University of Biskra
Biology Department
Module: Math-stat

First year license
2025/2026

Exam

Exercise 1 ***(8 pts)***

I) Let f be a function defined by:

$$f(x) = \begin{cases} \frac{\sin x}{x^2+x}, & -1 < x < 0, \\ \\ \frac{\ln(x+1)}{x+1}, & x \geq 0. \end{cases}$$

1. Give the domain of definition, D_f, of the function f.
2. Calculate the $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to -1} f(x)$.
3. Determine if the function f is continuous at the point $x_0 = 0$.
4. Determine if the function f is differentiable at the point $x_0 = 0$.
5. Determine $f'(x)$ where f is differentiable.

II) Evaluate the following integral:

$$\int \frac{\ln(x+1)}{x+1} dx.$$

Exercise 2 ***(10 pts)***

The following table shows a sample of the glycemia (mg/dL) of adult persons grouped in 5 classes having the same width:

Class	n_i	Middle point x_i	$n_i x_i$	$n_i x_i^2$
$[65, 75[$	15	.	.	.
$[75, 85[$	20	.	.	.
$[85, 95[$	30	.	.	.
$[95, 105[$	20	.	.	.
$[105, 115[$	15	.	.	.
Total	.			

1. Determine the variable studied and its nature.
2. Complete the table.
3. Draw the histogram of data with frequency polygon.
4. Calculate mean, mode, variance, standard deviation and range of the given sample.

Exercise 3 ***(2 pts)***

1. Classify these statistics according to their nature (indicator of **central tendency** or **dispersion**): mean, standard deviation, median and Q_3.
2. Demonstrate why the sum of the relative frequencies is equal to 1.

Good luck

Solution

Exercise 4 ***(8 pts)***

I) Let f be a function defined by:

$$f(x) = \begin{cases} \frac{\sin x}{x^2+x}, & \text{if } -1 < x < 0; \\ \frac{\ln(x+1)}{x+1}, & \text{if } x \geq 0. \end{cases}$$

1. $D_f =]-1, +\infty[$. (1p)
2. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin x}{x^2+x} = \lim_{x \to 0^-} \frac{\cos x}{2x+1} = 1.$ (0.5p)
   $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\ln(x+1)}{x+1} = 0.$ (0.5p)
   $\lim_{x \to -1} f(x) = \lim_{x \to -1} \frac{\sin x}{x^2+x} = \frac{\sin(-1)}{0^-} = +\infty.$ (1p)
3. We said that a function f is continuous at the point $x_0 = 0$ iff:

   $$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0).$$

   From question (2), we have $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$, so $f(x)$ is not continuous at the point $x_0 = 0$. (1p)
4. As f is not continuous at the point $x_0 = 0$ then we deduce that the function f is not differentiable at the point $x_0 = 0$. (1p)
   Note: We can also justify that f is not differentiable at the point $x_0 = 0$ by checking that $\lim_{x \to 0^+} \frac{f(x)-f(0)}{x-0} \neq \lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0}$.
5. $$f'(x) = \begin{cases} \frac{(x^2+x)\cos(x)-(2x+1)\sin(x)}{(x^2+x)^2}, & -1 < x < 0, \quad (0.5p) \\ \frac{(1-\ln(x+1))}{(x+1)^2}, & x>0. \quad (0.5p) \end{cases}$$

II) Evaluate the following integral:

$$\int \frac{\ln(x+1)}{x+1} dx.$$

To evaluate this integral, we use the substitution method.

