University of Biskra
Biology Department
Module: Math-stat

First year license
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EXO 05 :

$$1.\ \int \frac{x^2}{1-x^2}\,dx = \int -1 + \frac{1}{1-x^2}\,dx$$

$$= \underbrace{\int -1\,dx}_{I} + \underbrace{\int \frac{1}{1-x^2}\,dx}_{II}.$$

$$I = \int -1\,dx = -x + C_1$$

$$II = \int \frac{1}{1-x^2}\,dx = \int \frac{1}{(1-x)(1+x)}\,dx = \int \frac{A}{1-x} + \frac{B}{1+x}\,dx$$

$$\frac{A}{1-x} + \frac{B}{1+x} = \frac{A(1+x) + B(1-x)}{(1-x)(1+x)} = \frac{A+B+(A-B)x}{(1-x)(1+x)}.$$

$$\begin{cases} A+B=0 \\ A-B=1 \end{cases} \Rightarrow A = \frac{1}{2},\quad B = -\frac{1}{2}.$$

$$\int \frac{A}{1-x} + \frac{B}{1+x}\,dx = \int \frac{1/2}{1-x} + \frac{-1/2}{1+x}\,dx$$

$$= \frac{1}{2}\ln|1-x| + \left(-\frac{1}{2}\right)\ln|1+x| + C$$

$$= \frac{1}{2}\left(\ln|1-x| - \ln|1+x|\right) + C$$

$$= \frac{1}{2}\left(\ln\left|\frac{1-x}{1+x}\right|\right) + C$$

So :

$$\int \frac{x^2}{1-x^2}\,dx = \underbrace{-x}_{I} + \underbrace{\frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|}_{II} + C$$

$$2.\int \frac{dx}{(1+x)(1+x^2)} = \int \frac{A}{1+x} + \frac{Bx+C}{1+x^2}\,dx$$

$$\frac{A}{(1+x)} + \frac{Bx+C}{(1+x^2)} = \frac{A(1+x^2)+(Bx+C)(1+x)}{(1+x)(1+x^2)}.$$

$$= \frac{(A+B)x^2 + (B+C)x + A+C}{(1+x)(1+x^2)}$$

$$\begin{cases} A+B=0 \\ B+C=0 \\ A+C=1 \end{cases} \Rightarrow \quad A=\frac{1}{2}\ ,\ B=-\frac{1}{2}$$

$$C=\frac{1}{2}.$$

$$\int \frac{A}{1+x} + \frac{Bx+C}{1+x^2}\,dx = \int \frac{1/2}{1+x} + \frac{-1/2\,x+1/2}{1+x^2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{1+x} + \frac{-x+1}{1+x^2}\,dx$$

$$= \frac{1}{2}\int \frac{1}{1+x} - \frac{x}{1+x^2} + \frac{1}{1+x^2}\,dx$$

$$\int \frac{1}{(1+x)(1+x^2)}\,dx = \frac{1}{2}\left(\ln|1+x| - \frac{1}{2}\ln|1+x^2| + \arctan(x)\right) + C$$

(3) $\int \frac{dx}{x^2+2x-3} = \int \frac{1}{(x-1)(x+3)}\,dx$

(B) $= \int \frac{A}{(x-1)} + \frac{B}{x+3}\,dx$

$\frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)} = \frac{(A+B)x+3A-B}{(x-1)(x+3)}$

$\begin{cases} A+B=0 \\ 3A-B=1 \end{cases} \Rightarrow A=\frac{1}{4},\ B=-\frac{1}{4}$

$= \int \frac{\frac{1}{4}}{(x-1)} + \frac{-\frac{1}{4}}{(x+3)}\,dx$

$= \frac{1}{4}\int \frac{1}{(x-1)}dx + \left(-\frac{1}{4}\right)\int \frac{1}{x+3}\,dx$

$= \frac{1}{4}\ln|x-1| + \left(-\frac{1}{4}\right)\ln|x+3| + C$

$= \frac{1}{4}\left(\ln|x-1| - \ln|x+3|\right) + C$

$= \frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C.$

(3)

4) $\int \frac{4-2x}{(x^2+1)(x-1)^2}\,dx = \int \frac{Ax+B}{x^2+1} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}\,dx$

$\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2} = \frac{(Ax+B)(x-1)^2 + C(x^2+1)(x-1) + D(x^2+1)}{(x^2+1)(x-1)^2}$

$= \frac{(A+C)x^3 + (B-2A-C+D)x^2 + (A-2B+C)x + (B-C+D)}{(x^2+1)(x-1)^2}$

$\begin{cases} A+C=0 \\ B-2A-C+D=0 \\ A-2B+C=-2 \\ B-C+D=4 \end{cases} \Rightarrow \begin{matrix} A=1, & C=-1 \\ B=1, & D=2 \end{matrix}$

$\Rightarrow \int \frac{4-2x}{(x^2+1)(x-1)^2}\,dx = \int \frac{x+1}{x^2+1} + \frac{-1}{(x-1)} + \frac{2}{(x-1)^2}\,dx$

$= \int \frac{x+1}{x^2+1}\,dx + \int \frac{-1}{x-1}\,dx + \int \frac{2}{(x-1)^2}\,dx$

$\Rightarrow I = \left[\frac{1}{2}\ln|x^2+1| + \arctan(x) - \ln|x-1| - 2\left(\frac{1}{x-1}\right)\right] + C$

(4)