

TD(03)

Exercise 1 A sample of 15 children from a given city provided the following heights (in cm):

70	85	93	99	101	105	110	121	138	166	74	85	93	99	102
----	----	----	----	-----	-----	-----	-----	-----	-----	----	----	----	----	-----

Assume that the children's heights follows a normal distribution $N(\mu, \sigma^2)$

1. Determine point estimates of the mean and the standard deviation of the children's heights.
2. Based on the sample, can we conclude, at the 2% significance level, that the average height of the children is equal to 110 cm?

Exercise 2 After a 400-meter run, the pulse rates (in beats per minute) of 7 students attending a physical education class were measured as follows:

83	96	99	110	130	95	74
----	----	----	-----	-----	----	----

Assume that the pulse rate increase follows a normal distribution $N(\mu, \sigma^2)$. We assume that the variation in pulse is known $\sigma^2 = 324$. At a significance level $\alpha = 5\%$, can we conclude that:

- The mean pulse rate is less than 100 beats per minute?

Exercise 3 In order to compare two types of trees, we collected height measurements of several trees, presented in the following table.

							Sum
Tree 1	23.3	24.0	24.3	24.5	25.0	25.9	147
Tree 2	21.1	21.1	22.1	22.4	23.3		110

1. Determine a point estimate of the mean and the variance for each sample.
2. Suppose we want to know whether the two types of trees have the same average height.
 - a) Specify the form of the test to be performed in this case.
 - b) Give the conditions necessary to perform this test for a risk level of $\alpha = 2\%$.
3. Assuming that the conditions in 2.b) are satisfied, then:
 - a) Give the test statistic defined in 2.a).
 - b) Compute the value of the test statistic, knowing that $\hat{\sigma}_c^2$ is the common variance of the two samples defined by:

$$\hat{\sigma}_c^2 = \frac{(n_1 - 1)\hat{\sigma}_{c,1}^2 + (n_2 - 1)\hat{\sigma}_{c,2}^2}{n_1 + n_2 - 2} = \frac{(6 - 1)0.7880 + (5 - 1)0.8700}{6 + 5 - 2} = 0.8244.$$

- c) Give the critical value associated with this test for a significance level $\alpha = 2\%$.

Exercise 4 ($\alpha = 2\%$.)

We are interested in the yield of barley for four different varieties. We have four plots, each assigned to one barley variety. The experiment is repeated at different locations. The following data were obtained:

Variety 1	Variety 2	Variety 3	Variety 4
46; 43; 48	57; 53; 43; 54; 48	50; 41; 47; 42	39; 51; 45; 43

Do the four varieties have the same mean yield? To answer this question:

- a) Specify the form of the test to be performed in this case.
- b) Give the conditions necessary to perform this test.