

Guided Work Series Number 4

Exercise 8.1

Let A be a matrix in $\mathcal{M}_3(\mathbb{R})$ defined as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}.$$

- (1) Is the matrix A diagonalizable?
- (2) Calculate $(A - 2I_3)^2$, then $(A - 2I_3)^n$ for each $n \in \mathbb{N}$. Deduce A^n .

Exercise 8.2

Let the matrix

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{pmatrix}.$$

- (1) Find the characteristic polynomial of A .
- (2) Prove that A is diagonalizable, then find the diagonal matrix D and an invertible transition matrix P such that $A = PDP^{-1}$.
- (3) Calculate A^n for $n \in \mathbb{N}$.

Exercise 8.3

Let the matrix A :

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

- (1) Diagonalize the matrix A .
- (2) Express the solutions of the differential system $X' = AX$ in the eigenbasis and sketch their trajectories.

Exercise 8.4

Let the matrix A :

$$A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 3 & -1 \\ -2 & -1 & -3 \end{pmatrix}.$$

- (1) Factorize the characteristic polynomial of A and then find the eigenvalues of the matrix.
- (2) Find the eigenspaces of A .
- (3) Is A diagonalizable?

Exercise 9.1

Solve the following linear systems using the Gauss method:

$$\left\{ \begin{array}{l} x + y + 2z = 3 \\ x + 2y + z = 1 \\ 2x + y + z = 0 \end{array} \right., \quad \left\{ \begin{array}{l} x + 2z = 1 \\ -y + z = 2 \\ x - 2y = 1 \end{array} \right. .$$

Exercise 9.2

- (1) Find the solutions to the following system in four different ways (by substitution, by Gauss's pivot method, by inverting the coefficient matrix, and by using Cramer's rule):

$$\left\{ \begin{array}{l} 2x + y = 1 \\ 3x + 7y = -2 \end{array} \right.$$

- (2) Choose the method that seems fastest for solving, depending on the values of a , the following systems:

$$\left\{ \begin{array}{l} ax + y = 2 \\ (a^2 + 1)x + 2ay = 1 \end{array} \right. \quad \left\{ \begin{array}{l} (a + 1)x + (a - 1)y = 1 \\ (a - 1)x + (a + 1)y = 1 \end{array} \right.$$

Exercise 9.3

Find the solutions to the following system:

$$(S) = \left\{ \begin{array}{l} 3x + 2z = 0 \\ 3y + z + 3t = 0 \\ x + y + z + t = 0 \\ 2x - y + z - t = 0 \end{array} \right.$$