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Chapitre II:

Part I: Electrostatics

II. Introduction

Electrostatics is the branch of physics that studies the phenomena created by electric charges that are *static* with respect to the observer.

Let us consider a very simple experiment: if we rub a glass rod with a piece of silk, or an amber rod with a piece of fur, we can observe that due to friction, these materials acquire a new property that we may call **electricity** (from the Greek word *elektron*, meaning amber). This electrical property gives rise to an interaction that is much stronger than gravitation.

There exist two types of electricity, and only two: positive electricity and negative electricity. Two bodies carrying electric charges of the same sign (positive or positive, negative or negative) repel each other, while two bodies carrying opposite charges (one positive, the other negative) attract each other.

- **Medical application**

The laws of electrostatics have proven useful in many fields, including:

Biophysics (the application of physical principles to biological phenomena)

The study of proteins, for example through electrophoresis, which consists of making proteins migrate according to their mass and electric charge

II.1. Electrification Phenomenon

II.1.1 Definition:

The electrification phenomenon is the process by which a body gains or loses **electric charges** (electrons), becoming **electrically charged**.

There are three main ways:

1. **By friction:** When two neutral objects are rubbed together (e.g., a balloon rubbed on hair), electrons are transferred from one to the other.
2. **By conduction:** When a charged object touches a neutral one, charges flow between them.

3. **By induction:** A charged object brought near a neutral one causes a rearrangement of charges without touching.

The object can become:

- **Positively charged** → if it *loses electrons*.
- **Negatively charged** → if it *gains electrons*.

II.1.2. Electric Charge

- Definition:
An **electric charge** is a physical property of matter that causes it to experience a force when placed in an electric field.

-Types:

- **Positive (+)** (protons)
- **Negative (−)** (electrons)

- **Properties:**

- Like charges **repel** each other.
- Unlike charges **attract** each other.
- Charge is **quantized** (it exists in multiples of the elementary charge $e=1.6\times10^{-19}$ C.
- Charge is **conserved** (it cannot be created or destroyed, only transferred).

Unit: Coulomb (C)

II.1.3. Point Charge

- **Definition:**

A **point charge** is an idealized model of a charged particle whose size is very small compared to the distance between charges.
It allows us to treat the entire charge as if it were concentrated at a single point.

Examples:

An electron or proton can be approximated as a point charge in physics problems.

The electrostatic interaction between two charged particles (law Coulomb) is proportional to their charges and inversely proportional to the square of the distance between them, its direction lying along the line joining these two charges.

This can be expressed mathematically as:

$$F = K_e \frac{qq'}{r^2}$$

where r is the distance between the two charges q and q'.

F is expressed in vector form by Formula:

$$\vec{F} = K_e \frac{qq'}{r^3} \vec{r}$$

The direction of this force is given by the sign of the charges:

$qq' > 0 \Rightarrow$ Repulsive force

$qq' < 0 \Rightarrow$ Attractive force

F: Force exerted by charge q on charge q'.

Therefore: r: directed from charge q to charge q' If we agree that a repulsive force is positive and an attractive force is negative, then the constant K_e is positive. Its value depends on the units used to express the different quantities involved in the expression of the force. It is characteristic of the medium in which the electric charges are placed.

$$Ke = \frac{1}{4\pi\varepsilon}$$

$$\underset{(N)}{F} = \frac{1}{4\pi\varepsilon} \frac{qxq'}{r^2}$$

ε = Permittivity of the medium expressed in $c^2/N \cdot m^2$ (In the MKSA system)

$$\varepsilon = \varepsilon_o \varepsilon_r$$

ε_0 = permittivity of free space = 8.854×10^{-12} $c^2/N \cdot m^2$

ε_r = relative permittivity of the medium or dielectric constant.

For practical applications, we can take K_e equal to $\mathbf{9 \times 10^9}$.

Thus, if the distance is expressed in meters, the force in Newton's.

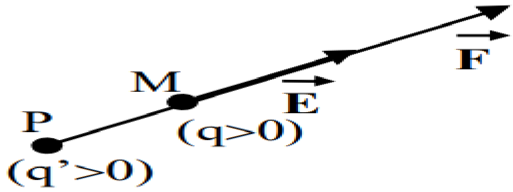
II.2. Electric Field. Example of application:

The electrostatic field influences the guidance of ion exchange at the level of the biological membrane. Proteins are made up of amino acids, some of which are charged. The surface of a protein exhibits a charge distribution that is not uniform, even though the total charge can be zero. This charge distribution on the protein surface guides intermolecular interactions during cell signaling pathways (which transmit messages within a cell to modulate its activity (growth, division, differentiation, death, etc.)).

