

Mohamed Khider University of Biskra

Faculty of FSES NV
Department of SM
University Year 2025/2026

Module: Series and Diff. Eq
Level: 2nd Year LMD
Specialty: Physics

Dirigated Work N°5

(NUMERICAL SERIES, FUNCTION SEQUENCES AND SERIES, INTEGER SERIES)

Exercise 1 Study the convergence of the following numerical series:

$$\sum_{n=1}^{+\infty} \frac{1}{n(n+1)}, \quad \sum_{n=2}^{+\infty} \ln\left(1 + \frac{1}{n}\right), \quad \sum_{n=0}^{+\infty} \frac{4n+3}{n+1}, \quad \sum_{n=1}^{+\infty} \frac{2^n n!}{n^n}, \quad \sum_{n=2}^{+\infty} \frac{(-1)^n}{\ln(n)},$$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n-1)^2}, \quad \sum_{n=1}^{+\infty} \frac{2n}{n+2^n}, \quad \sum_{n=1}^{+\infty} \left(\frac{n}{n+1}\right)^{n^2}, \quad \sum_{n=1}^{+\infty} \frac{2^{n+1}}{3^n},$$

Exercise 2 Consider the following sequence of functions:

$$f_n(x) = \frac{\sqrt{nx}}{1 + \sqrt{nx^2}}, \quad x \in]0, +\infty[.$$

- 1) - Study its simple convergence.
- 2) - Study its uniform convergence on $[b, +\infty[$, $b > 0$.

Exercise 3 Let the sequence of functions be defined by:

$$f_n(x) = \frac{e^{-x}}{1 + n^2 x}, \quad n \geq 1.$$

- 1) - Study its simple convergence and its uniform convergence on $[1, +\infty[$.
- 2) - Study the simple convergence of the series $\sum_{n \geq 1} f_n(x)$ on $[1, +\infty[$.
- 3) - Show that $\sum_{n \geq 1} f_n(x)$ converges normally on $[1, +\infty[$.

Exercise 4 1) - Determine the domain of convergence of the following integer series:

$$\sum_{n > 1} \frac{(-1)^n}{n^3} x^n.$$

- 2) - We consider the entiger series $f(x) = \sum_{n > 0} n (-2)^n x^n$.
 - a) - What is the radius of convergence of $f(x)$.
 - b) - What is the radius of convergence of $g(x) = \sum_{n > 0} (-2)^n x^n$.
 - c) - Deduce that $f(x) = xg'(x)$.

Charged of courses

Dr. OUAAR, F