For this, let's put $y = \ln(x+1) \Longrightarrow dy = \frac{1}{x+1} dx$. So,

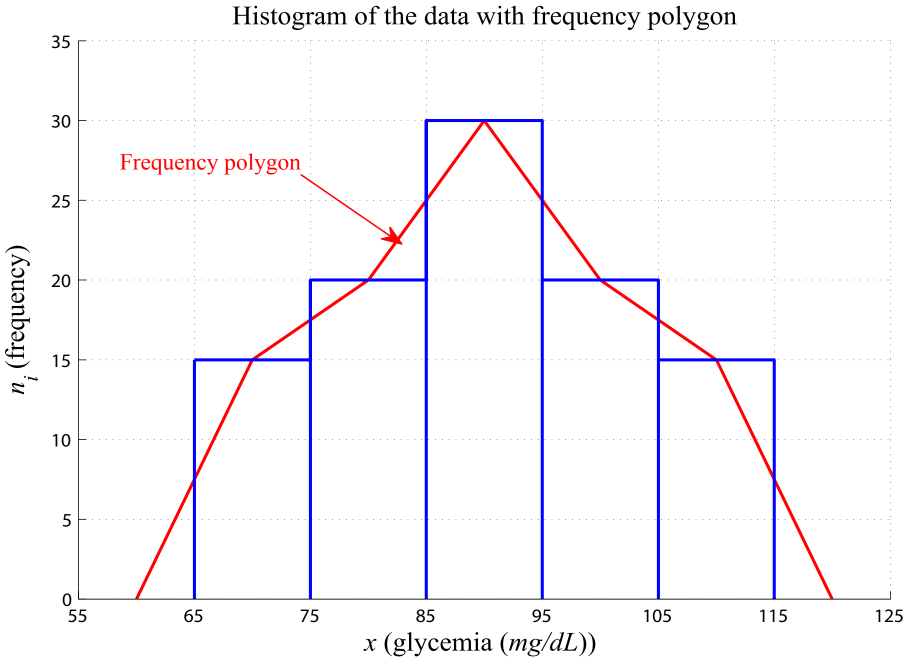
$$\begin{aligned}\int \frac{\ln(x+1)}{x+1}dx &= \int y dy, \\ &= \frac{1}{2}y^2 + c, \quad c \in \mathbb{R}, \qquad (2p) \\ &= \frac{1}{2}(\ln(x+1))^2 + c, \quad c \in \mathbb{R}.\end{aligned}$$

Exercise 5 ***(10 pts)***

1. the variable studied is the glycemia of adult persons and its nature is continuous variable. (1p)
2. Complete the table. (4p)

Class	n_i	Middle point x_i	n_ix_i	$n_ix_i^2$
$[65, 75[$	15	70	1050	73500
$[75, 85[$	20	80	1600	128000
$[85, 95[$	30	90	2700	243000
$[95, 105[$	20	100	2000	200000
$[105, 115[$	15	110	1650	181500
Total	100			

3. Draw the histogram of data with frequency polygon. (1p)



4. Calculate mean, mode, variance, standard deviation and range of the given sample.
   The arithmetic mean: (1p)

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{k} n_ix_i = \frac{1}{100}\sum_{i=1}^{5} n_ix_i = \frac{1}{100}(1050 + ... + 1650) = 90$$

Mode: (1p)
the modal class is $[85, 95[$.

$$M_o \in [85, 95[$$

$$M_o = e_{i-1} + a_i \frac{\Delta_1}{\Delta_1 + \Delta_2} = 85 + 10\frac{10}{10+10} = 90.$$

where $a_i = 10$ is the class width, $\Delta_1 = 30 - 20$, $\Delta_2 = 30 - 20$.
Note: We can also calculate the mode graphically.

Variance: (1p)

$$V(X) = \frac{1}{n}\sum_{i=1}^{k} n_i(x_i - \overline{x})^2 = \frac{1}{n}\sum_{i=1}^{k} n_i x_i^2 - \overline{x}^2 = \frac{1}{100}(73500 + ... + 181500) - 90^2 = 160.$$

Standard deviation denoted by σ_X: (0.5p)

$$\sigma_X = \sqrt{V(X)} = 12.65$$

The range, denoted by e: (0.5p)

$$e = \max_{1\leq i\leq n}(x_i) - \min_{1\leq i\leq n}(x_i) = 115 - 65 = 50$$

Exercise 6 *(2 pts)*

1. Classify these statistics according to their nature (indicator of **central tendency** or **dispersion**): mean, standard deviation, median and Q_3. (1p)

Class	central tendency	dispersion
Mean	X	
standard deviation		X
median	X	
Q_3	X	

2. Demonstrate why the sum of the relative frequencies is equal to 1. (1p)

   We have

$$\begin{cases} \sum_{i=1}^{k} f_i = f_1 + f_2 + \cdots + f_k, & \text{with } f_i = \frac{n_i}{n}; \\ \sum_{i=1}^{k} n_i = n_1 + n_2 + \cdots + n_k = n. \end{cases}$$

$$\begin{aligned} \sum_{i=1}^{k} f_i &= f_1 + f_2 + \cdots + f_k \\ &= \frac{n_1}{n} + \frac{n_2}{n} + \cdots + \frac{n_k}{n} \\ &= \frac{n_1 + n_2 + \cdots + n_k}{n} = \frac{n}{n} \\ &= 1. \end{aligned}$$

Good luck