Chapter 2

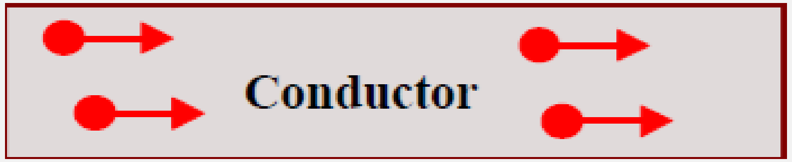
Conductors

Conductors

1- Conductors in Static Equilibrium

Conductor

In a conductor some electrons are free to move (without restraint) within the volume of the material (Exp: copper, silver, aluminum, gold)



Conductor in Static Equilibrium

When the charge distribution on a conductor reaches static equilibrium (i.e. nothing moving), the net electric field within the conducting material is exactly zero (and the electric potential is constant).

Conductor in static equilibrium

$E = 0$

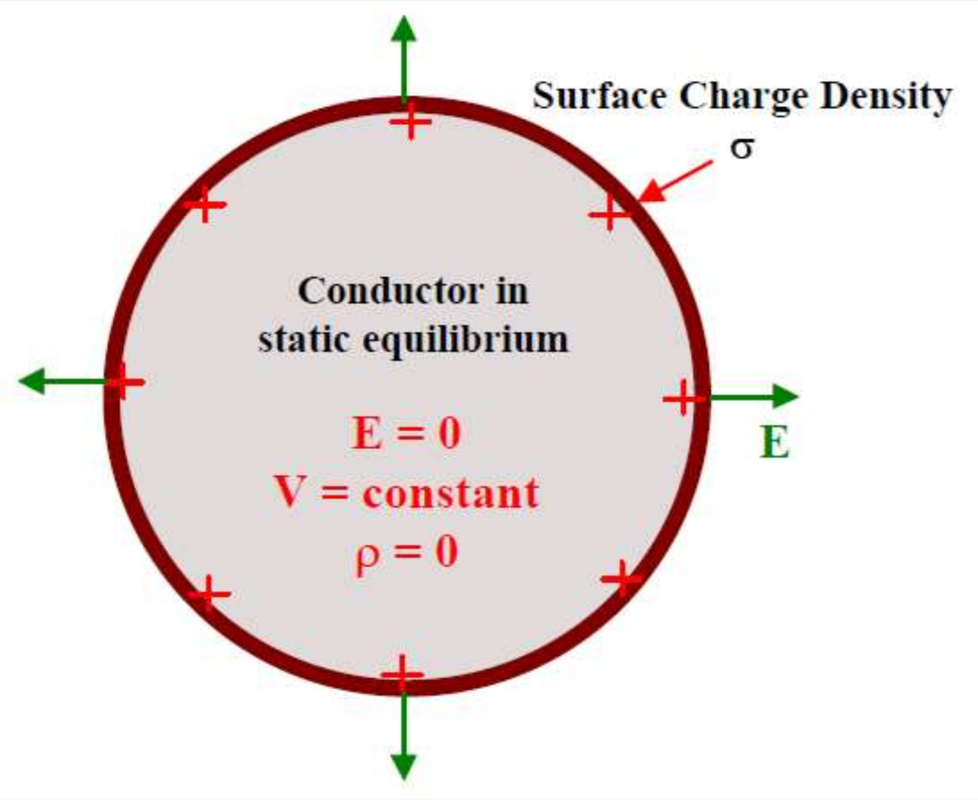
$V = \text{constant}$

Conductors

1- Conductors in Static Equilibrium

- **Excess Charge**: For a conductor in static equilibrium all the (extra) electric charge reside on the surface. There is no net electric charge within the volume of the conductor (i.e. **ρ = 0**).
- The electric field at the surface of a conductor in static equilibrium is **normal to the surface** and has a magnitude, $\mathbf{E = \sigma/\varepsilon_0}$, where σ is the surface charge density (i.e. charge per unit area) and the net charge on the conductor is:

