ELECTROKINETICS: PARTII

II. Introduction

The set of phenomena and laws related to moving electric charges. It studies the movement of electricity in material media, particularly electrical circuits. The whole set of phenomena and laws related to electric charges in motion.

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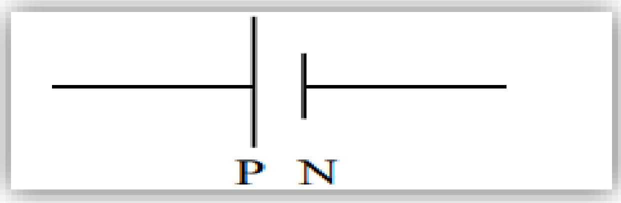
II.1 Production of electric current

If the plates of a capacitor that has been previously charged are connected by a conducting wire, it discharges, and the wire is crossed by an electric current. But this current lasts only for a very brief moment. This is what is called a transient regime. If we want to establish a constant current, that is, one whose intensity does not vary over time, it is necessary to use a generator, a device that maintains a potential difference between its terminals independent of time. Such a device converts any non-electric form of energy into electrical energy. A direct current (permanent) generator is represented by the symbol below: The pole P, whose potential is higher than that of pole N, is called the positive pole, the other pole.

If a generator is connected to an external circuit by conductive wires, by convention, the current flows from the positive pole to the negative pole. See figure



II.1.1 Nature **of electric current**

A metal is made up of roughly fixed positive ions surrounded by roughly free electrons. It is assumed that in steady state (constant current intensity), at every point of the conductor there exists an electric field given by the relation:

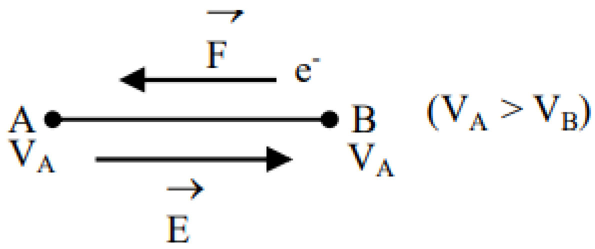
$$\vec{E} = \overrightarrow{grad}V$$

And that an electric charge (q) placed in this field is subjected to a force: $\vec{F}$ = q$\vec{E}$.

- If V_A - V_B is positive, the electric field is then directed from A to B (the field is directed from higher potentials to lower potentials).

- An electron (with negative charge) will then be subjected to a force: $\vec{F}$ = -e$\vec{E}$.

and will therefore move from B to A.



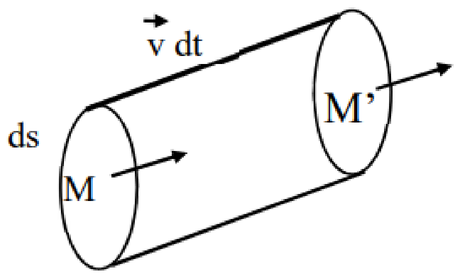
- Moreover, the conventional direction of current is from A to B. Thus, we see that the conventional direction of current is opposite to the actual direction of electron movement. Thus, we see that the conventional current direction is opposite to the actual movement direction of the electrons. Also, the conventional current direction is that of the electric field.

II.1.2 Electric current intensity

- Current density

Current lines are called the paths taken by mobile electric charges in a conductor. These charges are called charge carriers. Let M be a point on the conductor and ds an element of surface surrounding M, also taken in the conductor, ρ being the volumetric density of mobile charges at M.

During a time interval dt, the area ds moves by v.dt, M moving to M'. Look at figure.



By definition, the elementary current dI passing through ds is:

$$\mathbf{dI = \rho\, \vec{v} \cdot \vec{n}\, ds}$$

This last expression shows that dI is nothing other than the flux through ds of the vector:

$$\vec{\mathbf{j}} = \rho\vec{\boldsymbol{v}}$$

This vector $\vec{j}$ is called the current density vector.

- **General definition of intensity**:

Consider a surface (s) through which a current of intensity I flows. (s) can be decomposed into surface elements ds through which currents of intensity dI flow.

dI: flux through ds of the current density vector j.

$$\mathbf{I} = \iint \vec{\boldsymbol{j}}\ \vec{\boldsymbol{n}}\ \cdot \boldsymbol{ds}$$

The current I passing through (s) is therefore:

If the surface (s) is normal at each of its points to the current lines at these points:

$$\mathrm{I} = \iint \vec{j}\ \cdot ds$$

-Units:

- The current I is expressed in Amperes, the fundamental unit of the International System.

