Tutorial Series N°03

Exercise 1. Let

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & -3 & 1 \\ -3 & 4 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 5 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & -1 \\ 4 & 0 & 3 \end{pmatrix}.$$

1) Write the type and the size of each matrix.
2) Find, if possible:

$$A+B,\ 3B,\ 3B+A,\ B+C,\ AB,\ AI_3,\ BC,\ CB,\ 3(AB)C,\ A^2.$$

3) Find:

$$A^\top,\ B^\top,\ C^\top.$$

4) Find a matrix D with no zero entries such that AD is the 3×3 zero matrix.

Exercise 2. Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 2 \\ 1 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 1 & 1 \end{pmatrix}.$$

1) Compute $\det(A)$, $\det(B)$, and $\det(C)$.
2) Compute $\operatorname{tr}(A)$, $\operatorname{tr}(B)$, and $\operatorname{tr}(C)$.
3) Find, if possible, A^{-1}, B^{-1}, and C^{-1}.

Exercise 3. Consider the map

$$f:\mathbb{R}^2\to\mathbb{R}^3, \qquad f(x,y) = \big(2x+y,\ x-y,\ 3x+2y\big).$$

1) Is f a linear map?
2) Find $[f]_{\mathcal{B}_1,\mathcal{B}_2}$ with the standard bases

$$\mathcal{B}_1 = \left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}, \quad \mathcal{B}_2 = \left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}.$$

3) Find $[f]_{\mathcal{K}_1,\mathcal{K}_2}$ with

$$\mathcal{K}_1 = \left\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right\}, \quad \mathcal{K}_2 = \left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right\}.$$

Exercise 4. Let $f:\mathbb{R}^3\to\mathbb{R}^3$ be defined by

$$f(x,y,z) = \big(x+y+z,\ 2y+z,\ 3z\big).$$

1) Prove that f is an endomorphism of $\mathbb{R}^3$.
2) Give $A = M(f,\mathcal{B})$ in the standard basis $\mathcal{B} = (e_1,e_2,e_3)$.
3) For $v_1 = (1,0,0)$, $v_2 = (1,1,0)$, $v_3 = (1,1,1)$, compute $f(v_1)$, $f(v_2)$, $f(v_3)$ in the basis (v_1,v_2,v_3).
4) Deduce that $\mathcal{S} = (v_1,v_2,v_3)$ is a basis, then find $D = M(f,\mathcal{S})$.
5) Give the passage matrix G from $\mathcal{S}$ to $\mathcal{B}$.