Corrigé Type de l'examen de MMC

25/2

Ex n° 1 (12 pts)

$$\Sigma = \begin{pmatrix} 16 & 0 & 0 \\ 0 & 12 & 20 \\ 0 & 20 & -8 \end{pmatrix} daN/mm^2$$

1°) $\det(\Sigma - \lambda I) = 0 \Rightarrow (16-\lambda)(\lambda^2 - 4\lambda - 496) = 0$

1,5 $\sigma_I = 24,36\ daN/mm^2$ $\quad \sigma_{II} = 16,00\ daN/mm^2$ $\quad \sigma_{III} = -20,36\ daN/mm^2$

1,5 $(\Sigma - \sigma_i) X_i = 0 \Rightarrow X_1 = \begin{pmatrix} 0 \\ 0,85 \\ 0,52 \end{pmatrix} \quad X_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad X_3 = \begin{pmatrix} 0 \\ -0,52 \\ 0,85 \end{pmatrix}$

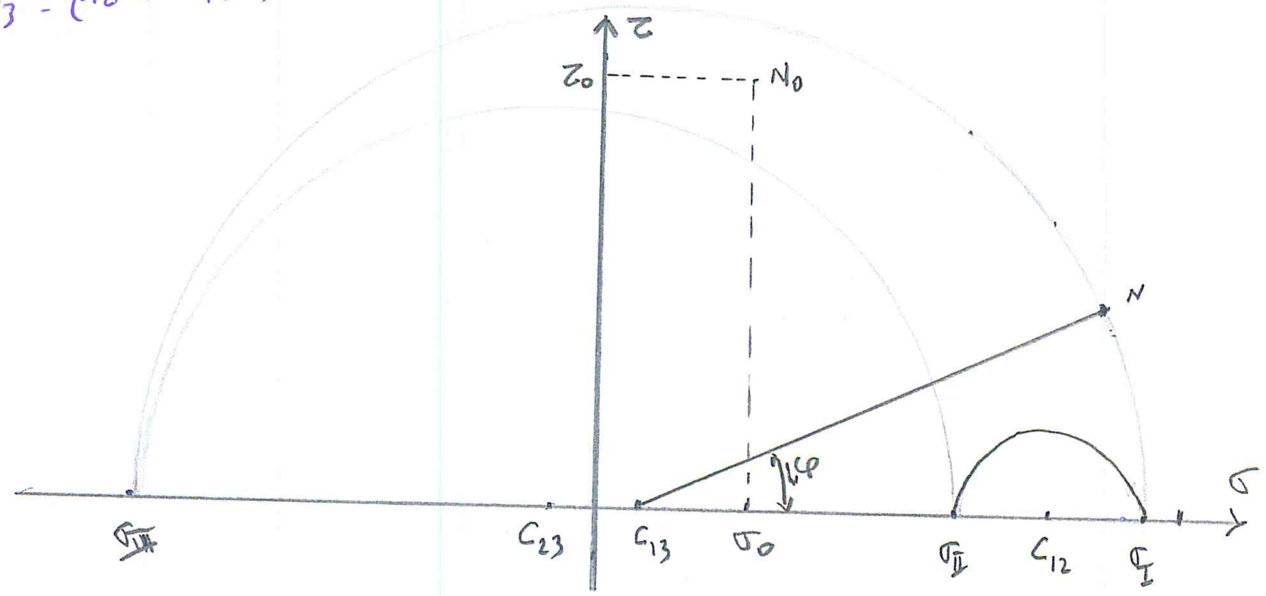
2°) $R_{ij} = \frac{1}{2}(\sigma_i - \sigma_j) \quad C_{ij} = \frac{1}{2}(\sigma_i + \sigma_j) \quad \sigma_i > \sigma_j$

1,0
$R_{12} = (24,36 - 16)/2 = 4,18 \quad C_{12} = 20,18$

$R_{13} = (24,36 + 20,36) = 22,36 \quad C_{13} = 2$

$R_{23} = (16 + 20,36) = 18,18 \quad C_{23} = -2,18$

1,0



0,5 3°) $\tau_{max} = R_{13} = (\sigma_I - \sigma_{III})/2 = 22,36\ daN/mm^2$

$$q = \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}_{X_1 X_3}$$

0,5 4°) $\sigma_0 = \frac{1}{3} tr(\Sigma) = 20/3 = 6,67\ daN/mm^2$

0,5 $\tau_0 = \frac{1}{3}\left[(\sigma_I - \sigma_{III})^2 + (\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2\right]^{1/2}$

$= 19,41\ daN/mm^2$

0,5 point représentatif : N_0

5/ $n = \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} \Rightarrow \quad \Sigma \cdot n = \Sigma \cdot \begin{pmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} = \begin{pmatrix} 0 \\ 16\sqrt{2} \\ 6\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 22,63 \\ 8,48 \end{pmatrix}$

0,5

$\sigma_n = n^t \cdot \Sigma \cdot n = \begin{pmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 16\sqrt{2} \\ 6\sqrt{2} \end{pmatrix} = 22 \ daN/mm^2$

