Exercice

$$\Sigma = \begin{pmatrix} 16 & 0 & 0 \\ 0 & 12 & 8 \\ 0 & 8 & -8 \end{pmatrix} \ daN/mm^2$$

1/ $\det(\Sigma - \lambda I) = 0 \Rightarrow (16-\lambda)\left[(12-\lambda)(-8-\lambda) - 64\right] = 0$ $\quad \Delta = 656$

0,5 0,5 $(16-\lambda)(\lambda^2 - 4\lambda - 160) = 0$

1,5 1,5 $\sigma_I = 16 \ daN/mm^2 \qquad \sigma_{II} = 14,8 \ daN/mm^2 \qquad \sigma_{III} = -10,8 \ daN/mm^2$

0,5 0,5

1,5 $(\Sigma - \sigma_i I)X_i = 0 \Rightarrow X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad X_2 = \begin{pmatrix} 0 \\ 0,94 \\ 0,33 \end{pmatrix} \qquad X_3 = \begin{pmatrix} 0 \\ -0,33 \\ 0,94 \end{pmatrix}$

1,5

2/ $R_{ij} = \frac{1}{2}(\sigma_i - \sigma_j) \qquad C_{ij} = \frac{1}{2}(\sigma_i + \sigma_j)$

$R_{12} = (16 - 14,8)/2 = 0,6 \qquad C_{12} = (16 + 14,8)/2 = 15,4$

1,5 1,5 $R_{23} = (14,8 + 10,8)/2 = 12,8 \qquad C_{23} = (14,8 - 10,8)/2 = 2,0$

$R_{13} = (16 + 10,8)/2 = 13,4 \qquad C_{13} = (16 - 10,8)/2 = 2,6$

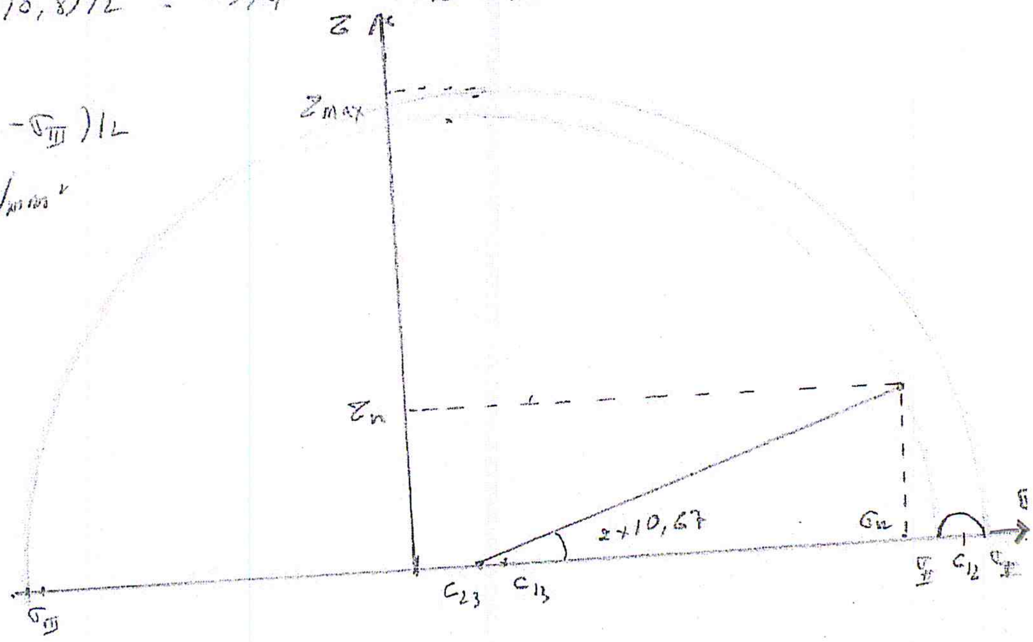
3/ $\tau_{max} = R_{13} = (\sigma_I - \sigma_{III})/2$

0,5 $= 13,4 \ daN/mm^2$

1,0

0,5 $q = \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}_{X_1 X_2 X_3}$

1,0 1,0



4/ $\sigma_0 = \frac{1}{3}\,tr(\Sigma) = \frac{1}{3}(16 + 12 - 8) = \frac{1}{3}(16 + 14,8 - 10,4) = \frac{20}{3} = 6,67 \ daN/mm^2$

1,0

2,0 $\tau_0 = \frac{1}{3}\left[(\sigma_I - \sigma_{II})^2 + (\sigma_I - \sigma_{III})^2 + (\sigma_{II} - \sigma_{III})^2\right]^{1/2}$