II.2.1 Electric Field Created by a Point Charge A

My region in which an electric charge experiences a force is called an electric field. The force is due to the presence of other charges in this region. This force is proportional to the electric charge. The proportionality factor is called the electric field. Therefore, the electrostatic field at a point M can be characterized by an electric field vector E (or electrostatic field vector) equal to the quotient of the force F by the charge q. Thus, we have the defining relation:

$$\vec{E} = \frac{\vec{F}}{q}$$



The electric field $\vec{E}$ does not depend on q. $\vec{E}$ has the following characteristics:

- A well-defined direction, that of the vector $\vec{F}$

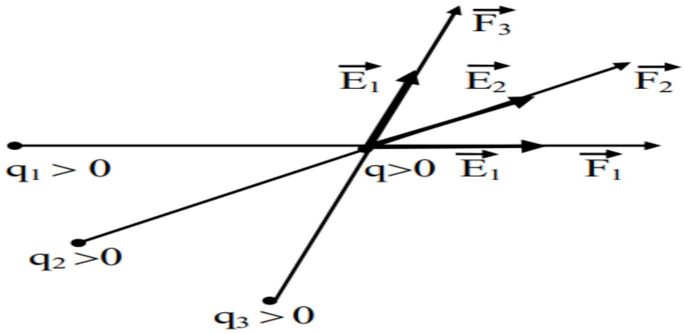
- The direction of $\vec{F}$ if the charge is positive ($q > 0$), the opposite direction if the charge is negative ($q < 0$).

- A magnitude equal to K_e We will define the unit of the electric field as N/C.

II.2.2 Field created by a discrete distribution of point charges.

A charge q placed in a region where other charges q_1, q_2, q_3, etc. are located. is subjected to a force $F = F_1 + F_2 + F_3 + ...$

(see figure).



We can say that the charge q is placed in a field created by the charges q_1, q_2, q_3, ... Indeed:

$qE = qE_1 + qE_2 + qE_3 + ...$ (*)

E_1: field created by q_1

E_2: field created by q_2

E_3: field created by q_3, etc.

E: resulting field (*)

$$(*) \Rightarrow \vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + ...$$

Conclusion:

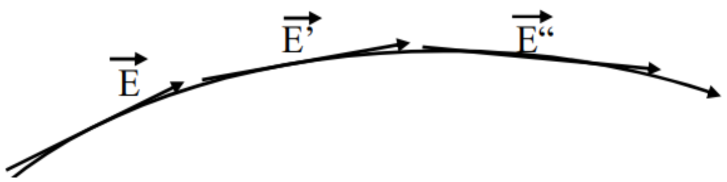
In a discrete distribution of N point charges, the total electric field E is the vector sum of the N electric fields created respectively by the N electric charges.

$$\sum_{i=1}^{n} \vec{E_I}$$

II.2.3 Field Lines :

A field line is defined as a curve that, at each of its points, is tangent to the electric field vector E.

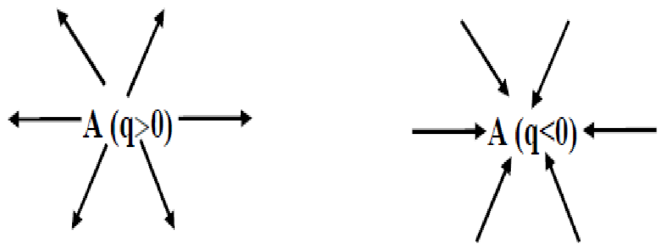
The direction of the line is that of the field vector (see figure).



When a magnetic field is created by a point charge placed at point A, the field lines are straight lines passing through A.

- They originate from A if the charge is positive (q > 0)

- They point towards A if it is negative (q < 0) (see figure below)



II.2.4 Uniform Field

An electric field is uniform within a region of space if the field vector is the same at all points in that region (same direction, same sense, same magnitude). When the field is uniform, the field lines are straight and parallel.

II.3 Electrostatic Potential

II.3.1 Definition of Electric Potential

The electric potential at a point in an electric field is defined as the potential energy of a unit charge placed at that point. If we denote the electric potential at a given point by V and the potential energy of a unit charge placed at that same point by Ep, we have:

$$V = \frac{E_p}{q} \quad \Rightarrow \quad E_P = qV$$

The zero of the electric potential is chosen to coincide with the zero of the potential energy. In most cases, the zero of the potential energy is chosen to be at infinity.