0,5

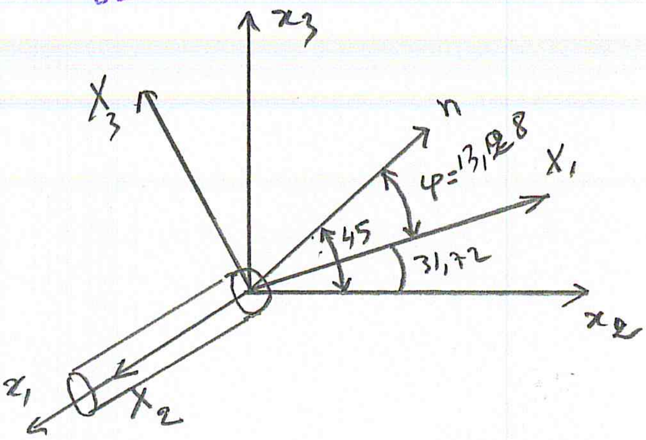
$\tau_n = \sqrt{|\Sigma \cdot n|^2 - \sigma_n^2} = \sqrt{512 + 72 - 484} = 10 \ daN/mm^2$

0,5

$X_1 = \begin{pmatrix} 0 \\ 0,85 \\ 0,52 \end{pmatrix} \Rightarrow \quad (x_2, X_1) = \operatorname{arctg}\left(\frac{0,526}{0,85}\right) = 31,72°$

$\varphi = 45 - 31,72 = 13,28°$

0,5 $\quad n \in \text{plan}(X_1, X_3)$



$\sigma_n = C_{13} + R_{13} \cos(2\varphi)$

0,5 $\quad = 2 + 22,36 \cos(2 \times 13,28)$

$= 22 \ daN/mm^2$

0,5 $\quad \tau_n = R_{13} \sin(2\varphi) = 10 \ daN/mm^2$

6/ $\varepsilon_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij}$

$\varepsilon_I = \frac{10^{-3}}{21,5}(1,3 \times 24,36 - 0,3 \cdot 20) = 1,19 \cdot 10^{-3}$

0,5 $\quad \varepsilon_{II} = 0,69 \cdot 10^{-3}$ $\qquad \mathcal{E} = \begin{pmatrix} 1,19 & 0 & 0 \\ 0 & 0,69 & 0 \\ 0 & 0 & -1,51 \end{pmatrix} \cdot 10^{-3}$

$\varepsilon_{III} = -1,51 \cdot 10^{-3}$

a/ La longueur

$L = (1 + \varepsilon_{II}) l_0 = (1 + 0,69 \cdot 10^{-3}) 4000 = 4002,75$ mm.

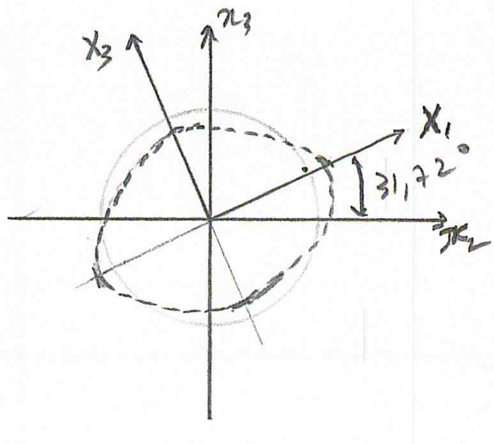
1,0 $\quad d_1 = (1 + \varepsilon_I) d_0 = (1 + 1,19 \cdot 10^{-3}) 200 = 200,24$ mm

$d_2 = (1 + \varepsilon_{III}) d_0 = (1 - 1,51 \cdot 10^{-3}) 200 = 199,70$ mm

La section transversale devient un ellipse d'axes d_1 et d_2

$(x_2, X_1) = 31,72°$

0,5



EX N° 2 (8pts)

$$\phi(x_1,x_2) = A\left(x_1^2 x_2^3 - \frac{1}{5}x_2^5\right) + \frac{1}{2}B x_1^2 x_2 + \frac{1}{2}C x_1^2$$

1°) Les conditions aux limites :

0,5 $\sigma_{22} = -q \quad$ si $\quad x_2 = h \quad (1)$

0,5 $\sigma_{22} = 0 \quad$ si $\quad x_2 = -h \quad (2)$

0,5 $\sigma_{12} = 0 \quad$ si $\quad x_2 = \pm h \quad (3)$

0,5

2°)

0,5 $\sigma_{11} = \dfrac{\partial^2\phi}{\partial x_2^2} = A(6x_1^2 x_2 - 4x_2^3)$

0,5 $\sigma_{22} = \dfrac{\partial^2\phi}{\partial x_1^2} = 2Ax_2^3 + Bx_2 + C$

0,5 $\sigma_{12} = -6Ax_1x_2^2 - Bx_1 = -\dfrac{\partial^2\phi}{\partial x_1\partial x_2}$

$$\left.\begin{aligned}(1) &\Rightarrow 2h^3A + hB + C = -q\\ (2) &\Rightarrow -2h^3A - hB + C = 0\end{aligned}\right\} \Rightarrow C = -q/2 \quad 0{,}5$$

0,5

$$\left.\begin{aligned}(3) &\Rightarrow 6h^2A + B = 0\\ (2) &\Rightarrow 2h^2A + B = -q/2h\end{aligned}\right\} \Rightarrow A = \frac{q}{8h^3} \quad 0{,}5$$

0,5 ⇒

$$B = -\frac{3q}{4h} \quad 0{,}5$$

0,5 $\sigma_{11} = \dfrac{q}{4h^3}(3x_1^2x_2 - 2x_2^3)$

0,5 $\sigma_{22} = \dfrac{q}{4h^3}(x_2^3 - 3h^2 q x_2 - 2h^3 q)$

+0,5 $\sigma_{12} = \dfrac{q}{4h^3}(-3x_1x_2^2 + 3h^2x_1)$