, each having at least one branch not shared with another loop, and apply Kirchoff's second law.

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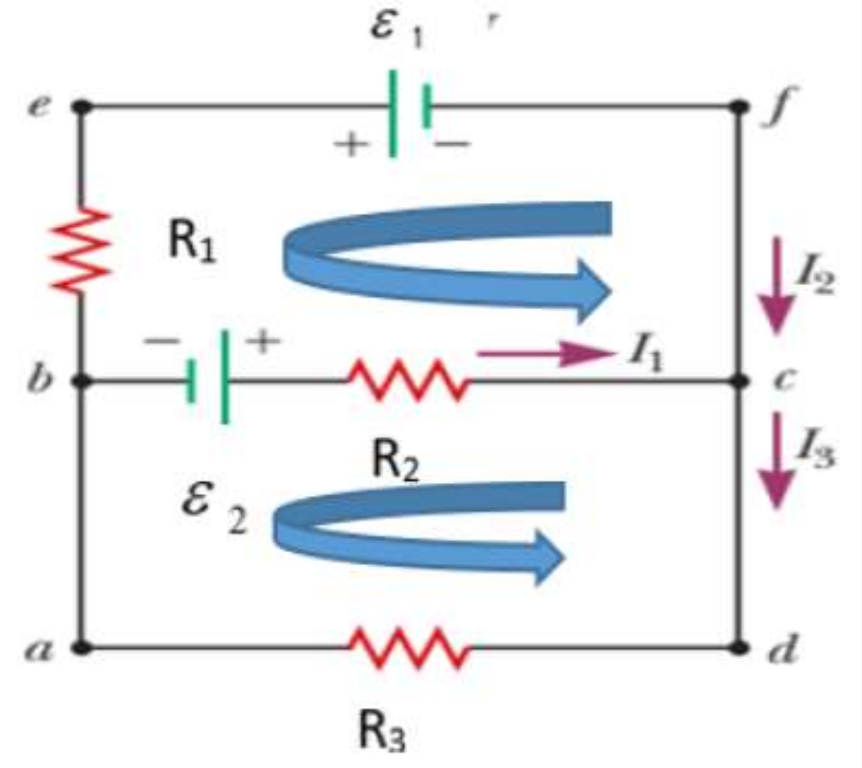
6- Kirchhoff’s Rules

Example

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure:

$$\begin{cases} R_1 = 4\Omega \\ R_2 = 6\Omega \\ R_3 = 2\Omega \\ \varepsilon_1 = 14V \\ \varepsilon_2 = 10V \end{cases}$$



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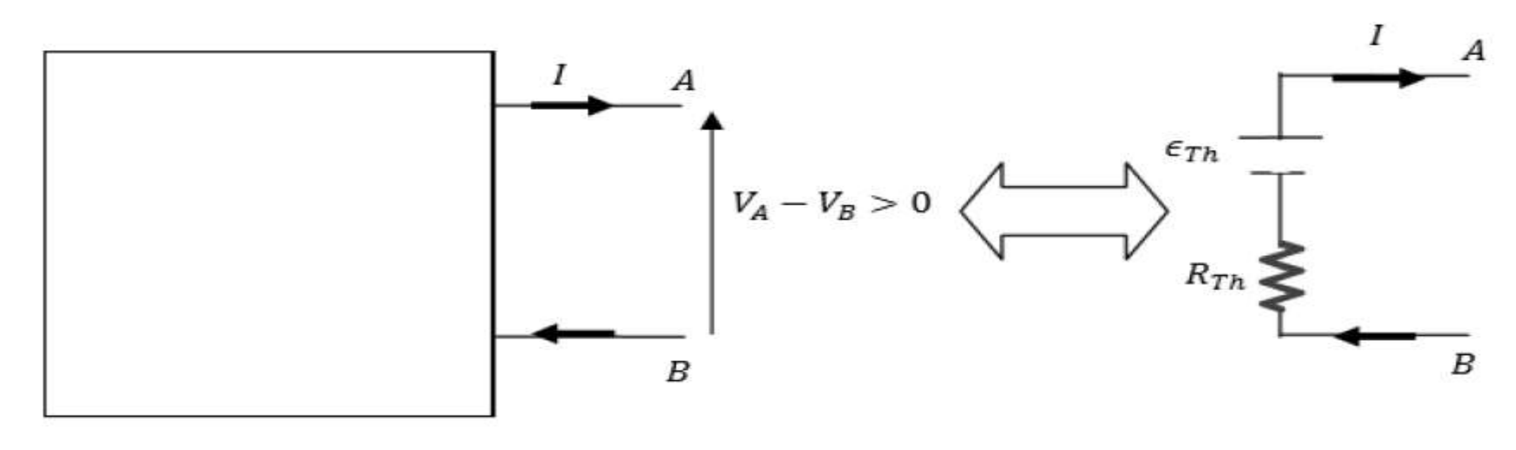
7- Thevenin's Theorem

Any linear network enclosed between two terminals A and B, no matter how complex it is, is equivalent to a single generator with an electromotive force E and internal resistance.



E_{th}: the potential difference measured between terminals A and B when the connection between A and B is removed (open circuit).

R_{th}: the equivalent resistance between terminals A and B after removing the connection between A and B, as well as all voltage and current sources.