Mohamed Khider University of Biskra

Faculty of FSESNV
Department of SM
University Year 2025/2026

Module: Series and Diff. Eq
Level: 2^{nd} Year LMD
Specialty: Physics

Dirigated Work N°6

(Fourier series, Laplace Transform)

Exercise 1 *(Trigonometric form of Fourier series)*

1- *Calculate the Fourier series, in trigonometric form, of the 2π-periodic function $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that: $f(x) = |x|$ on $]-\pi, \pi[$. Does the series converge to f?*

2- *Calculate the Fourier series, in trigonometric form, of the 2π-periodic function $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that: $f(x) = x^2$ on $[0, 2\pi[$. Does the series converge to f?*

Exercise 2 *(Complex form of Fourier series)*
Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be the 2π-periodic function such that $f(x) = e^x$ for all $x \in]-\pi, \pi]$.

1. *Calculate the complex Fourier coefficients of the function f.*
2. *Study the (simple, uniform) convergence of the Fourier series of f.*

Exercise 3 *(Explicit calculations of Laplace transforms)*

Calculate the Laplace transforms of the following functions:

$$f(t) = 1, \qquad f(t) = t^n, \qquad f(t) = e^{-at},$$
$$f(t) = \sin(\omega t), \qquad f(t) = \cos(\omega t), \qquad f(t) = t\sin(\omega t),$$

Exercise 4 *(Explicit calculations of inverse Laplace transforms)*
For each of the following functions, find a function $f(t)$ such that $\mathcal{L}[f(t)] = F(p)$:

$$F(p) = \frac{1}{(p+2)(p-1)}, \qquad F(p) = \frac{p}{(p+1)(p^2+1)},$$

Exercise 5 *(Application of Laplace transforms in the resolution of Diff Eqs)*
We consider the differential equation:

$$\begin{cases} \ddot{y} + 2\acute{y} + y = e^{-t} \\ y(0) = 0 \\ \acute{y}(0) = 2 \end{cases}, \qquad t \geq 0.$$

Charged of courses

Dr. $\mathbb{OUAAR}, \mathbb{F}$