$$Q = \int_{Surface} \sigma \, dA$$



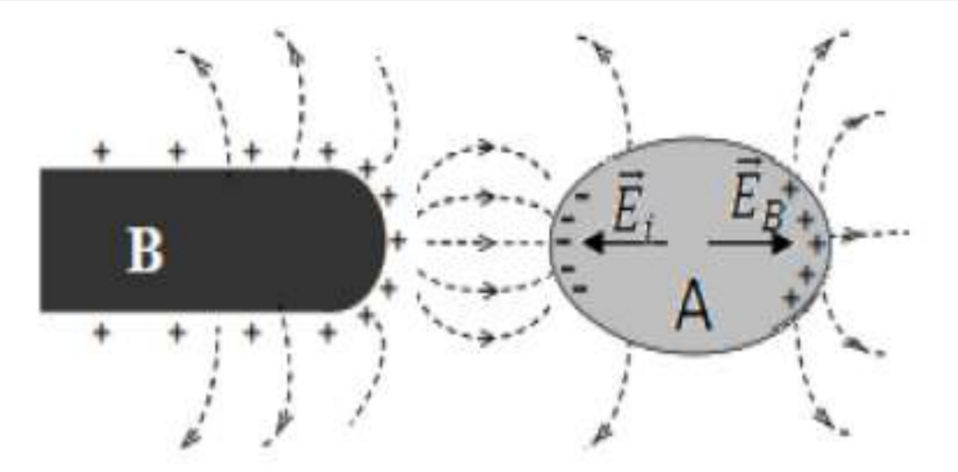
Conductors

2- Influence phenomenon

Partial electrostatic influence

Consider an electrically neutral conductor A. We approach the latter with a positively charged conductor B as shown in the figure.

➔ The free electrons of conductor A will migrate towards conductor B under the action of the electrostatic field which results from electrification by **influence**, which contributes to the creation of an electric field inside the conductor. Electrostatic influence divides the charge inside the conductor. This phenomenon is called **partial influence**.



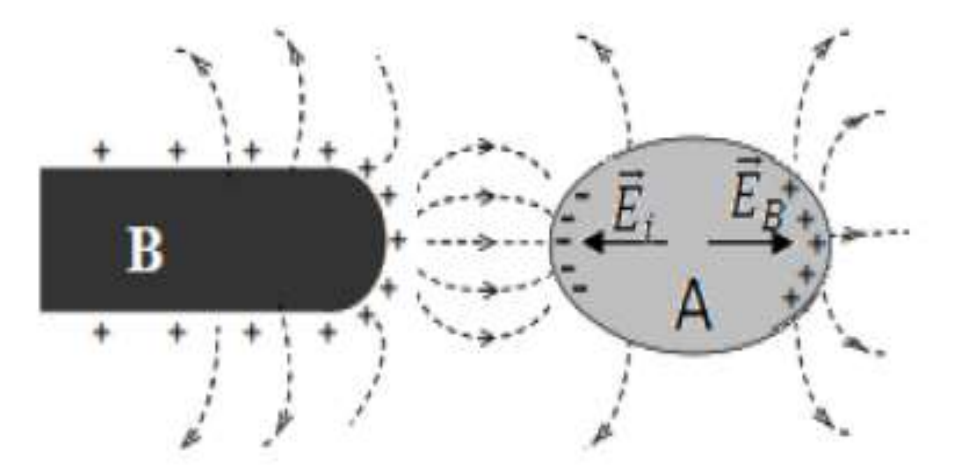
Conductors

2- Influence phenomenon

Partial electrostatic influence

Consider an electrically neutral conductor A. We approach the latter with a positively charged conductor B as shown in the figure.

- Conductor has tons of free electrons and under the influence of E_{ext} they will run to the left surface leaving positive charges near the right surface and creating $E_{internal}$. The electrons will keep moving until the internal field cancels out the external field inside the conductor.



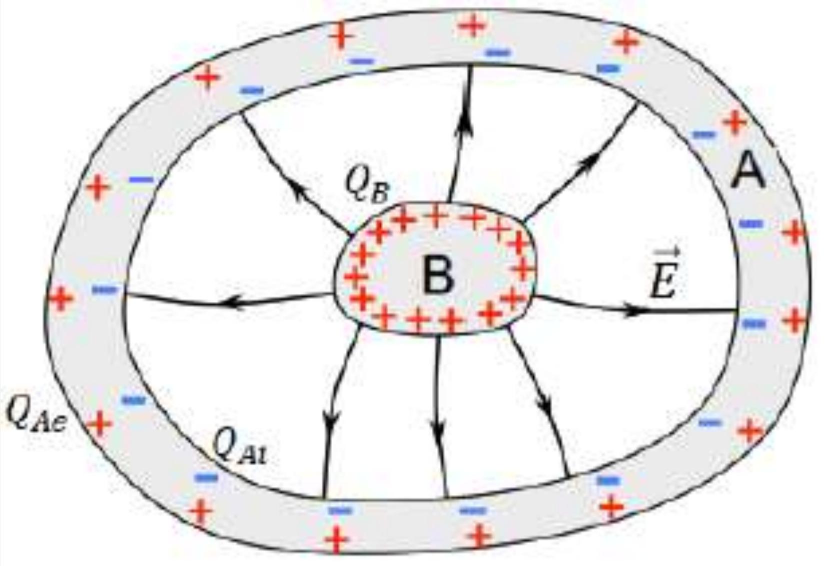
- As we have seen previously, not all the field lines originating from conductor B terminate on conductor A; therefore, the effect is said to be **partial**.