The unit of electric potential is [Joule/Coulomb] or [volt].

II.3.2 Relationship between the electric field and potential.

Suppose a charge q moves from A to B in an electric field. Give the change in potential energy of the charge: $E_{PA} - E_{PB} = q(V_A - V_B)$ (*)

Now: $E_{PA} - E_{PB} = w_{A\rightarrow B}$, $w_{A\rightarrow B}$ = work done on the charge when it moves

from A to B: Therefore: $w_{A\rightarrow B} = q(V_A - V_B)$ Since the electric force acting on the charge q is:

$$\boldsymbol{F} = q\boldsymbol{E}$$

where E is the electric field

We can write:

$$W_{A \to B} = \int_A^B \vec{F}\,.\vec{dl} = \int_A^B q\vec{E}\,.\vec{dl}$$
$$= q(V_A - V_B).$$

Simplifying by q

$$\int_A^B \vec{E}\,.\vec{dl} = V_A - V_B$$

provides a relationship between the electric field A and the potential difference. If A and B coincide, the integration path is a closed curve and therefore:

$$\int \vec{E}\,.\vec{dl} = 0$$

the work done by the electric field on a charge completing a closed circuit is zero.

The electric field thus corresponds to a force derived from a potential.

$$\int_A^B \vec{E}\,.\vec{dl} = \int_A^B E_l\,.dl = V_A - V_B = -(V_B - V_A) = -\int_A^B dV$$

E_l: component of E along the trajectory.

$$\boldsymbol{E_1\,dl = -dv} \quad \textbf{or} \quad \boldsymbol{E_1 = -\frac{\partial V}{\partial l}}$$

In a more compact form, equation (∠) is written:

E = - grad V

E = - grad V: indicates that the electric field vector E is therefore a vector directed from higher potentials to lower potentials.

In Cartesian coordinates:

$$\overrightarrow{grad}\,V = \frac{\partial V}{\partial X}\,\vec{\imath} + \frac{\partial V}{\partial y}\,\vec{\jmath} + \frac{\partial V}{\partial z}\,\vec{k}$$

In Polar coordinates:

$$\overrightarrow{\text{grad}}\, V = \frac{\partial V}{\partial r}\,\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\vec{u}_\theta$$

The electric potential V of a point charge is equal to the work done by E to bring a unit positive charge from infinity to point M.

$$V = \int_{\infty}^{M} \vec{E}\,.\vec{dl}$$

If E is created by a point charge q placed at a point P, then

$$V(M) = \frac{1}{4\pi\varepsilon}\,\frac{q}{\|\overrightarrow{PM}\|}$$

If E is created by a set of N point charges qi, placed at points Pi, the total potential is equal to the algebraic sum of the potentials created by each charge:

$$V(M) = \frac{1}{4\pi\varepsilon}\sum_i \frac{q_i}{\|\overrightarrow{P_i M}\|}$$

II.4 Electric field vector flux.

The flux Φ of the electric field E through the elemental surface ds is called the quantity:

$$d\varnothing = \vec{E}\,.\,\vec{ds}$$

The total flux Ø through the surface s = ∬ ds is given by:

$$\varnothing = \iint_s \vec{E}\,.\,\vec{ds}$$

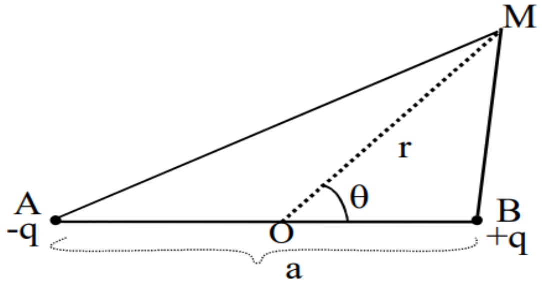
II.4.1 Electric dipole:

Two equal electric charges of opposite signs, separated by a distance a, form an electric dipole.

Let +q and -q be these charges; an electric dipole moment p is defined as follows:

$$\vec{p} = \overrightarrow{AB}\,q.$$

The dipole moment is a vector directed from the negative charge toward the positive charge.



