MOHAMED KHIDER UNIVERSITY – BISKRA
Faculty of Exact Sciences
Department of Mathematics

First Year (L1) – Mathematics

Module: Descriptive Statistics

Course Summary: Position Parameters

Mode – Median – Means – Quartiles – Deciles – Percentiles

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1) What are position parameters?

Position parameters are measures that **locate** the center (or a given rank) of a distribution:

- **Mode** (most frequent value/category)
- **Median** (middle value)
- **Quantiles**: **quartiles**, **deciles**, **percentiles**
- **Means**: arithmetic, weighted, geometric, harmonic (when appropriate)

2) Valid measures for each variable type

Variable type	Position measures that make sense
Nominal qualitative (no order)	Mode only. Median/quantiles/means are not defined (no ordering, no numeric scale).
Ordinal qualitative (ordered categories)	Mode, median category, quartile/decile/percentile categories using cumulative frequencies. Arithmetic mean is usually not meaningful.
Quantitative (discrete/continuous)	Mode, median, quantiles, arithmetic mean, weighted mean. Geometric mean for growth factors; harmonic mean for rates.

3) Mode

Discrete or qualitative data

Mode is the value/category with the **largest frequency**. A dataset may be unimodal, bimodal, or multimodal.

Grouped continuous data (histogram)

Modal class = class with the largest bar (largest **density** if class widths are unequal).

Grouped mode (equal class widths):

$$Mo = L + \frac{(f_1 - f_0)}{(f_1 - f_0) + (f_1 - f_2)}\, h$$

where L is the lower bound of the modal class, h its width, f_1 its frequency, f_0 the previous class frequency, f_2 the next class frequency.

If widths are unequal, use densities $d_i = f_i/h_i$ in place of f_i.

4) Median

Raw (ungrouped) data

Order the data.

$$\text{If } N \text{ is odd: } Me = x_{\frac{N+1}{2}}, \qquad \text{If } N \text{ is even: } Me = \frac{x_{\frac{N}{2}} + x_{\frac{N}{2}+1}}{2}.$$

Frequency table (discrete)

Compute cumulative frequencies and locate the position $N/2$.

Grouped continuous data (interpolation)

If the median position $N/2$ falls in a class with lower bound L, width h, frequency f, and cumulative frequency before the class C_{prev}:

$$Me = L + \frac{\left(\frac{N}{2} - C_{\text{prev}}\right)}{f}\, h.$$

5) Quantiles: Quartiles, Deciles, Percentiles

Quantiles locate values according to a percentage of observations.

Key definitions

$$Q_1 = P_{25}, \quad Q_2 = Me = P_{50} = D_5, \quad Q_3 = P_{75}, \quad D_k = P_{10k}.$$

Positions (table method)

For total size N, common positions are:

$$Q_1 :\ 0.25N, \qquad Q_3 :\ 0.75N,$$

$$D_k :\ \frac{kN}{10}, \qquad P_p :\ \frac{pN}{100}.$$

Then locate the class/value using cumulative frequencies.

Grouped continuous interpolation

If a quantile position falls in a class $(L, h, f, C_{\text{prev}})$:

$$Q = L + \frac{(\text{position} - C_{\text{prev}})}{f}\, h.$$

6) Arithmetic mean

Discrete data / frequency table

$$\bar{x} = \frac{\sum x_i n_i}{N}.$$

Grouped continuous data (approximation)

Using class midpoints m_i:

$$\bar{x} \approx \frac{\sum m_i f_i}{N}.$$

7) Weighted mean

Used when scores have different weights w_i (with $\sum w_i = 1$):

$$\bar{x}_w = \sum w_i x_i.$$

8) Geometric mean

Used for multiplicative changes (growth rates, indices). If factors are $g_1, \ldots, g_n$:

$$G = (g_1 g_2 \cdots g_n)^{1/n}.$$

If rates are r_i, use factors $g_i = 1 + r_i$.

9) Harmonic mean

Used for rates when the numerator is the same (e.g., equal distances).

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}.$$

Example: average speed over equal distances.

10) Key equivalences to remember

$$Me = Q_2 = D_5 = P_{50}, \qquad Q_1 = P_{25}, \qquad Q_3 = P_{75}, \qquad D_k = P_{10k}.$$

End of Course Summary