Conductors

2- Influence phenomenon

Total Electrostatic influence

When the field lines carrying B end up on A, the influence in this case is total. Total Electrostatic influence between two conductors occurs when all the field lines starting from a conductor end in the other conductor. Surfaces with total influence have the same charge but different sign.



$$Q_B = -Q_{Ai}$$

- If conductor A is initially isolated and neutral, we have: $$Q_{Ai} + Q_{Ae} = 0$$

- If conductor A is initially isolated and and charged with an initial charge Q_0, we have:

$$Q_{Ai} + Q_{Ae} = Q_0 \Rightarrow Q_{Ae} = Q_B + Q_0$$

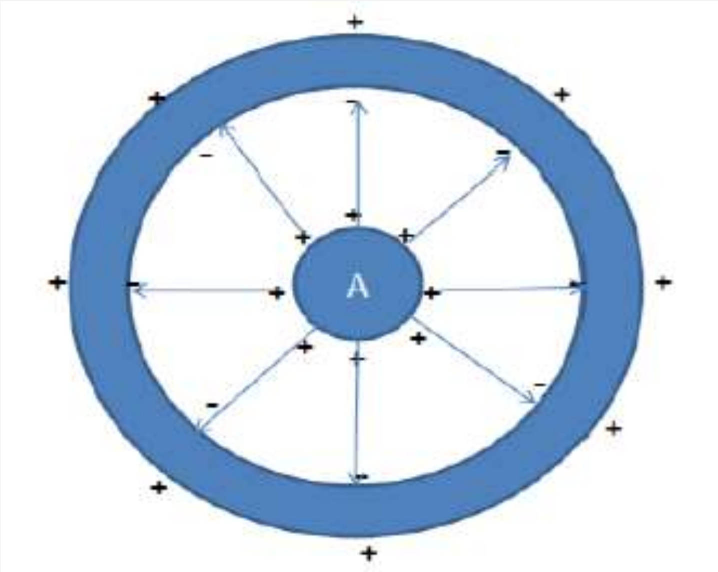
Conductors

2- Influence phenomenon

Example1:

Consider a conductor A of radius R carrying a charge Q (positive for example) We take a hollow conductor B of inner radii R_{int} and outer radius R_{ext} which surrounds the conductor A.

✓ There are field lines which go from A towards B and therefore creation of an electric field that separates the charges of B, attracting the negative ones and repelling the positive ones.



Relationship between the charge Q , of conductor A and the charge Q_{int} and Q_{ext} of conductor B:

- Inside the conductors A the field is zero. We take a sphere as a Gaussian surface of radius R_{int} <r< R_{ext}

Conductors

2- Influence phenomenon

Example1:

$$\vec{E}=\vec{0}\Rightarrow\sum q_{\text{int}}=0\Rightarrow Q+Q_{\text{int}}=0\Rightarrow Q_{\text{int}}=-Q$$

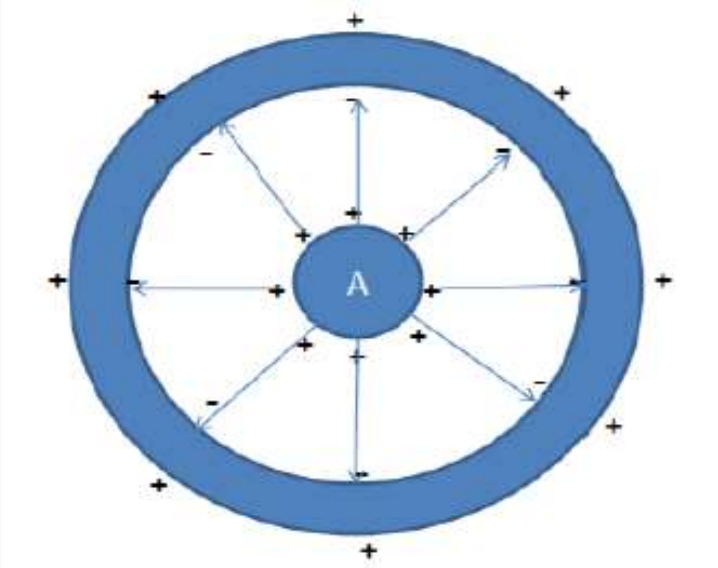
- If B is initially neutral, there is conservation of charge:

$$Q_{\text{int}}+Q_{ext}=0\Rightarrow Q_{ext}=-Q_{\text{int}}=Q$$

- If B initially has a charge q, there is conservation of charge:

$$Q_{\text{int}}+Q_{ext}=q\Rightarrow Q_{ext}=q-Q_{\text{int}}=q+Q$$

- If B is connected to the ground, the charges on the exterior face disappear and there remains only Q and Q_{int}



