Worksheet N°1

Exercise 1 Calculate the derivatives of the following univariate functions:

1. $f(x,y) = e^{x^2+y}$, $\quad g(x,y) = x^2 + y^2$, $\quad h(x,y) = x^y$.
2. $\arcsin\left(\frac{x}{\sqrt{a}}\right)$, $\quad \arccos\left(\frac{x}{\sqrt{a}}\right)$, $\quad \arctan\left(\frac{x}{a}\right)$, with $a > 0$.

Exercise 2 Let's consider the following functions:

$$\begin{aligned}
f_1(x) &= ln\left(\frac{\alpha x + \beta}{ax + b}\right). \\
f_2(x) &= \frac{\alpha}{2a}\ \ln(ax^2 + b) + \frac{\beta}{\sqrt{ab}}\ \arctan\left(\sqrt{\frac{a}{b}}\ x\right),\ \text{with } a, b > 0. \\
f_3(x) &= \frac{\alpha}{\sqrt{a}}\ \arcsin\left(\sqrt{\frac{a}{b}}\ x\right) - \frac{\beta}{a}\ \sqrt{b - ax^2},\ \text{with } a, b > 0. \\
f_4(x) &= \sum_{k=0}^{q} \frac{(-1)^k\ C_q^k}{(n + 2k + 1)} \cos(x)^{n+2k+1}, \qquad \text{with } n, k, q \in \mathbb{N} \text{ and } C_q^k = \frac{q!}{(q-k)!\ k!}.
\end{aligned}$$

1. Give the derivatives of the functions f_1, f_2 and f_3 in their simplified form.
2. Check that f_4' can be expressed as a product of $\cos^n$ and $\sin^m$ (n and m are two natural numbers).
3. Check that the proposition of the second question remains true when we replace *cos* with *sin* in the expression of f_4.

Exercise 3 Considering the polynomial $P_2(x) = x^2 + ax + b$, with $a, b \in \mathbb{R}$. Show that:

1. If $a^2 - 4b \geq 0$ then $P_2(x)$ can be rewritten $P_2(x) = (x + A)^2 - B^2$.
2. If $a^2 - 4b < 0$ then $P_2(x)$ can be rewritten $P_2(x) = (x + A)^2 + B^2$.

Exercise 4 Determine the expression of the function f that must be differentiated to obtain the following:

$$\begin{aligned}
f'(x) &= \cos(x), & f'(x) &= x + 2x^{\sqrt{2}} + 1, & f'(x) &= \tfrac{x+1}{1+x^2}, \\
f'(x) &= \tfrac{x+1}{\sqrt{1-x^2}}, & f'(x) &= e^{\frac{x}{2}} + 1
\end{aligned}$$

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Worksheet N° 1 & 2

Exercise 1

1. Calculate the following:

1) $\int 2dx$	2); $\int x^2+2x-1dx;$	3) $\int \frac{\theta^2+3}{\sqrt{\theta}}d\theta;$
4) $\int xy^2+y^2\sqrt[3]{y}dy;$	5) $\int x^2+\sqrt{x}+\frac{1}{x^4\sqrt{x}}dx;$	6) $\int \sin(\theta)-\cos(\theta)d\theta.$

2. Find the real function f that satisfying the following conditions

1) $f'(x)=3x^2+e^{-x}$ and $f(0)=1;$	3) $f''(x)=sin(x)-e^{-2x},\quad f'(0)=\frac{5}{2}$ and $f(0)=0;$
2) $f'(x)=\sqrt[3]{x^2}-\frac{1}{x^2}$ and $f(1)=3;$	4) $f''(x)=sin(x)-cos(x),\quad f'(0)=0$ and $f(\frac{\pi}{2})=0.$

Exercise 2 *(by substitution)* Find the antiderivatives of the following functions:

$f(x)=\tan(x),\quad f(x)=\sin^3(x)\cos^2(x),\quad f(x)=\frac{\sin(x)cos(x)}{(1+\cos(2x))^2},\quad f(x)=\frac{\arcsin(x)}{\sqrt{1-x^2}},$

$f(x)=\frac{x}{x^4+a^2},\quad f(x)=\frac{1}{x\ln(x)},\quad f(x)=xa^{x^2},\quad f(x)=\frac{(a^x-b^x)^2}{a^xb^x}$ with $a,\ b>0.$

$\int \sin^{2p+1}(x)\cos^q(x)dx$ and $\int \sin^p(x)\cos^{2q+1}(x)dx$, with $p,q\in\mathbb{N}$

$\int \frac{1}{x\ln^n(x)}$ with $n\in\mathbb{Z}^*$

Exercise 3 *(by parts)* Let $n\in\mathbb{N}^*$. Calculate the following using integration by parts.