1,0 $= \frac{1}{3}\left[(16 - 14,8)^2 + (16 + 10,8)^2 + (14,8 + 10,8)^2\right]^{1/2}$

$= 37,08/3 = 12,36 \ daN/mm^2$

5°) $n = \begin{pmatrix} 0 \\ \sqrt{3}/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0{,}8 \\ 0{,}5 \end{pmatrix} \Rightarrow \Sigma n = \Sigma \begin{pmatrix} 0 \\ \sqrt{3}/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 0 \\ 6\sqrt{3}+4 \\ 4\sqrt{3}-4 \end{pmatrix}$

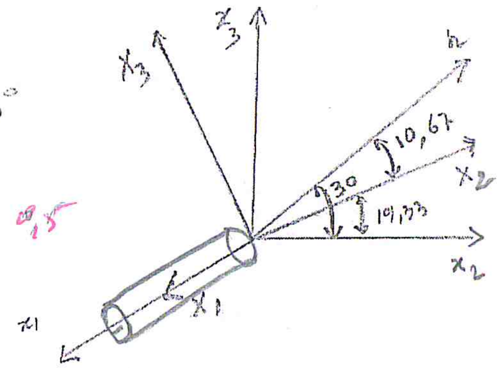
$\sigma_n = n^t \Sigma . n = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 6\sqrt{3}+4 \\ 4\sqrt{3}-4 \end{pmatrix} = 7 + 4\sqrt{3} = 13{,}93 \text{ daN/mm}^2$

$\tau_n = \sqrt{|\Sigma . n|^2 - \sigma^2} = \sqrt{(6\sqrt{3}+4) + (4\sqrt{3}-4)^2 - (7+4\sqrt{3})^2} = \sqrt{91 - 40\sqrt{3}} = 4{,}66 \text{ daN/mm}^2$

$X_2 = \begin{pmatrix} 0 \\ 0{,}94 \\ 0{,}33 \end{pmatrix} \Rightarrow (x_2, X_2) = \operatorname{arctg}\left(\frac{0{,}33}{0{,}94}\right) = 19{,}33°$

$\varphi = 30 - 19{,}33 = 10{,}67°$

$n \in$ au plan (X_2, X_3)



$\sigma_n = C_{23} + R_{23} \cos 2\varphi$
$= 2{,}0 + 12{,}8 \cos(2 \times 10{,}67) = 13{,}92 \text{ daN/mm}^2$

$\tau_n = R_{23} \sin 2\varphi = 12{,}8 \sin(2 \times 10{,}67) = 4{,}66 \text{ daN/mm}^2$

6°) $\varepsilon_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} tr(\Sigma) \delta_{ij}$

$\varepsilon_I = \frac{10^3}{20} (1{,}25 \times 16 - 0{,}25 \cdot 20) = 0{,}750 . 10^{-3}$

$\varepsilon_{II} = \frac{10^3}{20} (1{,}25 \times 14{,}8 - 0{,}25 \times 20) = 0{,}675 . 10^{-3}$

$\varepsilon_{III} = \frac{10^3}{20} (1{,}25 (-10{,}8) - 0{,}25 \times 20) = -0{,}925 . 10^{-3}$

$\mathcal{E} = \begin{pmatrix} 0{,}750 & 0 & 0 \\ 0 & 0{,}675 & 0 \\ 0 & 0 & -0{,}925 \end{pmatrix} 10^{-3}$

a) la longueur

$L = (1 + \varepsilon_I) l_0 = (1 + 0{,}750 . 10^{-3}) 3000 = 3002{,}25 \text{ mm}$

section transversale

$d_1 = (1 + \varepsilon_{II}) d_0 = (1 + 0{,}675 . 10^{-3}) 300 = 300{,}20 \text{ mm}$

$d_2 = (1 + \varepsilon_{III}) d_0 = (1 - 0{,}925 . 10^{-3}) 300 = 299{,}72 \text{ mm}$

b) volume

$\frac{\Delta V}{V} = \frac{V - V_0}{V_0} = tr(\mathcal{E}) \Rightarrow V = (1 + tr(\mathcal{E})) V_0 = (1 + tr(\mathcal{E})) \frac{\pi d_0^2}{4} l$

$V = [1 + (0{,}75 + 0{,}675 - 0{,}925) . 10^{-3}] \times \pi . 300^2 \times 3000/4 = 212\,163\,532{,}87 \text{ mm}^3$