- The magnitude of the current density vector j is expressed in $A \cdot m^{-2}$

-Note

The electric current I also corresponds to the amount of electric charge transported per unit of time

$$\boldsymbol{I} = \frac{\boldsymbol{dq}}{\boldsymbol{dt}}$$

II.2 Ohm's Law:

Consider a metallic conductor; the flow of current in this conductor is caused by electrons. These are subjected to:

An electric force $\vec{f_e}$ = - e $\vec{E}$, where $\vec{E}$, is the electric field existing in the portion of the conductor under consideration.

A resistive force $\vec{f'}$ = - λ v, v being the velocity of the mobile electrons. This force is due to the various collisions of electrons with the fixed ions of the metal's crystal lattice.

In a steady state, the velocity of a given electron is constant.

Therefore: $\vec{f_e} + f' = 0$

Let: $\overline{v}$ = - e $\frac{\vec{E}}{\lambda}$

The current density is: $\overline{j}$= ρ $\overline{v}$

ρ: being the volume density of mobile charges, that is, mobile electrons.

If N is the number of electrons per unit volume: ρ = - Ne

Thus:

$$\overline{j} = -Ne\,\overline{v}$$

or $\boldsymbol{\overline{j}=-N\frac{e^2}{\lambda}}$

We set:

$$\boldsymbol{-N\frac{e^2}{\lambda} = \gamma}$$, the conductivity of the material.

Thus: $\overrightarrow{J} = \gamma \vec{E}$

This expression is called the local form of Ohm's law.

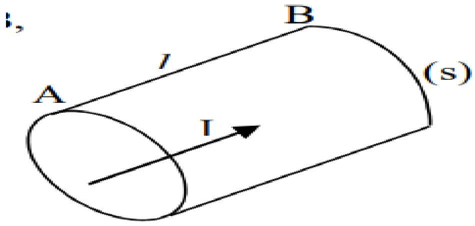
In the International System, **γ** is expressed in Siemens per meter [$S \cdot m^{-1}$].When the conductor is linear, homogeneous, and isotropic, γ is constant, and then the current lines coincide with the electric field lines.

II.3 Resistance of a Conductor

Consider a conductor of length l, with ends A and B, carrying a current of intensity I.

Let VA and VB be the potentials of the conductor at A and B:

$$V_A - V_B = \int_A^B E . dl$$



If we multiply E by any scalar, $V_A - V_B$ is multiplied by k, and the same happens to I, and consequently, the ratio $V_A - V_B$ / I remains unchanged.

By definition, the resistance of the conductor is then called R.

$$\mathbf{R = \frac{V_A - V_B}{I}}$$

The resistance of the conductor depends only on the material and the geometry of the conductor.

* A resistance is represented in either of the following ways:

Units:

- R is expressed in [Ohm] [Ω]

* γ is expressed in [Siemens per meter] [$S.m^{-1}$], using the resistivity of the material defined by

- ρ = 1 / γ (not to be confused with the volume density of mobile charges)
- ρ is expressed in [Ohm . meter] [Ω.m]

Note:

The conductance of the conductor, denoted G, is also defined

$$\mathbf{G} = \frac{\mathbf{1}}{\boldsymbol{R}} = \frac{\boldsymbol{I}}{\boldsymbol{V_A - V_B}} \qquad \text{(Unit of conductance: Siemens).}$$

II.4 Resistance of a wire conductor with constant cross-section

- A conductor is called a wire conductor when its transverse dimensions are small compared to its length.
- The field lines are parallel to the generators of the wire. Let l be its length and S its cross-section:

$$E = \frac{VA - VB}{l}; \qquad J = I / S; \qquad J = \gamma$$

Thus:

$$I / S = \gamma\, (V_A - V_B) / l$$

hence:

$$\mathbf{R} = \frac{1}{\gamma}\,\frac{l}{S} = \frac{\rho l}{S}$$

II.5 Joule's Law

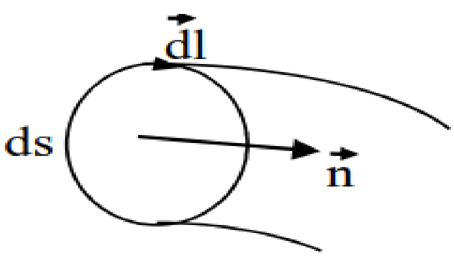
Consider a metallic conductor and calculate the work during a time interval (dt), where the electron moves by dl under electric forces fe and braking forces f'.

