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Tutorial Series N°01: Vector Spaces

Exercise 0.1:

Let $E = \mathbb{R}^3$. Determine which of the following subsets are vector subspaces of $\mathbb{R}^3$:

a) $F_1 = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = 0\}$
b) $F_2 = \{(x, y, z) \in \mathbb{R}^3 \mid x + y = 1\}$
c) $F_3 = \{(x, y, z) \in \mathbb{R}^3 \mid xy = 0\}$
d) $F_4 = \{(x, y, z) \in \mathbb{R}^3 \mid x = 2y,\ z = 0\}$
e) $F_5 = \{(x, y, z) \in \mathbb{R}^3 \mid x \geq 0\}$
f) $F_6 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 0\}$

Exercise 0.2:

Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = (1, 2, 1), \quad v_2 = (2, 9, 0), \quad v_3 = (3, 3, 4)$$

1. Show that the family $\mathcal{B} = \{v_1, v_2, v_3\}$ forms a basis of $\mathbb{R}^3$.
2. Find the coordinates of the following vectors relative to the basis $\mathcal{B}$:

$$u = (5, 14, 5) \quad \text{and} \quad w = (0, -4, 5)$$

Exercise 0.3:

Let $v_1 = (1, -1, 2)$, $v_2 = (2, m, 1)$, and $v_3 = (4, 1, 5)$ be vectors in $\mathbb{R}^3$.

1. Determine the values of the parameter $m \in \mathbb{R}$ for which the family $\{v_1, v_2, v_3\}$ is **linearly dependent** (linked).
2. For the value $m = 1$, express the vector v_3 as a linear combination of v_1 and v_2.

Exercise 0.4:

Consider the subspaces F and G of $\mathbb{R}^3$ defined by:

$$F = \text{span}\big((1, 1, 0), (0, 1, 1)\big) \quad \text{and} \quad G = \text{span}\big((1, 0, 1)\big)$$

1. Determine the dimension of F and the dimension of G.
2. Show that $F + G = \mathbb{R}^3$.
3. Is the sum direct ($F \oplus G = \mathbb{R}^3$)? Justify your answer by calculating $F \cap G$.

Exercise 0.5:

Let $E = \mathbb{R}_2[X]$ be the space of polynomials of degree ≤ 2. Consider the family $\mathcal{B}' = \{P_1, P_2, P_3\}$ where:

$$P_1(X) = 1 + X, \quad P_2(X) = 1 + X^2, \quad P_3(X) = X + X^2$$

1. Show that $\mathcal{B}'$ forms a basis of $\mathbb{R}_2[X]$.
2. Express the following polynomials as linear combinations of $\mathcal{B}'$:

$$Q(X) = 1 + X + X^2 \quad \text{and} \quad R(X) = 2 - X + 3X^2$$

Good Luck!