

Tutorial Series N°04: Systems of Linear Equations

Exercise 1. Consider the following linear system:

$$(S_1) : \begin{cases} x + 2y + 3z = 1 \\ -2x - 4y - 5z = 2 \\ 3x + 5y + 6z = 3 \end{cases}$$

- 1) Write the system (S_1) in matrix form $A\mathbf{x} = \mathbf{B}$.
- 2) Calculate the determinant of the coefficient matrix A .
- 3) Find the solutions of the system using the **Cramer method**.

Exercise 2. Let A and B be the following matrices:

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ 5 \\ 12 \end{pmatrix}$$

- 1) Is the matrix A invertible? Justify your answer.
- 2) Calculate the inverse matrix A^{-1} using the adjugate matrix method.
- 3) Use the inverse matrix to solve the linear system $A\mathbf{x} = B$.

Exercise 3. Solve the following linear systems using the indicated methods:

- 1) Solve using the **Cramer method**:

$$(S_2) : \begin{cases} 2x + y - 3z = 5 \\ 3x - 2y + 2z = 5 \\ 5x - 3y - z = 16 \end{cases}$$

- 2) Solve the following system using the **Cramer method** and compare with the **Inversion method**:

$$(S_3) : \begin{cases} x + y + 2z = 3 \\ x + 2y + z = 1 \\ 2x + y + z = 0 \end{cases}$$