1. • $\int \arctan(x)dx,$ • $\int \arcsin(x)dx,$ • $\int \frac{\arcsin(\sqrt{x})}{\sqrt{x}}dx,$ • $\int \frac{x\arctan(x)}{(1+x^2)^2}dx$

2. • $\int \ln(x)dx,$ • $\int x\ln(x)dx,$ • $\int (\ln(x))^2dx,$ • $\int x^n\ln(x)dx$

3. • $\int x^2e^{-x}dx,$ • $\int x^ne^{-x}dx,$

4. • $\int \sin(x)e^xdx,$ • $\int \cos(\beta x)e^{\alpha x}dx,$ • $\int \sin(\beta x)e^{\alpha x}dx,$

5. • $\int \sqrt{a^2-x^2}dx,$ • $\int \cos^2(x)dx,$ • $\int \sin^4(x)dx$

Exercise 4 *(Rational and irrational functions)*Calculate the following integrals

- $\int \frac{x+1}{x^2+x+2}dx,$
- $\int \frac{x+1}{x^2+x-2}dx,$
- $\int \frac{x+1}{(x-1)(x-2)}dx,$
- $\int \frac{x+1}{(x-1)^2(x-2)}dx,$
- $\int \frac{x3+2x^2+3x-1}{(x^2+1)(x^2-4)}dx,$
- $\int \frac{x^5+x^4-8}{x^3-4x}dx,$
- $\int \frac{x+1}{\sqrt{(x-1)(x-2)}}dx,$
- $\int \frac{x+1}{\sqrt{(1-x)(x-2)}}dx,$
- $\int \frac{x3+2x^2+3x-1}{x^2+2x+1}dx,$
- $\int \frac{1}{2\cos^2(x)+\cos(x)\sin(x)+\sin^2(x)}dx$
- $\int \ln(x^2+2x-3)dx,$
- $\int \frac{1}{(x^2-4)^2}dx,$
- $\int \frac{1}{(x^2+4)^2}dx,$
- $\int \frac{1}{(x^2-4)^4}dx,$
- $\int \frac{1}{(x^2+4)^4}dx,$

$\int \frac{2x^{1/2}+3x^{1/4}}{1+x^{1/4}}dx$, • $\int \frac{2x^{1/2}+3x^{1/3}}{1+x^{1/3}}dx$, • $\int \sqrt[3]{5x-1}dx$, • $\int \frac{dx}{\sqrt{\sqrt{x}-2}}$, • $\int \sqrt[3]{\frac{x+1}{x-1}}dx$.

Exercise 5 Let consider the following integrals

$$I(x) = \int e^x \cos^2(x/2)dx \text{ and } J(x) = \int e^x \sin^2(x/2)dx.$$

1. Compute $F(x) = I(x) + J(x)$.
2. Compute $G(x) = I(x) - J(x)$.
3. Deduce the expressions of I and J.

Exercise 6 Calculate the following integrals:

1. $\int_0^{+\infty} e^{-x}dx$ and $\int_0^{+\infty} x^2e^{-x}dx$.
2. $\int_1^e \ln(x)dx$ and $\int_e^{2e} x^2\ln(x)dx$.
3. $\int_0^{2\pi} \sin(x)cos^2(x)dx$ and $\int_0^{\frac{\pi}{2}} sin(x)e^x dx$.

Exercise 7 Let f a real function defined by

$$f(x) = \frac{5(x-2)}{x(x-5)}.$$

1. Calculates the antiderivative of f.
2. Calculates the value of the surface delimited by the curve of f, the axe $y = 0$, the axe $x = 1$ and $x = 3$.

Exercise 8 Let's consider the function f defined by:

$$f(x) = \begin{cases} \alpha\left(\frac{1}{x}\right)^{\alpha+1}; & \text{if } x > 1, \\ 0; & \text{else,} \end{cases}$$

with α is a positive real number.
Question: Discuss the existence of the integral $\int_a^{+\infty} x^n f(x)dx$ according to n with $n \in N^*$.

Exercise 9 Consider the real function f on D_f defined by:

$$f(x) = \sqrt{a^2 - x^2}, \quad \textit{with } a > 0.$$

1. Give the domain D_f.
2. Calculate the value of the area delimited by the curve of f and the axis $y = 0$ over the domain D_f.
3. Deduce the expression for the surface delimited by a circle with center $(0, 0)$ and radius R.