Per unit time, this work corresponds to dissipated power

$\mathbf{P = \gamma E^2}$ $\quad \gamma = (Ne^2 / \lambda)$ and also; $\quad$ **P= (1/γ)** $\quad$ $j^2 = \rho\, j^2$,

(ρ: resistivity of the material). These relations constitute the local form of Joule's law.

Consider a tube of elemental field with area ds, and length dl. By integrating over the field tube with cross-section ds between the ends A and B of the conductor, we get:



$$\mathbf{dP = dI\,(V_A - V_B)}$$

by integrating over the entire cross-section of the conductor, we finally obtain:

$$\mathbf{P = I\,(V_A - V_B)}$$ **Since** $\mathbf{V_A - V_B = RI}$, we also get:

$$\mathbf{P = RI^2} \quad \textbf{and} \qquad \mathbf{P = (V_A - V_B)^2 / R}$$

This last expression constitutes a possible form of the finite (non-locally expressed) Joule's law.

Finally, the energy dw dissipated as Joule heating in the conductor is such that:

$$\mathbf{dw = P\ dt\ dl\ ds}$$

During a time dt: $dw = RI^2\,dt$ The current being assumed constant in intensity, over a finite time τ:

$$\mathbf{w = RI^2.\tau}$$

II.6 Combinations of resistances

Different resistances can be grouped in different ways

- Series combination

- Parallel combination

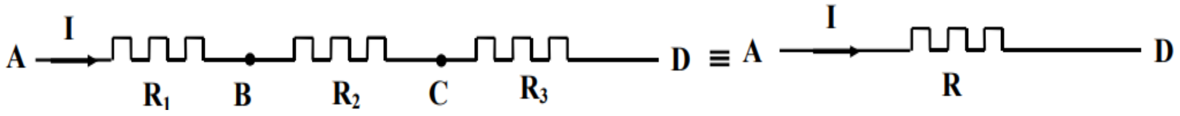
-Mixed group

II.6.1 series combination

Let us consider three resistors R1, R2, and R3 connected in series. In these conditions, the resistors are crossed by the same current of intensity *I*.

We can write Ohm's law across each resistor:

$$V_A - V_B = R_1\ I$$
$$V_B - V_C = R_2\ I \qquad\qquad V_A - V_D = (R_1 + R_2 + R_3)\ I$$
$$V_C - V_D = R_3\ I$$



We call the equivalent resistance of the grouping the single resistance R which, placed between points A and D, subjected to the same potential difference, is traversed by the same current:

$$V_A - V_D = R\,I$$

From this, we deduce that the equivalent resistance of a series circuit is:

$$R = R_1 + R_2 + R_3$$

And, in general:

$$R = \Sigma\, R_i$$

II.6.2 Grouping in parallel

Consider three resistors R_1, R_2, R_3 connected in parallel. In this situation the resistors are subjected to the same potential difference (p.d.), but they are crossed by currents of different magnitudes.

The powers dissipated by Joule heating in each resistor are:

$$P_1 = \frac{(VA - VB)^2}{R_1} \qquad P_2 = \frac{(VA - VB)^2}{R_2} \qquad P_3 = \frac{(VA - VB)^2}{R_3}$$

The total power dissipated by the whole network is:

$$P = (VA - VB)^2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

Using the equivalent resistance Req of the parallel group, defined by

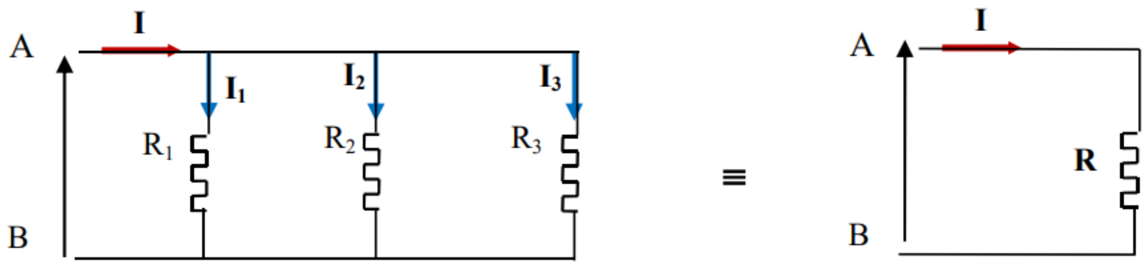
$$P = \frac{(VA - VB)^2}{R}$$

you can also write :

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

We generaliize

$$\frac{1}{R} = \sum_i \frac{1}{R_i}$$



II.6.3 Mixed group

A certain number of resistances are mounted in series, the different derivations being mounted in parallel .We begin by calculating the resistance of the different branches, and then we calculate the resistance of the whole system.

