

Guided Work Series Number **3** Matrices

Exercise 7.1

Let

$$A = \begin{pmatrix} -7 & 2 \\ 0 & -1 \\ 1 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}, \quad D = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},$$

and

$$E = \begin{pmatrix} 1 & 2 \\ -3 & 0 \\ -8 & 6 \end{pmatrix}.$$

(A Calculate all possible sums of two of these matrices.

(B Calculate all possible products of two of these matrices.

(C Calculate $3A + 2E$ and $5B + 4EA^T$.

(D Find α such that $A - \alpha E$ is the null matrix.

Exercise 7.2

1) Calculate the products AB and BA when they are defined, in each of the following cases:

$$(a) \quad A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}$$

$$(b) \quad A = \begin{pmatrix} 0 & 3 & 6 \\ -2 & 0 & 0 \\ 2 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$(c) \quad A = \begin{pmatrix} 1 & 3 \\ 1 & 1 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

2) Calculate the transpose of the matrices given above.

Exercise 7.3

Let $A, B \in M_2(\mathbb{R})$ be the matrices defined by:

$$A = \begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}.$$

Compare the two matrices $(A + B)^2$ and $A^2 + 2AB + B^2$. Then compare $(A + B)^2$ with $A^2 + AB + BA + B^2$.

Exercise 7.4

Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Find all matrices

$$B = \begin{pmatrix} c & d \\ e & f \end{pmatrix} \in \mathcal{M}_2(\mathbb{R})$$

that commute with A , i.e., $AB = BA$.

Exercise 7.5

Let a and b be non-zero real numbers and the matrix

$$A = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}.$$

Find all matrices $B \in \mathcal{M}_2(\mathbb{R})$ that commute with A , i.e., $AB = BA$.

Exercise 7.6

Find A and B in $\mathcal{M}_2(\mathbb{R})$ such that $AB = 0$ and $BA \neq 0$.

Exercise 7.7

Let the matrix

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

(1 Is there a matrix $B \in M_{2,3}(\mathbb{R})$ such that $AB = I_3$? If yes, give the general form of B .

(2 Is there a matrix $C \in M_{2,3}(\mathbb{R})$ such that $CA = I_2$? If yes, give the general form of C .

Exercise 7.8

Let the following matrices:

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}.$$

(1 Calculate A^2 , A^3 . Then deduce A^n for every $n \geq 1$.

(2 Answer the same question for the matrix B .

Exercise 7.9

Calculate, using the Gauss method and then the adjugate matrix method, the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$$

Exercise 7.10

Prove that

$$D = \begin{vmatrix} 1+a & a & a \\ b & 1+b & b \\ c & c & 1+c \end{vmatrix} = 1+a+b+c.$$