VI-1 Potential created at M by the dipole:

$$V = \frac{q}{4\pi\varepsilon}\left(\frac{1}{BM} - \frac{1}{AM}\right)$$

When point M is sufficiently far from the dipole (r >> a), we have the following approximations:

$1/BM - 1/AM = a\cos\theta / r^2$

$$V = \frac{q}{4\pi\varepsilon}\frac{a\cos(\theta)}{r^2} \quad \text{and} \quad V = \frac{q}{4\pi\varepsilon}\frac{\vec{p}.\vec{r}}{r^3}$$

II.4.2 Electric force couples acting on a dipole placed in an electric field

Consider a dipole with dipole moment P, placed in an external electrostatic field E. Let E be the value of the field at point A where the charge (-q) is located, and $\vec{E}$ + (d$\vec{E}$) be the value of the field at point B where the charge (+q) is located.

The charge (-q) is subject to the force $\vec{F}_A$ = -q $\vec{E}$.

The charge (+q) is subject to the force $\vec{F}_A$ = q $\vec{E}$ + (d$\vec{E}$).

Let: $\vec{F}_B$ = - $\vec{F}_A$ + q(.dE)$^{\rightarrow}$.

We can therefore consider that the dipole is subjected to a couple:

$\vec{F}_A$ - $\vec{F}_A$ with a general resultant:
df = $\vec{F}_B + \vec{F}_A$ = q$\overline{.\,dE}$, si $\vec{E}$ is uniform, $\overrightarrow{dE}$ = 0, so df = 0

II.5 Electrical Conductor

II.5.1 Definition:

In a conductor, charges can move because there are approximately free electrons.
Examples: (metal, biological body, etc.)
The addition of excess charges will create movements of charges.
We will consider in our study a conductor in equilibrium.

II.5.2 Definition of the equilibrium of a conductor:

By definition, a conductor is said to be in equilibrium if the electrons it contains are, on average, at rest.

II.5.3 Properties of a conductor in equilibrium:

The distribution of electric charges in a conductor in equilibrium can only be on the surface.
Let a point be located within the bulk of a conductor in equilibrium,
according to Gauss's theorem:

$$\phi = \sum qi/\varepsilon = \iint \vec{E}.\vec{ds} = \iiint \rho dv/\varepsilon$$

$$\text{or : } \vec{E} = \vec{0} \Rightarrow \rho = 0$$

Now: E = 0 ⇒ ρ = 0
The positive and negative charges in the mass of a conductor in equilibrium cancel each other out, so that statistically, at any point, the volume charge density is zero. If excess charges are placed on a conductor, when it is in equilibrium, these can only reside on the surface.

The volume of a conductor in equilibrium is equipotential: Since at every point in the mass of a conductor in equilibrium $\vec{E} = \vec{0}$, this means that $\overrightarrow{grad\, V} = \vec{0} \Rightarrow$
V = constant, the potential takes the same value at every point in a conductor in equilibrium. Therefore, the surface of this conductor is equipotential, and the field lines are perpendicular to the surface of the conductor.

II.5.4 Field in the vicinity of a conductor in equilibrium:

The electrostatic field created in a vacuum in the immediate vicinity of a conductor in equilibrium is perpendicular to the surface of the conductor and has a magnitude equal to the surface charge density on the conductor divided by ε_0.

VII- 5 Capacitance of a conductor in equilibrium: The capacitance of an isolated conductor is called the always positive proportionality coefficient:

$$\boxed{C = Q/V}$$

Units:
C: is expressed in the International System in Farads (F).
The capacitance C depends only on the geometric characteristics of the conductor.

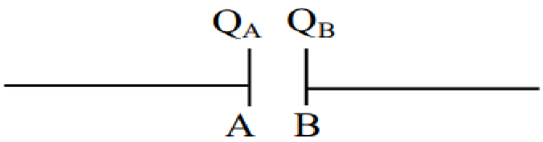
II.5.(Energy of a conductor: The energy of a conductor in equilibrium is.

$$\xi = \frac{1}{2} CV^2 = \frac{1}{2} Q^2/C$$

II.6 Capacitor

II.6.1 Definition:

A capacitor is defined as a set of two conductors A and B separated by an insulator.
A: internal plate of the capacitor
B: external plate of the capacitor
The [AB] system forms a capacitor, schematically represented by:



$$Q_A = -Q_B$$

II.6.2 Capacitance of a capacitor:

The plates are short-circuited by a conducting wire.
The connecting wire carries the charge Q when the plates are short-circuited.
This charge is called the charge of the capacitor.
Thus: The charge of the capacitor is the charge carried by the internal plate.
The capacitance of a capacitor is defined as the quantity denoted by C such that:

$$C = Q/(V_1 - V_2)$$

Or V1 - V2 = potential difference between the internal and external electrodes. The capacitance of a capacitor depends only on its geometry.