II.7 Generator

The generator is made up of a series of conductors through which current flows from the

(-) pole to the (+) pole. Inside the generator, electrons therefore go from the (+) pole, which has the highest potential, to the (-) pole, which has the lowest potential. Its role is to raise the potential of the charges it receives through one terminal and release them through the other terminal.

There are several types of generators:

- Electrostatic generators.

- Electrochemical generators (Daniel cell based on the oxidation of Zinc and the reduction of Copper).

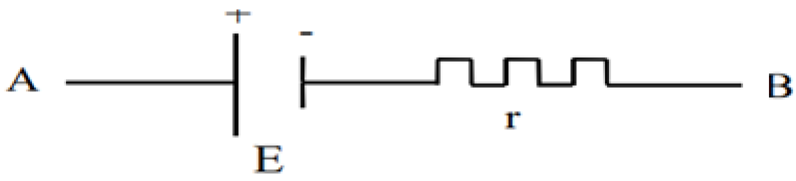
Since the generator is made up of several conductors, it has a certain resistance called internal resistance that slows down the movement of charges.

II.7.1 The electromotive force of a generator:

We consider generators whose electromotive force (emf: E) is a constant.

$$\mathbf{E = U = V_A - V_B \ \ (volts)}$$

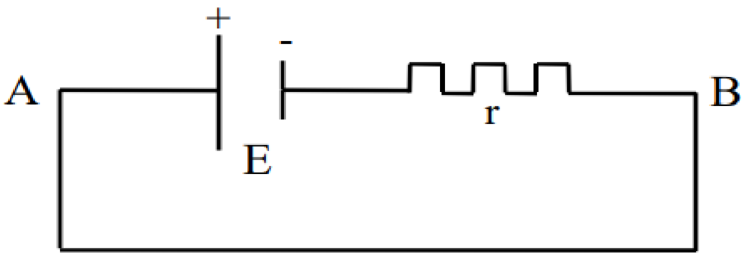
- **Open circuit:**



The voltage difference (potential difference) across the generator terminals is by definition the generator's (emf)

In this case, the internal resistance does not come into play.

- Closed circuit:



Let there be a voltage source: E, with internal resistance: r, delivering a current of intensity I. It provides a power **P = E I**. But part of it is dissipated by the Joule effect inside the generator: its

value is $\mathbf{p = r\,I^2}$.

The power available at the terminals of the generator is:

$$\mathbf{P' = P - p = E\,I - r\,I^2}$$

- If $V_A - V_B$ is the voltage difference across the generator, P' also equals:

$$\mathbf{P' = (V_A - V_B)\,I}$$

By equating these two expressions:

$$\mathbf{V_A - V_B = E - r\,I \Leftrightarrow E = (V_A - V_B) + r\,I}$$

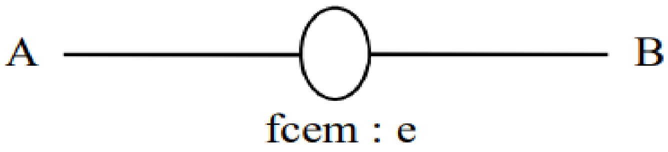
(v) (Ω) (A)

II.8 Receivers.

Devices that convert electrical energy into mechanical or chemical energy are called receivers.

II.8.1 The counter-electromotive force of a receiver.

- **Open circuit**.

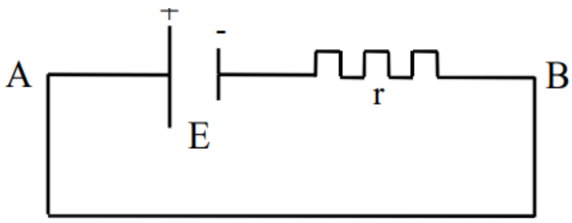


The voltage (potential difference) across the receiver is by definition the counter-electromotive force (cemf: e) of the receiver.

$$\mathbf{e = U = V_A - V_B \ (volts)}$$

- Closed circuit.

u = volts



Let's consider a receptor with an EMF: e and a resistance: r, through which a current of intensity I flows.

Let V_A - V_B be the voltage across its terminals.

The electrical power supplied by the current is: $\mathbf{(V_A - V_B)\ I}$

It is used to:

- provide the receptor with the energy it converts into mechanical energy or chemical, i.e. eI.

to produce the Joule effect, i.e. rI^2.

