Corrigé type de l'examen d'élasticité

25/2

Ex n° 1 : (8pts)

$$u_1 = -12.10^{-3} x_1$$
$$u_2 = 8.10^{-3} x_2 + 4.10^{-3} x_3$$
$$u_3 = 2.10^{-3} x_2 + 4.10^{-3} x_3$$

1) 0,5 $\mathrm{Grad}(u) = \begin{pmatrix} -12 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 2 & 4 \end{pmatrix} .10^{-3}$

$\varepsilon = \frac{1}{2}\left(\mathrm{grad}(u) + \mathrm{grad}^T(u)\right) = \begin{pmatrix} -12 & 0 & 0 \\ 0 & 8 & 3 \\ 0 & 3 & 4 \end{pmatrix} 10^{-3}$

0,5

$\Omega = \frac{1}{2}\left[\mathrm{grad}(u) - \mathrm{grad}^T(u)\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} 10^{-3}$

0,5

$\Delta = 52$

2) $\det(\varepsilon - \lambda I) = 0 \Rightarrow (-12-\lambda)(\lambda^2 - 12\lambda + 23) = 0$

$\varepsilon_I = 9,6.10^{-3}$ $\quad \varepsilon_{II} = 2,4.10^{-3}$ $\quad \varepsilon_{III} = -12.10^{-3}$

1,5

3) $n = \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} \Rightarrow \varepsilon(n) = \begin{pmatrix} 0 \\ 11\sqrt{2}/2 \\ 7\sqrt{2}/2 \end{pmatrix}$

0,5

$\varepsilon = n^t \varepsilon . n = \left(0 \ \frac{\sqrt{2}}{2} \ \frac{\sqrt{2}}{2}\right) \begin{pmatrix} 0 \\ 11\sqrt{2}/2 \\ 7\sqrt{2}/2 \end{pmatrix} = 9 \Rightarrow \varepsilon = 9.10^{-3}$

0,5

$g = \sqrt{|\varepsilon . n|^2 - \varepsilon^2} = \sqrt{121/2 + 49/2 - 81} = 2 \Rightarrow g = 2.10^{-3}$

0,5

4) a) The sphere becomes an ellipsoid.

0,5

b) $\varepsilon = \begin{pmatrix} 9,6 & 0 & 0 \\ 0 & 2,4 & 0 \\ 0 & 0 & -12 \end{pmatrix} 10^{-3}$

$d_1 = (1 + \varepsilon_I) d_0 = (1 + 9,6.10^{-3})\, 2000 = 2019,2$ mm

1,5 $d_2 = (1 + \varepsilon_{II}) d_0 = (1 + 2,4.10^{-3})\, 2000 = 2004,7$ mm

$d_3 = (1 + \varepsilon_{III}) d_0 = (1 - 12.10^{-3})\, 2000 = 1976,0$ mm

c) $V = [1 + tr(\varepsilon)] V_0 = [1 + tr(\varepsilon)] \frac{4}{3}\pi R^3$

0,5

0,5 $tr(\varepsilon) = 0 \Rightarrow V = V_0 = 4,189.10^9$ mm^3

0,5

Ex n° 2 : (12 pts)

$$\Sigma = \begin{pmatrix} 16 & -9 & 0 \\ -9 & 10 & 0 \\ 0 & 0 & 8 \end{pmatrix} daN/mm^2$$

$\Delta = 360$

1°/ $det(\Sigma - \lambda I) = 0 \Rightarrow (8-\lambda)(\lambda^2 - 26\lambda + 79) = 0$

0,5

$\sigma_I = 22,49\ daN/mm^2 \qquad \sigma_{II} = 8,00\ daN/mm^2 \qquad \sigma_{III} = 3,51\ daN/mm^2$

1,5

$(\Sigma - \sigma_i I) X_i = 0 \Rightarrow X_1 = \begin{pmatrix} -0,81 \\ 0,58 \\ 0 \end{pmatrix} \quad X_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad X_3 = \begin{pmatrix} 0,58 \\ 0,81 \\ 0 \end{pmatrix}$

0,5 1,5

2°/ $n = \begin{pmatrix} \sqrt{3}/3 \\ \sqrt{3}/3 \\ \sqrt{3}/3 \end{pmatrix}$

1,0 $\sigma_0 = n^T \Sigma_x \cdot n = tr(\Sigma)/3 = 34/3 = 11,33\ daN/mm^2$

1,0 $\tau_0 = \sqrt{|\Sigma_x n|^2 - \sigma^2} = \frac{1}{3}\sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_I - \sigma_{III})^2 + (\sigma_{II} - \sigma_{III})^2} = 8,09\ daN/mm^2$

3°/ $\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \qquad \sigma_{kk} = 34$

0,5

1,0 $\mathcal{E} = \begin{pmatrix} 0,505 & 0,557 & 0 \\ 0,557 & 0,133 & 0 \\ 0 & 0 & 0,009 \end{pmatrix}_{e_i} 10^{-3} \qquad \mathcal{E} = \begin{pmatrix} 0,906 & 0 & 0 \\ 0 & 0,009 & 0 \\ 0 & 0 & -0,268 \end{pmatrix}_{X_i} 10^{-3}$

1,0

4°/ $d_i = (1 + \varepsilon_i) d_0$

cercle appartient au plan principal $(X_1\ X_3)$

1,0 $d_1 = (1 + \varepsilon_I) d_0 = (1 + 0,906 \cdot 10^{-3}) 400 = 400,36$ mm

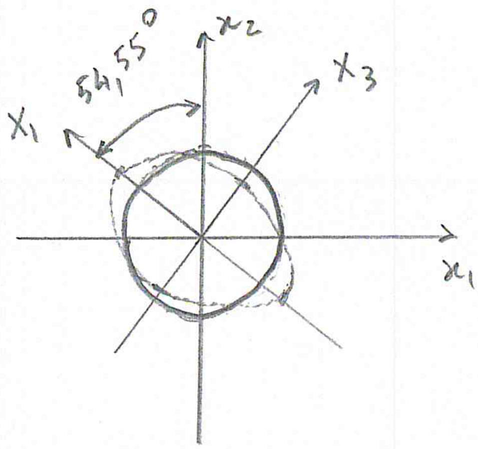
$d_2 = (1 + \varepsilon_{III}) d_0 = (1 - 0,268 \cdot 10^{-3}) 400 = 399,89$ mm

1,0

direction du grand axe

$(x_2, X_1) = arcos(0,58)$

0,5 $= 54,55°$



1,0