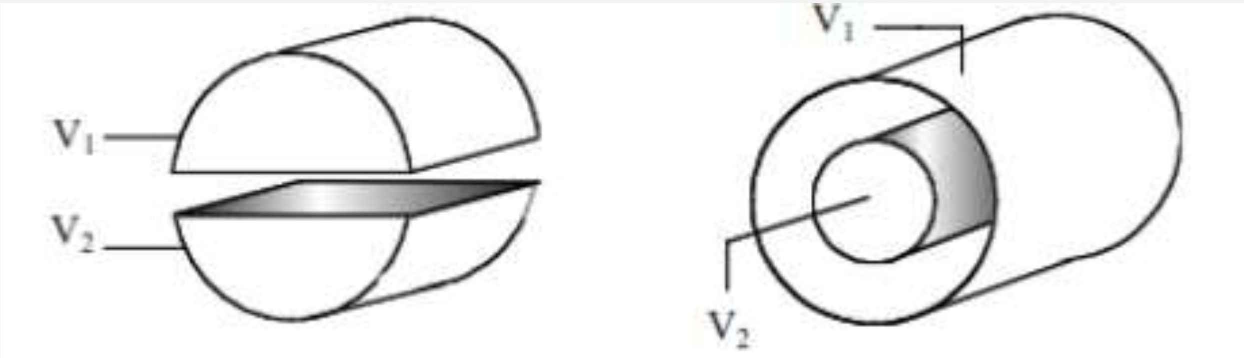
Conductors

3- Capacitors

In this part we introduce capacitors, which are one of the simplest circuit elements. Capacitors are charge-storing devices that can store energy in the form of an electric potential energy, and are commonly used in a variety of electric circuits.

Definition of a capacitor:

A capacitor is a system composed of two electrical conductors in electrostatic interaction with each other.

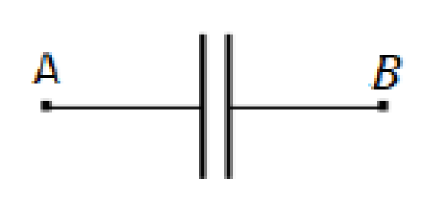


Conductors

3- Capacitors

Definition of a capacitor:

- There are two types of capacitors:
  - → Capacitors with two closely spaced conductors.
  - → Capacitors with total influence.
- The space separating the two conductors may be either vacuum or a dielectric medium.
- In electrical circuits, a capacitor is represented as shown in Figure. Each conductor is called a **plate** of the capacitor; therefore, a capacitor has two plates.



Conductors

3- Capacitors

Definition of a capacitor:

- When a potential difference is applied between the plates of a capacitor, for example by connecting it to a power source, the capacitor becomes charged. The two plates then acquire equal and opposite charges.

Capacitance of a Capacitor

The concept of electrical capacitance, which was introduced in the case of a single conductor, can be extended to a capacitor (two conductors). The capacitance of a capacitor is defined by:

$$C = \frac{Q}{V} = \frac{Q}{V_1 - V_2}$$

- Q: the charge carried by each of the two plates(+Q on one plate and −Q on the other).
- $V = V_1 - V_2$: the potential difference between the two plates.

Conductors

3- Capacitors

Capacitance of a Capacitor

The capacitance is a constant value characteristic of each capacitor. Its value depends on:

- the shape, dimensions, and relative position of the two plates forming the capacitor.
- the nature of the medium separating them.