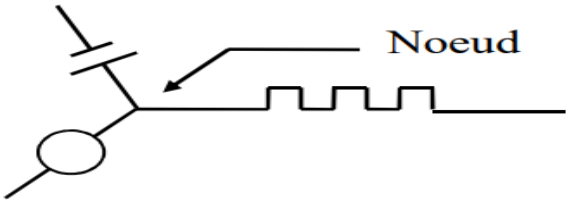
Therefore, we must have:

$$(V_A - V_B)\, I = eI + rI^2$$
$$(V_A - V_B) = e + rI$$

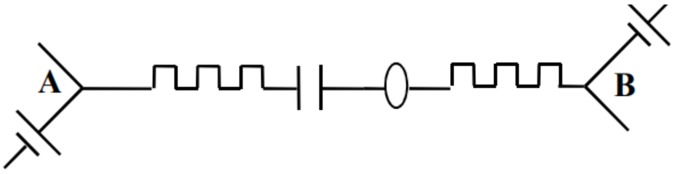
II.9 KIRCHHOFF's Laws.

They allow the study of complex electrical networks, particularly those made up of multiple electrical sources-

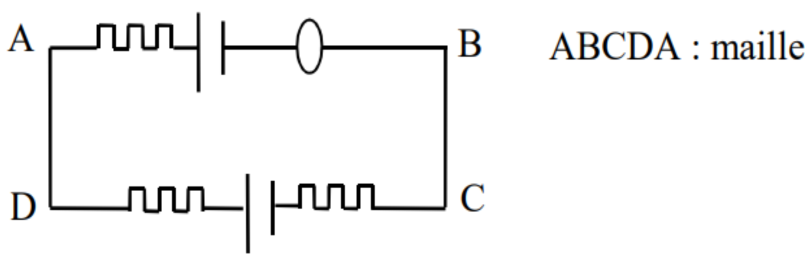
- Definitions.: Network: It is a set of conductive connections linking generators and receivers through resistances
- .Node: It is a point where more than two elements meet.



- Branch: A set of elements connected in series between two nodes is called a branch.



- Loop: It is a set of branches whose succession forms a closed circuit.
  ABCDA: loop



- **Node Law**:
  The sum of currents arriving at a node is equal to the sum of currents leaving it.

- **Loop Law**:

Consider a network containing only generators with internal resistances ri, receivers with internal resistances ρi, and resistances Ri. Knowing all these elements, we want to determine the current flowing through each branch. For this, we use the loop law.

$$\sum_{i} Ri\ Ii = \sum_{j} \varepsilon j\ Ej$$

∑ εj Ej: receivers and generators

∑ Ri Ii: element of branch i

Choose a direction for traversing the loop:

when the current direction coincides with the traversal direction, RiIi is counted positively; otherwise, RiIi is counted negatively.

εj = ±1 is determined by the traversal direction of the loop

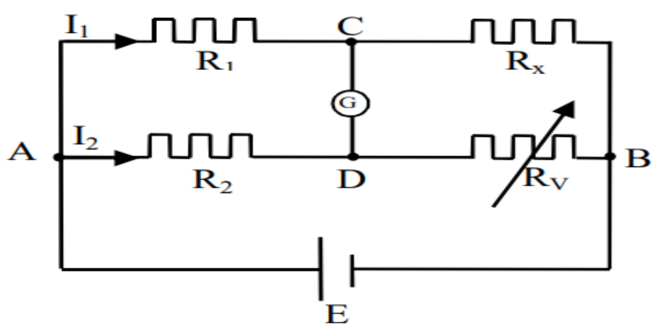
εj = +1 if during the traversal we leave through the positive (+) terminal

εj = -1 if during the traversal we leave through the negative (-) terminal

II.10 Application

- **(Wheatstone Bridge)**

The Wheatstone bridge consists of an electrical circuit with three known resistances and a fourth to be determined, powered by a DC generator E (see figure).



R_1 and R_2 are resistances with a known ratio, Rv is a known adjustable resistance, and Rx is the unknown resistance. By adjusting the resistances R_1, R_2, and Rv, it is possible to nullify the current in the galvanometer, in which case the bridge is said to be balanced.

