Mohamed Khider University of Biskra
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Module: Mathematics2

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Guided Work Series Number 1

Limited Expansions

Exercise 5.1 (Basic Limited Expansions)

Find the limited expansion at the specified order for each function:

1. $f(x) = e^{2x}\cos x$ at order 3 around $x_0 = 0$
2. $f(x) = \ln(1+\sin x)$ at order 4 around $x_0 = 0$
3. $f(x) = \sqrt[3]{1+x^2}$ at order 6 around $x_0 = 0$

Exercise 5.2 (Limits using Limited Expansions)

Calculate the following limits using limited expansions:

1. $\lim\limits_{x\to 0} \dfrac{e^x - \cos x - x}{x^2}$
2. $\lim\limits_{x\to 0} \dfrac{\ln(1+x) - x + \frac{x^2}{2}}{x^3}$
3. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x} - 1 - \frac{x}{2} + \frac{x^2}{8}}{x^3}$

Exercise 5.3 (Composition of Functions)

Find limited expansions for the following composite functions:

1. $f(x) = e^{\tan x}$ at order 3 around $x_0 = 0$
2. $f(x) = \sin(\ln(1+x^2))$ at order 6 around $x_0 = 0$
3. $f(x) = \arctan(e^x - 1)$ at order 4 around $x_0 = 0$

Exercise 5.4 (Division and Rational Functions)

Find limited expansions for the following rational functions:

1. $f(x) = \frac{e^x}{\cos x}$ at order 4 around $x_0 = 0$
2. $f(x) = \frac{\ln(1+x)}{\sqrt{1+x}}$ at order 3 around $x_0 = 0$
3. $f(x) = \frac{1-\cos x}{\sin x}$ at order 4 around $x_0 = 0$

Exercise 5.5 (Taylor Polynomials at Non-zero Points)

Find Taylor polynomials around the specified points:

1. 3rd order Taylor polynomial of $f(x) = \ln x$ around $x_0 = 1$
2. 2nd order Taylor polynomial of $f(x) = \sqrt{x}$ around $x_0 = 4$
3. 3rd order Taylor polynomial of $f(x) = e^{-x}$ around $x_0 = 2$

Exercise 5.6 (Applications and Error Estimation)

Approximate the following values and estimate the error:

1. $\sqrt{1.02}$ using a 2nd order Taylor polynomial of $\sqrt{1+x}$ at $x_0 = 0$
2. $\sin(0.1)$ using a 3rd order Taylor polynomial
3. $\ln(1.1)$ using a 4th order Taylor polynomial