

Guided Work Series Number 2

DIFFERENTIAL EQUATIONS

Exercise 6.12 (Separation of Variables)

Solve the differential equation:

$$\frac{dy}{dx} = \frac{x^2}{y} \quad \text{with initial condition } y(0) = 2$$

Exercise 6.13 (Exact Differential Equation)

Verify that the following equation is exact and find its general solution:

$$(2xy + y^2)dx + (x^2 + 2xy)dy = 0$$

Exercise 6.14 (Homogeneous Differential Equation)

Solve the homogeneous differential equation:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$$

Exercise 6.15 (First Order Linear Differential Equation)

Solve the linear differential equation:

$$\frac{dy}{dx} + 2xy = x$$

Exercise 6.16 (Bernoulli Differential Equation)

Solve the Bernoulli differential equation:

$$\frac{dy}{dx} - \frac{2y}{x} = x^2y^3$$

Exercise 6.17 (Riccati Differential Equation)

Solve the Riccati differential equation:

$$\frac{dy}{dx} = 1 + x - y - xy + y^2$$

given that $y_1 = x$ is a particular solution.

Exercise 6.18 (Another Bernoulli Equation)

Solve the Bernoulli differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = y^2 \ln x$$

Exercise 6.19 (Bernoulli Differential Equation)

Solve the Bernoulli differential equation:

$$\frac{dy}{dx} + y = xy^2$$

Exercise 6.20 (Second Order Linear Differential Equation with Constant Coefficients)

Solve the initial value problem:

$$y'' - 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

Exercise 6.21

Find in the domain I of \mathbb{R} the solutions of the following differential equations:

- 1) $x \ln xy' + y = x, \quad I =]1, +\infty[$
- 2) $x(xy' + y - x) = 1, \quad I =]-\infty, 0[$
- 3) $2xy' + y = x^4, \quad I =]-\infty, 0[$

Exercise 6.22

Find on the set $] - \infty, 0[$ and the set $]0, +\infty[$ the solutions of the differential equation:

$$|x|y' + (x - 1)y = x^3$$