In this case: $V_C - V_D = 0 \Rightarrow V_C = V_D$

Applying Ohm's law to the terminals of R_1 and R_2:

$$V_A - V_C = R_1 I_1 \text{ and } V_A - V_D = R_2 I_2$$

Thus: $R_1 I_1 = R_2 I_2 \Rightarrow I_2 = (R_1 / R_2)\, I_1$

Also: $V_C - V_B = R_x I_1$ **and** $V_D - V_B = R_v I_2$

Thus: $R_x I_1 = R_v I_2 \Rightarrow I_2 = (R_x / R_v)\, I_1$

At the balance of the bridge, the four resistances are such that:

$$\frac{R1}{R2} = \frac{Rx}{Rv} \Rightarrow Rx = Rv\,\frac{R1}{R2}$$

- **(Pacemaker)**

  The pacemaker is a pulse generator that has an electronic circuit called an RC circuit, which is a parallel circuit (below). It includes a lithium battery that delivers small low-voltage electrical pulses at a predetermined frequency to force the heart to beat at a regular rhythm. This stimulator is made up of two electrical circuits. In the circuit below, a clock controls the switch. Depending on its position, either the heart circuit or the pacemaker circuit is activated. Our circuit is described by constants (values) of the following components:

  C: is the capacitance of the electrical circuit.

  E : is the electric potential of the battery (in volts), a quantity defining the electrical state of the stimulator.

  r: is the resistance in the pacemaker.

  R: is the natural electrical resistance of the heart.

  II.10.1 Daily Operation of the Pacemaker

  The main components involved in the operation of the pacemaker are, for reference:

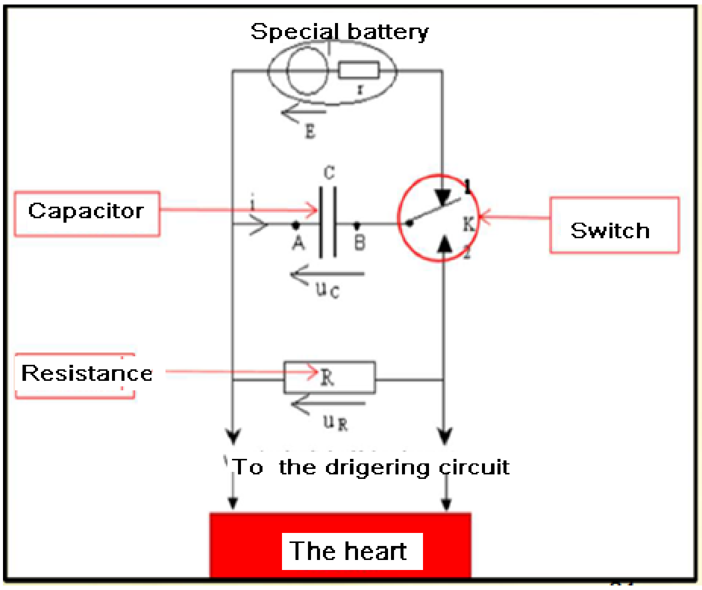
  - The casing, which provides electrical energy thanks to its battery
  - The leads, which distribute the energy supplied

  II.10.2 Supply of Electrical Energy by the Casing

The energy delivered by the battery passes through a capacitor that stores the electricity produced by the battery. The resistance value r is very low so that the capacitor charges very quickly when the switch is in position 1. When the charge is complete, the switch moves to position 2 and the capacitor slowly discharges into the resistor R, which has a high value. Thus, when the voltage across R reaches the given limit value, the pacemaker sends the electrical impulse to the heart through the leads or electrodes. The battery has an average voltage of 5 to 6 volts and a current of 1 to 2 ampere-hours. The switch is located

II.10.3 Pacemaker Electrical Circuit

In physics, Ohm's law is a formula used to calculate the voltage U of an electric current. To find U, the following formula is used: U = R x I, with U in Volts, R in Ohms, and I in Amperes.



II.11Dangers of Electricity on the Human Body

Electricity is defined by three inseparable quantities: current, voltage, and resistance.

- Current (I):

is measured in amperes (A). It is the flow of electrons colliding within the wires. The greater this flow, the higher the risk of electrocution. It can be compared to the flow of a river: the greater the flow, the more likely one is to be swept away.

- Voltage (U):

is expressed in volts (V). It is the force with which electrons are set in motion within the electric wires. The higher it is, the greater the risk of electrocution. It can be compared to height: the higher it is, the greater the risk of being knocked out by falling.

- The resistance (R):

offered to the passage of electric current by any material is expressed in ohms. The higher it is, the lower the risk of electrocution. These three quantities are linked by the formula $U=R*I$. Finally, a fourth quantity, power, P,