The method for calculating the capacitance of a capacitor is based on the relation: $Q = CV$

First, we calculate the electric field at any point inside the capacitor. Since the circulation of the electric field between the plates makes it possible to determine the expression of the potential, the capacitance of the capacitor can then be obtained from the following ratio: $C = \frac{Q}{V}$

Conductors

3- Capacitors

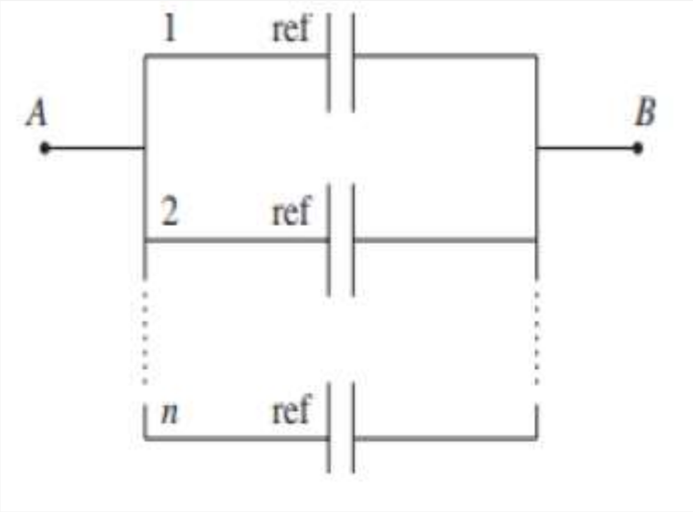
Combinations of Capacitors

We will now study the equilibrium of several capacitors. The description of this equilibrium depends on how they are connected. They can be in series, in parallel, in circuit.

- ✓ In practice, we take total influence conductors such that Q_A = Q_B= Q

a- Parallel Combination:

n capacitors with capacities C1, C2,......Cn, connected as shown in Figure, are known as a parallel combination of capacitors. The figure shows a circuit diagram for this combination of capacitors.



Conductors

3- Capacitors

a- Parallel Combination:

All the internal plates of these capacitors to the same point A with potential V_A and all the external plates to another point B with potential V_B , the potential difference $(V_A - V_B)$ is the same for all the capacitors. We can therefore write:

$$V_A - V_B = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \dots\dots = \frac{Q_n}{C_n}$$

The total charge Q supported by all the internal plates is obviously equal to the sum of the charges accumulated on all the capacitors:

$$Q = Q_1 + Q_2 + Q_3 + \dots\dots + Q_n = (C_1 + C_2 + C_3 + \dots\dots + C_n)(V_A - V_B) = C_{eq}(V_A - V_B)$$

We can then define the equivalent capacity of a parallel combination as follows:

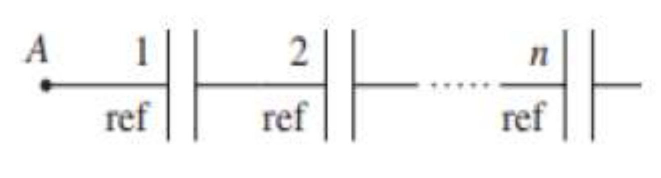
$$C_{eq} = C_1 + C_2 + C_3 + \dots\dots + C_n = \sum_{i=1}^{n} C_i$$

Conductors

3- Capacitors

b- Series Combination:

n capacitors with capacities C1, C2,......Cn, connected as shown in Figure, are known as a series combination of capacitors. The figure shows a circuit diagram for this combination of capacitors.



the charges on capacitors connected in series are the same We can therefore write: $Q_1 = Q_2 = Q_3 = \ldots\ldots\ldots\ldots\ldots = Q_n$

The individual voltages V_1 , V_2 , ……. and V_n across the capacitors add to give the total voltage V_{Tot} across the combination:

$$\Delta V_{Tot} = \Delta V_1 + \Delta V_2 + \ldots\ldots\ldots + \Delta V_n = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} + \ldots\ldots\ldots + \frac{Q_n}{C_n}$$

Conductors

3- Capacitors

b- Series Combination:

Applying the definition of capacitance to the circuit: $\Delta V_{Tot} = \frac{Q}{C_{eq}}$

We can then define the equivalent capacity of a series combination as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}$$

Internal energy of a conductor in electrostatic equilibrium isolated in space

Given an initially neutral conductor, we charge it until its final charge becomes Q. The internal energy of the capacitor is defined as the work necessary to carry the charge from 0 to Q.

$$dE_p = qdV \Rightarrow E_p = \int_0^Q qdV = \frac{1}{C}\int_0^Q qdq = \frac{1}{2C}Q^2 \Rightarrow E_p = \frac{1}{2C}Q^2 = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Conductors

3- Capacitors

Example1:

1- Calculate the equivalent capacity in the figure opposite.

We apply between A and B a potential difference V_{AB}=48 $Volts$.

We give: $C_1 = 3\ \mu F$; $C_2 = 6\ \mu F$; $C_3 = 2\ \mu F$

2- Calculate the charge of each capacitor and the difference of potential between its armatures.

3- Calculate the potential energy stored.