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Worksheet N° 3

Remark: Treat only the ***first four examples*** of each exercise and leave the rest to the students.

Exercise 1 *(Separated variables equations)* Find the solution of the following equations and, if possible, express your solution in the form $y = f(x)$.

$y' = \frac{e^{x-y}}{1+e^x}$ *with* $y(1) = 0$ and $y' = \frac{x^2y-y}{1+y}$ *with* $y(3) = 1$.

- $y' - 2xy = 2x$
- $y' = yx^2 - 2x^2$
- $xy + y' = y^2\ln(x)$
- $y' = xy^2 - x - y^2 + 1$
- $y' = xe^{x+y}$
- $y' = \frac{1+y^2}{1+x^2}$
- $y' = \frac{y^2+1}{xy+y}$
- $y\ln(x)y' = \frac{y^2+1}{x}$
- $\ln(x)y' = \frac{y}{x}$
- $(2xy^3 + 4x)y' = xy^2 + y^2$
- $(xy + y)y' = x - xy$
- $y' = \frac{\sin(1/x)}{x^2\cos(y)}$

Exercise 2 *(Linear differential equations)* Solve the following first order differential equations

- $xy' - 2y = -x$
- $xy' - 2y = x^3e^x$ with $y(1) = 0$
- $xy' - y = 2x\ln(x)$
- $xy' + 2y = \frac{cos(x)}{x}$
- $(x+1)y' + 2y = (x+1)^{5/2}$
- $y' + y\ tan(x) = \cos^2(x)$
- $y' + 2\frac{y}{x} = \frac{4}{x}$ with $y(1) = 6$
- $xy' - 2y = x^4e^x$
- $xy' + y = (1+x)e^x$

Exercise 3 *(Homogeneous equations)* Show that each of the following differential equations is homogeneous and find the general solution of the equation.

- $y' = \frac{x^2e^{y/x}+y^2}{xy}$
- $y' = \frac{\sqrt{x^2-y^2}+y}{x}$
- $y' = \frac{x^4+2y^4}{xy^3}$

Exercise 4 *(Bernoulli's equations)* Solve the following Bernoulli's differential equations:

- $y' + \frac{y}{x} = 3x^2y^2$
- $y' - 4y = 2e^x\sqrt{y}$
- $y' - \frac{3}{4}y = (9x-3)y^5$
- $3y' + \frac{3}{x}y = 2x^2y^4$

Exercise 5 *(Riccati's equations)* Solve the following differential equations:

- $x^3y' + y^2 + yx^2 + 2x^4 = 0$
- $y' + \frac{y}{x} = 3x^2y^2 + xe^x$
- $(x^2+1)y' = y^2 - 1$
- $(y' - y^2)\cos x + y(2\cos^2 x + \sin x) = \cos^3 x$
- $2xyy' = 1 + y^2 + cos(x)$

Exercise 6 *(Second order differential equations)* Solve the following differential equations :

- $y'' - 3y' + 2y = 0$
- $y'' - 2y' + y = 0$
- $y'' + y = 2\cos(x)$
- $y'' - 3y' + 2y = xe^x$
- $y'' + yy' + y^2 = xy^3$
- $y'' + 2y' + 2y = 0$
- $y'' + y = 2\cos(x)$
- $y'' - 3y' + 2y = \sin(x) + 2e^{-x} + xe^{-2x} + x^2 + x + 1$
- $y'' - 2y' + y = x^2e^x$
- $y'' + 2y' - 3y = 3x + 4 - 4e^{-3x} + 5\sin(2x+1)$.

Exercise 7 Let us consider the following linear differential equation:

$$y' - \frac{y}{(x+1)^n} = \frac{1}{(x+1)^n}, \quad \text{with } x,\ y \in \mathbb{R}_+ \text{ and } n \in \mathbb{R}. \tag{1}$$

1. Solve the equation (1) according to the values of n.
2. Determine the solution of (1) that satisfies the condition $y(0) = 0$ when $n = 1$ and $n = 2$.

Exercise 8

Discuss the solutions of the following differential equation according to the real parameter n.

$$y' - y = xy^n, \text{ with } n \in \mathbb{R},\ x > 0 \text{ and } y > 0.$$

Exercise 9

1. Solve the following second order differential equation.

$$y'' + 2y' + 5y = 4e^{-x}.$$

2. Determines the solution that passed from the origin $(0, 0)$ and the point $(\pi/4, 0)$.