

Mohamed Khider University of Biskra

Faculty of Exact Sciences
Department of SM
University Year 2025/2026

Module: Series and Diff. Eq
Level: 2nd Year LMD
Specialty: Physics

Dirigated Work N°3

(IMPROPER INTEGRALS)

Exercise 1 Calculate the following integrals:

$$\int_0^{+\infty} \frac{1}{x^2 + 4x + 9} dx, \quad \int_0^1 \ln x dx, \quad \int_1^{+\infty} \frac{\ln x}{x^2} dx, \quad \int_0^{+\infty} e^{-2x} \sin x dx, \quad \int_0^1 \frac{\ln x}{\sqrt{1-x}} dx,$$
$$\int_0^{+\infty} \frac{\ln x}{\sqrt{x}(1-x)^{3/2}} dx, \quad \int_0^{+\infty} \frac{x \ln x}{(1+x^2)^2} dx, \quad \int_0^{+\infty} \frac{\arctan x}{(1+x^2)^{3/2}} dx, \quad \int_0^a \frac{x^2}{\sqrt{a^2-x^2}} dx,$$

Exercise 2 Study the nature of the following integrals:

$$\int_1^{+\infty} \frac{dx}{x^\alpha}, \quad \int_0^1 \frac{dx}{x^\alpha}, \quad \int_1^{+\infty} \frac{\sqrt{x}}{(1+x)^\alpha} dx, \quad \int_0^\pi \frac{dx}{(1-\cos x)^\alpha},$$
$$\int_1^{+\infty} \frac{\ln x}{x + e^{-x}} dx, \quad \int_0^{+\infty} \frac{e^{\sin x}}{\sqrt{x}} dx, \quad \int_0^{+\infty} \frac{\arctan x}{x^\alpha} dx.$$

Exercise 3 Using the variable change, calculate the following integrals:

$$\int_0^{\frac{\pi}{2}} \sqrt{\tan x} dx, \quad \int_0^{+\infty} \cos(e^x) dx, \quad \int_0^{+\infty} \sin(x^2) dx,$$

Exercise 4 Study the absolute convergence and the semi-convergence of the following integrals:

$$\int_1^{+\infty} \frac{\sin x}{x^2} dx, \quad \int_0^1 \frac{\sqrt{x} \sin\left(\frac{1}{x^2}\right)}{\ln(1+x)} dx, \quad \int_1^{+\infty} \frac{\sin x}{x} dx, \quad \int_0^1 \frac{\sqrt{x} \sin x}{x+1} dx.$$

Exercise 5 Determine the set of pairs (α, β) for which the generalized integral is convergent:

$$\int_1^{+\infty} \frac{dx}{x^\alpha (1+x^\beta)}$$

Charged of courses

Dr. OUAAR, F