Ex 04: by substitution:

① $\int \frac{1}{\sqrt{2-5x}}\,dx = ?$

$y = 2-5x \quad , \quad dy = -5\,dx \Rightarrow dx = -\frac{1}{5}dy.$

$-\frac{1}{5}\int \frac{1}{\sqrt{y}}\,dy = -\frac{1}{5}\int (y)^{-1/2}\,dy.$

$= -\frac{1}{5}\left[\frac{1}{-\frac{1}{2}+1}\,y^{-1/2+1}\right] + C$

$= -\frac{1}{5}\left(2\,y^{1/2}\right) + C.$

So:

$\int \frac{1}{\sqrt{2-5x}}\,dx = -\frac{2}{5}(2-5x)^{1/2} + C$

$= \frac{-2}{5}\sqrt{2-5x} + C.$

② $\int \frac{x}{\sqrt{4x+5}}\,dx = ?$

$y^2 = 4x+5 \Rightarrow 2y\,dy = 4\,dx$

$\Rightarrow \boxed{\frac{1}{2}y\,dy = dx}$

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$y^2 = 4x+5 \Rightarrow x = \frac{y^2-5}{4}$.

$y = \sqrt{4x+8}$

$$\int \frac{x}{\sqrt{4x+5}}\,dx = \int \frac{\frac{y^2-5}{4}}{\cancel{y}} \left(\tfrac{1}{2}\cancel{y}\,dy\right)$$

$$= \int \frac{y^2-5}{8}\,dy$$

$$= \frac{1}{8}\int (y^2-5)\,dy = \frac{1}{8}\left(\int y^2\,dy - \int 5\,dy\right)$$

$$= \frac{1}{8}\left(\frac{1}{3}y^3 - 5y\right) + C$$

$$= \frac{1}{8}\left(\frac{1}{3}\sqrt{4x+5}^{\,3} - 5\sqrt{4x+5}\right) + C$$

$$= \boxed{\frac{1}{24}(4x+5)\sqrt{4x+5} - \frac{5}{8}\sqrt{4x+5} + C}$$

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(3) $\int x\sqrt{x-1}\,dx$.

$\boxed{y^2 = x-1} \Rightarrow \boxed{x = y^2+1}$, $y = \sqrt{x-1}$

$\boxed{2y\,dy = dx}$

$$\int x\sqrt{x-1}\,dx = \int (y^2+1)\cdot y \cdot 2y\,dy.$$

$$= 2\int (y^2+1)y^2\,dy$$

$$= 2\int (y^4+y^2)\,dy$$

$$= 2\left[\frac{1}{5}y^5 + \frac{1}{3}y^3\right] + C$$

$$= \frac{2}{5}(x-1)^2\sqrt{x-1} + \frac{2}{3}(x-1)\sqrt{x-1} + C$$

④ $\int \frac{x}{\sqrt{1-x^2}} dx = ?$

$\int \frac{1}{\sqrt{1-x^2}} \cdot x\,dx = ?$

$\boxed{y^2 = 1 - x^2} \Rightarrow 2y\,dy = -2x\,dx$

$\Rightarrow \boxed{-y\,dy = x\,dx}$

$\boxed{y = \sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{1-x^2}} x\,dx = \int \frac{1}{y} \cdot (-y)\,dy$

$= -\int 1\,dy$

$= -y + C$

$$\boxed{\int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} + C.}$$

⑤ $\int \frac{e^x}{2+e^x} dx.$

$y = e^x \Rightarrow \ln y = x$

$\Rightarrow \frac{1}{y} dy = dx.$

$\int \frac{y}{2+y} \cdot \left(\frac{1}{y}\right) \cdot dy = \int \frac{1}{2+y} dy$

$= \ln|2+y| + C$

$\int \frac{e^x}{2+e^x} \Rightarrow \ln|2+e^x| + C.$

6) $\int \frac{(\ln(x))^2}{x}\, dx = ?$

$y = \ln(x) \Rightarrow x = e^{y}.$

$\Rightarrow dx = e^{y} dy.$

$\int \frac{(y)^2}{\cancel{e^y}} \cancel{e^y} dy = \int y^2 dy$

$= \frac{1}{3} y^3 + C.$

$$\boxed{\int \frac{(\ln(x))^2}{x}\, dx = \frac{1}{3}(\ln(x))^3 + C.}$$