Units of capacitance: The unit of capacitance is the Farad (F**).**

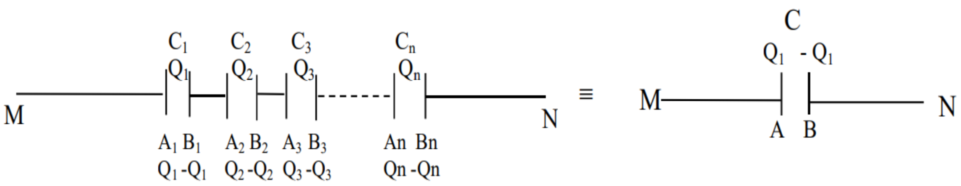
II.6.3 Energy of a capacitor: It is the energy that passes through the connecting wires. The armatures are short-circuited.

$$\xi = Q^2/2C = \frac{1}{2} Q(V_1 - V_2)$$

II.6 .4 Capacitor groupings: When several capacitors are available, they can be grouped in different ways:

- Either in series.
- Or in parallel.
- Or in series-parallel or mixed grouping.

II.6.1 Series Connection:
The internal plate of one of the capacitors is connected to the external plate of the other in this type of connection.
Let us consider the case of n initially uncharged capacitors placed in series.



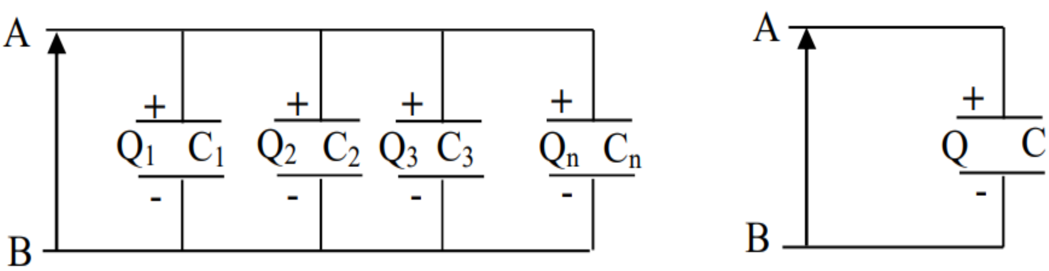
C_1, C_2, C_3,.....,C_n are the capacitances of the capacitors.
Between points M and N, a potential difference $V_M - V_N$ is applied, so that C_1 acquires a charge Q_1, C_2 acquires a charge Q_2, C_3 acquires a charge Q_3,......, and C_n acquires a charge Qn.

The whole system being initially neutral:

$$- Q_1 + Q_2 = 0 \; ; \quad - Q_2 + Q_3 = 0 \quad \ldots\ldots\ldots \; - Q_{n-1} + Q_n = 0$$
$$\text{d'où :} \quad Q_1 = Q_2 = Q_3 = \ldots = Q_{n-1} = Q_n$$
$$V_M - V_N = Q_1/C_1 + Q_2/C_2 + Q_3/C_3 + \ldots\ldots + Q_n/C_n.$$
$$V_M - V_N = Q_1 (1/C_1 + 1/C_2 + 1/C_3 + \ldots\ldots + 1/C_n).$$

The equivalent capacitor placed between M and N, subjected to the voltage $V_M - V_N$, will take the charge Q_1. If C is the capacitance of this equivalent capacitor:
$V_M - V_N = Q_1 / C$. Hence, by identifying: $1/C = 1/C_1 + 1/C_2 + 1/C_3 + \ldots + 1/C_n$.
Therefore, for any number of capacitors:

II.6.2 Parallel Grouping: In this type of grouping, all the internal reinforcements are connected together, as are the external reinforcements.



In the case of n capacitors (see figure), each capacitor is subjected to the same voltage difference

$$V_M - V_N = Q_1 / C_1 = Q_2/C_2 = Q_3/C_3 = ... = Q_n/C_n.$$

The sum of the charges taken by each capacitor gives the total charge taken by the same set

$Q = Q_1 + Q_2 + Q_3 + ... + Q_n$, hence $C = C_1 + C_2 + C_3 + ... + C_n$.

The equivalent capacitor taken between M and N, having a charge Q and capacitance C, is such that:

$$Q = C(V_M - V_N).$$
$$C = C_1 + C_2 + C_3 + ... + Cn$$

So, for any number of capacitors: $\boxed{C = \sum Ci}$

II.6.3 Series-parallel grouping or (mixed grouping): It is the combination of the two cases above:
We start by calculating the capacitance of a section (in series) and then deduce the capacitance of the whole.

Dr Boudour